Graph Theory

Lectures 1–2

Main definitions
- undirected graphs
- vertex degree
- degree sequence, graph generation from the degree sequence

Isomorphism
- automorphisms, orbits

Lectures 3–4

Particular graphs
- complete graphs
- bipartite graphs
- stars and wheels

Graphs from other graphs
- Complementary graphs
- Collapsed graphs
- Subgraphs
- Line graphs
- Graph products

Lectures 5–6

Important vertex subsets
- Cliques
- Stable subsets of vertices
- Dominating subsets of vertices
- Vertex covers
- Chromatic number

Lectures 7–8

Paths and circuits
- Connected graphs
- Shortest paths
- Eccentricity
- Radius and diameter
- Cuts
- Trees and forests

Planar graphs and chromatic number of planar graphs

Lectures 9–10

Directed graphs
- In- and out-degree of a vertex
- Acyclic graphs
- Strong connectivity

What is a matrix? (basic reminder on matrices)
- Linear operators between spaces
- Determinant. Trace
- Eigenvalues and eigenvectors

Lectures 11-12

Graph matrices

- Incidence matrix for undirected graphs
- Adjacency matrix for undirected graphs
- Matrix powers and paths

Lectures 13–14

- Laplacian matrix
- Incidence matrix for directed graphs
- Adjacency matrix for directed graphs
- Graph spectrum: ordinary and Laplacian
  - spectrum of a complete graph

Lectures 15–16

  - spectrum of a bipartite graph
  - spectrum of a circuit
  - spectrum of the complementary graph

Lectures 17–18

- Diffusion equation
- Markov chains
  - stationary probability

Lectures 19–20

  - uniform random sampling
  - Google page rank
- Electrical networks

Lectures 21–22

Graphs for subset families

- Hypergraphs
- Unimodal projections
- Cocitation graphs
- Bibliographic coupling graphs

Lectures 23–24

Minimum cuts

- Max flow problem
- Min cut in a graph
- Randomized algorithm

Lectures 25–26

- Spectral techniques for min and max cut

Lectures 27–28

Modularity

- Introduction

Lectures 29–30

Maximum modularity with fixed number of edges

Lectures 31–32

Maximum modularity with fixed degrees
Exchange heuristics
Graph clustering

Lectures 33–34

Random graphs
Introduction

Lectures 35–36

$G(n, p)$ model
Degree probabilities
Random graphs with fixed degree: the configuration model

Lectures 37–38

Size of the connected components
Phase transition
Giant component
Graph analysis via generating functions
Results for Poisson graphs

Game Theory

Lectures 39–40

Main definitions
Normal form
Equilibrium solutions
Extended form
Total and partial information
Analysis of the Game of Nim

Lectures 41–42

Zero sum games
pure and mixed stategies
existence of a solution

Lectures 43–44

Non constant sum games. Non cooperative games
Nash equilibria
the prisoners' dilemma
mixed stategies
conditions for the existence of the equilibrium

Lectures 45–46

Cooperative games

bargaining
status quo
status quo from a zero sum game
Braess paradox
network flows and Nash equilibria: the price of anarchy

Lectures 47–48

Multi player games

Characteristic function
Imputations. Core
Shapley value