Problem 1: Let $\langle P_\alpha, Q_\alpha \mid \alpha < \epsilon \rangle$ be an $I$-supported iteration, where $\epsilon$ is a limit ordinal, and let $\gamma < \epsilon$. Verify the following:

1. $P_{\gamma+1}$ is isomorphic to $P_\gamma * Q_\gamma$.
2. $p \in P_\epsilon$ iff $[\text{dom } p \in I \cap P(\epsilon) \text{ and } \forall \alpha < \epsilon \ p \upharpoonright \alpha \in P_\alpha]$.
3. For $p, q \in P_\epsilon$, $q \leq p$ iff $\forall \alpha < \epsilon \ q \upharpoonright \alpha \leq p \upharpoonright \alpha$.
4. For $p, q \in P_\epsilon$, $q \leq^* p$ iff $\forall \alpha < \epsilon \ q \upharpoonright \alpha \leq^* p \upharpoonright \alpha$.

Problem 2: Let $\langle P_\alpha, Q_\alpha \mid \alpha < \epsilon \rangle$ be an $I$-supported iteration, and fix an ordinal $\alpha < \epsilon$. Let $p \in P_\epsilon$ and $q \in P_\alpha$ be such that $q \leq_\alpha p \upharpoonright \alpha$. Verify the following:

1. $p \land q \in P_\epsilon$.
2. $p \land q \leq p, q$.
3. If $p_2 \leq p_1$, and $q \leq_\alpha p_2 \upharpoonright \alpha$, then $p_2 \land q \leq p_1 \land q$.
4. If $q_2 \leq q_1$, and $q_1 \leq_\alpha p \upharpoonright \alpha$, then $q_2 \land p \leq q_1 \land p$.
5. $p \perp q$ iff $p \upharpoonright \alpha \perp q$.
6. $P_\alpha$ is a complete subforcing of $P_\epsilon$. 
Problem 3: Verify the following: If $\kappa$ is a regular uncountable cardinal, $P$ is a partial order with the $\kappa$-cc, and $\sigma$ is a $P$-name for a subset of $V$ of size less than $\kappa$ (that is, $\sigma^G \subseteq V$ and $|\sigma^G| < \kappa$ whenever $G$ is $P$-generic), then there is a set $x$ of size less than $\kappa$ in $V$ such that $\models_P \sigma \subseteq \check{x}$.

Hint: A slightly weaker result was shown in Philipp’s lecture in the proof that $\kappa$-cc forcings preserve cardinals $\geq \kappa$.

Problem 4:

1. State and verify a result analogous to Corollary 6.12 for the $\kappa$-cc when $\kappa$ is a regular cardinal greater than $\omega_1$.

2. Provide a counterexample to the following statement: If $P$ is $\omega_2$-cc and $\models_P \dot{Q}$ is $\omega_2$-cc, then $P * \dot{Q}$ is $\omega_2$-cc.

Hint: Let $P$ be the standard forcing that collapses $\omega_1$ to become countable.