Problem 1: Let $P$ be a notion of forcing that is $\sigma$-closed. Verify the following:

1. Forcing with $P$ does not add new countable sequences of elements of $V$.

2. If $T \in V$ is a tree, then $\sigma$-closed forcing does not add new branches of cofinality $\omega$ to $T$.

Problem 2: Let $\kappa$ be a regular and uncountable cardinal. Show that if there exists a $\kappa$-Aronszajn tree, then there exists a normal $\kappa$-Aronszajn tree.

Problem 3: Show that if $\kappa$ is an infinite cardinal, then $\text{Add}(\kappa^+, 1, 2)$, the forcing that adds a single new Cohen subset of $\kappa^+$, yields a forcing extension in which $2^\kappa = \kappa^+$ holds, because it collapses $2^\kappa$ to become of size $\kappa^+$ while not adding new subsets of $\kappa$. 
**Definition:** We define the *minimal counterexample iteration* $P_\kappa$ for PFA of length $\kappa$ with collapses as a countable support iteration $\{P_\alpha, \dot{Q}_\alpha \mid \alpha < \kappa\}$, where we (inductively) define $\dot{Q}_\alpha$ as for the usual minimal counterexample iteration for PFA of length $\kappa$ from the lecture in case $\alpha$ is an even ordinal, but we let $\dot{Q}_\alpha$ be such that $\models_\alpha \dot{Q}_\alpha = \text{Fn}(\omega_1, 1, \omega_2)$ when $\alpha$ is an odd ordinal, so we simply demand that at every odd stage in our iteration, the $\omega_2$ of our intermediate model is collapsed to become of size $\omega_1$ by the above $\sigma$-closed forcing.

**Problem 4:** Show that if $\kappa$ is supercompact, then $P_\kappa$ as defined above satisfies the following:

1. $P_\kappa$ forces the PFA (by the very same argument as for the iteration used to force PFA in the lecture).
2. $P_\kappa$ is $\kappa$-cc and hence preserves $\kappa$ (by the same argument that I tried to give for Lemma 13.2 – the part of the argument that actually worked showed that the iteration used to force PFA in the lecture satisfies the $\kappa$-cc).
3. $P_\kappa$ forces that $\check{\kappa} = \omega_2$, because it collapses all cardinals of the ground model strictly between $\omega_1$ and $\kappa$.
4. $P_\kappa$ forces that $2^{\aleph_0} = \aleph_2$, using nice names.

*Remark:* Hence, the above shows that starting from a supercompact cardinal, PFA is consistent with $2^{\aleph_0} = \aleph_2$. As I already remarked, PFA in fact implies $2^{\aleph_0} = \aleph_2$.

*Remark 2:* The argument that I wanted to do in the lecture in fact cannot work, for example if starting with a supercompact cardinal $\kappa$, however also assuming that PFA already held in our ground model, then there wouldn’t be any counterexamples to PFA and the minimal counterexample iteration $P_\kappa$ for PFA of length $\kappa$ that we used in the lecture would just be the trivial forcing, so it would certainly not force that $\kappa$ becomes $\omega_2$. 