Appendice alle
Soluzioni dei Problemi del Compitino del 6 aprile 2004

Soluzione alternativa dell'esercizio 2, Tema A

L'esercizio poteva essere risolto applicando de L'Hôpital 3 volte:

$$\lim_{x\to 0}\frac{\ln(1+\operatorname{sen}(2x-x^2)-2x)+x^2e^{3x}}{x\cos(3x)-e^{2x^2}\operatorname{sen}x}=$$

$$\overset{H}{=}\lim_{x\to 0}\frac{\frac{\cos(2x-x^2)(2-2x)-2}{1+\operatorname{sen}(2x-x^2)-2x}+2xe^{3x}+3x^2e^{3x}}{\cos(3x)-3x\operatorname{sen}(3x)-4xe^{2x^2}\operatorname{sen}x-e^{2x^2}\cos x}=\left[\frac{0+0+0}{1-0-0-1}\right]=\left[\frac{0}{0}\right]$$

$$\overset{H}{=}\lim_{x\to 0}\Bigg(-\frac{(\cos(2x-x^2)(2-2x)-2)^2}{(1+\operatorname{sen}(2x-x^2)-2x)^2}+\frac{-\operatorname{sen}(2x-x^2)(2-2x)^2-2\cos(2x-x^2)}{1+\operatorname{sen}(2x-x^2)-2x}+$$
$$+2e^{3x}+12xe^{3x}+9x^2e^{3x}\Bigg)\Big/\Bigg(-6\operatorname{sen}(3x)-9x\cos(3x)-4e^{2x^2}\operatorname{sen}x-16x^2e^{2x^2}\operatorname{sen}x-$$
$$-4xe^{2x^2}\cos x-4xe^{2x^2}\cos x+e^{2x^2}\operatorname{sen}x\Bigg)=\left[\frac{-0-2+2+0+0}{-0-0-0-0-0-0+0}\right]=\left[\frac{0}{0}\right]$$

$$\overset{H}{=}\lim_{x\to 0}\Bigg(2\frac{(\cos(2x-x^2)(2-2x)-2)^3}{(1+\operatorname{sen}(2x-x^2)-2x)^3}-$$
$$-\frac{2(\cos(2x-x^2)(2-2x)-2)(-\operatorname{sen}(2x-x^2)(2-2x)^2-2\cos(2x-x^2))}{(1+\operatorname{sen}(2x-x^2)-2x)^2}+$$
$$-\frac{(\cos(2x-x^2)(2-2x)-2)(-\operatorname{sen}(2x-x^2)(2-2x)^2-2\cos(2x-x^2))}{(1+\operatorname{sen}(2x-x^2)-2x)^2}+$$
$$+\frac{-\cos(2x-x^2)(2-2x)^3-\operatorname{sen}(2x-x^2)2(2-2x)(-2)+2\operatorname{sen}(2x-x^2)(2-2x)}{1+\operatorname{sen}(2x-x^2)-2x}+$$
$$6e^{3x}+12e^{3x}+36xe^{3x}+18xe^{3x}+27x^2e^{3x}\Bigg)\Big/\Bigg(-18\cos(3x)-9\cos(3x)+27x\operatorname{sen}(3x)-$$
$$-3e^{2x^2}\cos x-12xe^{2x^2}\operatorname{sen}x-32xe^{2x^2}\operatorname{sen}x-64x^3e^{2x^2}\operatorname{sen}x-16x^2e^{2x^2}\cos x-$$
$$-8e^{2x^2}\cos x-32x^2e^{2x^2}\cos x+8xe^{2x^2}\operatorname{sen}x\Bigg)$$

$$=\frac{0-0-0-8+6+12+0+0+0}{-18-9+0-3-0-0-0-0-8-0+0}=\frac{10}{-38}=-\frac{5}{19}.$$

Confrontate la difficoltà di questa soluzione con quella proposta precedentemente!

Soluzione alternativa dell'esercizio 5c, Tema A

Per calcolare il polinomio di Taylor di ordine 5 di h_3 si potevano alternativamente calcolare le derivate di h_3 fino all'ordine 5:

$$\begin{aligned}h_3'(x)&=2xe^{2x^2}+x^2e^{2x^2}4x+x^2\operatorname{sen}(x-2x^2)(1-4x)-2x\cos(x-2x^2)\\&=(2x+4x^3)e^{2x^3}+(x^2-4x^3)\operatorname{sen}(x-2x^2)-2x\cos(x-2x^2),\end{aligned}$$

