Lectures 1–2

Presentation of the course

Lectures 3–4

Mathematical models:

- quantitative values entering the decisional process
  - data (external values out of control of the decision maker)
  - decisional values
    - decisional parameters (values fixed directly by the DM)
    - decisional variables (values computed by the model)
- constraints
  - structural constraints (depending on the problem structure)
  - flexible constraints (introduced by the DM to better direct his/her choice)
- objectives (they make explicit the preference structure of the DM)

Some examples of real problems

- Diet problem:
  - definition of the quantitative values: foods, food ingredients
  - constraints among values
  - objectives: costs, preferences
  - Linear programming model, refinement of the model

Lectures 5–6

- Activity planning problem
  - First and second model
- Staffing problem
  - Integer linear programming model

Lectures 7–8

Multi-objective optimality

- Non dominated solutions - Pareto optima
- Linear aggregation of the objectives
- Turning objectives into constraints
- Minimizing distance from utopian points

Lectures 9–10

Properties of linear programming

- Geometry
  - feasible set - polyhedron
  - optima on vertices
  - vertex representation - degeneracy
- Simplex method (basic facts)
  - bases and vertices
  - initialization

Lectures 11–12

- Duality
  - formal definition
  - strong duality
  - shadow price as value-added

Lectures 13–14

shadow price from supply-demand equilibrium
complementarity

Integer linear programming

partitioning the feasible set
lower bounds
upper bounds
branch-and-bound

Lectures 15–16

Computer demonstrations

Linear programming in Excel: examples
Linear programming in Lingo: examples
Integer linear programming in Lingo: examples
branch-and-bound simulation

Lectures 17–18

Routing models – shortest paths

Directed graphs: optimality principle, Bellman's equation
Solving Bellman's equation:
for acyclic graphs

Lectures 19–20

for generic graphs with non-negative costs: Dijkstra algorithm
for generic graphs: Bellman-Ford algorithm
for generic graphs: Floyd-Warshall algorithm
Shortest paths on very large graphs

Lectures 21-22

Solving Bellman's equation by Linear programming
Dual problem as a flow problem
Flows and paths

Lectures 23–24

Routing models – shortest paths with capacities

minimum cost flows

Max flow and min cut capacity

Ford-Fulkerson algorithm
max flow–min cut theorem
Min cut (without source-sink)
deterministic algorithm via max flow

Lectures 25–26

direct deterministic algorithm
randomized algorithm: probabilistic analysis, data structures

Lectures 27–28

Allocation models – Assignment
- Cardinality assignment
- Weighted assignment via LP
- Marriage theorem

Allocation models – Matching
- Weighted matching via LP
- Adding cutting planes

Lectures 29–30

Examples of matching models for sport scheduling
- round-robin scheduling
- home-away matches: modeling as max-cut

Routing models – Minimal spanning trees
- Kruskal's algorithm
- Prim's algorithm
- Borůvka algorithm

Lectures 31–32

Routing models – constraints on vertices
- Traveling salesman problem
- ILP model (symmetric and asymmetric)
- subtour inequalities; separation problem as min cut
- branch-and-cut method
- bad and good models for TSP

Lectures 33–34

Routing models – constraints on edges
- Eulerian graphs: undirected, directed and mixed
- Chinese postman problem
  - on undirected graphs
  - on directed graphs
  - on mixed graphs

Lectures 35–36

Allocation models – Knapsack
- Dynamic programming model for 0-1 Knapsack
- Dynamic programming model for integer Knapsack

Lectures 37–38

- Continuous Knapsack
- ILP model for 0-1 Knapsack
- ILP model for integer Knapsack

Allocation models – Bin packing
- problem definition and variants (Multi processor scheduling, Cutting stock)
- ILP model
- Column generation model

Very large linear programming: column generation

- Primal-dual optimality conditions: dual feasibility
- Master problem and Column generator problem

Lectures 39–40

- Max flow example
- Multi-commodity flow example

Lectures 41–42

Heuristics

- Greedy methods
- Local search
- Simulated annealing

Lectures 43–44

- Tabu search
- Genetic algorithms

Routing models – Vehicle routing

- Direct ILP model
- Column generation model
- Clark and Wright heuristics

Lectures 45–46

Scheduling

- One machine models
  - Algorithm for minimum weighted sum (Smith's rule)
  - Algorithm for minimum maximum delay (Jackson's rule)

Lectures 47–48

- Multi-machine models
  - Job Shop, Flow Shop, Open Shop
  - Two machine Flow Shop
  - No wait Flow Shop
  - Job Shop by Shifting Bottleneck heuristics
  - Open Shop with preemption