$$
\begin{aligned}
h_3''(x) &= (2+12x^2)e^{2x^2} + (2x+4x^3)e^{2x^2}4x + (2x-12x^2)\,\mathrm{sen}(x-2x^2) + \\
&\quad + (x^2-4x^3)\cos(x-2x^2)(1-4x) - 2\cos(x-2x^2) + 2x\cos(x-2x^2)(1-4x) \\
&= (2+20x^2+16x^4)e^{2x^2} + \big((x-4x^2)^2-2\big)\cos(x-2x^2) + (4x-20x^2)\,\mathrm{sen}(x-2x^2),
\end{aligned}
$$

$$
\begin{aligned}
h_3'''(x) &= (40x+64x^3)e^{2x^2} + (2+20x^2+16x^4)e^{2x^2}4x + 2(x-4x^2)(1-8x)\cos(x-2x^2) - \\
&\quad - \big((x-4x^2)^2-2\big)\,\mathrm{sen}(x-2x^2)(1-4x) + (4-40x)\,\mathrm{sen}(x-2x^2) + \\
&\quad + (4x-20x^2)\cos(x-2x^2)(1-4x) \\
&= 16(3x+9x^3+4x^5)e^{2x^2} + 6(x-10x^2+24x^3)\cos(x-2x^2) + \\
&\quad + (6-48x-x^2(1-4x)^3)\,\mathrm{sen}(x-2x^2),
\end{aligned}
$$

$$
\begin{aligned}
h_3^{iv}(x) &= 16\big((3+27x^2+20x^4) + (3x+9x^3+4x^5)4x\big)e^{2x^2} + 6(1-20x+72x^2)\cos(x-2x^2) - \\
&\quad - 6(x-10x^2+24x^3)\,\mathrm{sen}(x-2x^2)(1-4x) + (-48-2x(1-4x)^3 - \\
&\quad - x^2 3(1-4x)^2(-4))\,\mathrm{sen}(x-2x^2) + (6-48x-x^2(1-4x)^3)\cos(x-2x^2)(1-4x) \\
&= 16(3+39x^2+54x^4+16x^6)e^{2x^2} + 6(1-20x+72x^2)\cos(x-2x^2) - \\
&\quad - 6(x-10x^2+24x^3)(1-4x)\,\mathrm{sen}(x-2x^2) + (-48-2x(1-4x)^3 - \\
&\quad + 12x^2(1-4x)^2)\,\mathrm{sen}(x-2x^2) + (6-48x-x^2(1-4x)^3)(1-4x)\cos(x-2x^2),
\end{aligned}
$$

$$
\begin{aligned}
h_3^{v}(x) &= 16\big((78x+216x^3+96x^5) + (3+39x^2+54x^4+16x^6)4x\big)e^{2x^2} + 6(-20+144x)\cos(x-2x^2) - \\
&\quad - 6(1-20x+72x^2)\,\mathrm{sen}(x-2x^2)(1-4x) - 6\big((1-20x+72x^2)(1-4x) - \\
&\quad - 4(x-10x^2+24x^3)\,\mathrm{sen}(x-2x^2) - 6(x-10x^2+24x^3)(1-4x)^2\cos(x-2x^2) + \\
&\quad + (-2(1-4x)^3 - 2x3(1-4x)^2(-4) + 24x(1-4x)^2 + 12x^2 2(1-4x)(-4))\,\mathrm{sen}(x-2x^2) \\
&\quad + (-48-2x(1-4x)^3+12x^2(1-4x)^2)(1-4x)\cos(x-2x^2) + \big((-48-2x(1-4x)^3 + \\
&\quad + 12x^2(1-4x)^2)(1-4x) - 4(6-48x-x^2(1-4x)^3)\big)\cos(x-2x^2) + \\
&\quad - (6-48x-x^2(1-4x)^3)(1-4x)^2\,\mathrm{sen}(x-2x^2).
\end{aligned}
$$

Si ha

$$
\begin{aligned}
&h_3'(0) = 0+0-0 = 0, \\
&h_3''(0) = 2-2+0 = 0, \\
&h_3'''(0) = 0+0+0, \\
&h_3^{iv}(0) = 16\cdot 3 + 6 - 0 + 0 + 6 = 60, \\
&h_3^{v}(0) = 0 + 6(-20) - 0 - 0 - 0 + 0 - 48 - 72 - 0 = -240,
\end{aligned}
$$

perciò

$$
P_5(x) = \frac{60}{4!}x^4 + \frac{-240}{5!}x^5 = \frac{5}{2}x^4 - 2x^5.
$$

Confrontate la difficoltà di questa soluzione con quella proposta precedentemente!