Reactive Systems over Directed Bigraphs*

Davide Grohmann   Marino Miculan

Dept. of Mathematics and Computer Science, University of Udine, Italy.
grohmann@dimi.uniud.it, miculan@dimi.uniud.it

Abstract. We study the construction of labelled transition systems from reactive systems defined over *directed bigraphs*, a computational meta-model which subsumes other variants of bigraphs. First we consider wide transition systems whose labels are all those generated by the IPO construction; the corresponding bisimulation is always a congruence. Then, we show that these LTSs can be simplified further by restricting to a subclass of labels, which can be characterized syntactically.
We apply this theory to the Fusion calculus: we give an encoding of Fusion in directed bigraphs, and describe its simplified wide transition system and corresponding bisimulation.

1 Introduction

Bigraphical reactive systems (BRSs) are an emerging graphical meta-model of concurrent computation introduced by Milner [5, 6]. The key structure of BRSs are *bigraphs*, which are composed by a hierarchical *place graph* describing locations, and a *link (hyper-)graph* describing connections. The dynamics of a system is represented by a set of reaction rules which may change both these structures. Remarkably, using a general construction based on the notion of *relative pushout* (RPO), from a BRS we can obtain a labelled transition system such that the associated bisimulation is always a congruence [3].

Several process calculi for Concurrency can be represented in bigraphs, such as CCS and (using a mild generalization), also the π-calculus and the λ-calculus. Nevertheless, other calculi, such as Fusion [7], seem to escape this framework. On the other hand, a "dual" version of bigraphs introduced by Sassone and Sobociński [8] seem suitable for Fusion calculus, but not for the others.

In order to overcome this limitation, in previous work we have introduced a generalization of both Milner's and Sassone-Sobociński's variants, called *directed bigraphs* [2, 1]. Intuitively, in directed bigraphs edges represent *resources* which are *accessed* by controls, and *indicated* by names. Connections from controls to edges (through names) represent "resource requests", or accesses. Directed bigraphs feature an RPO construction which generalizes and unifies both Jensen-Milner's and Sassone-Sobociński's variants [2].

In this paper, we consider reactive systems built over directed bigraphs, hence called *directed bigraphical reactive system* (DBRS). Given a DBRS, we can readily obtain a labelled transition system (called *directed bigraphical transition system*, DBTS) by taking as observations all the contexts generated by the IPO

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construction. We show that the bisimilarity associated to this DBTS (called "standard") is always a congruence. However, this LTS is still quite large, and may contain transitions not strictly necessary. In fact, we show that, under a mild condition, standard bisimilarity can be characterized also by a smaller, more tractable DBTS, whose transitions are only those really relevant for the agents.

In order to check the suitability of this theory, we apply it to the Fusion calculus. We present the first encoding of the Fusion calculus as a DBRS; then, we discuss the DBTS and associated bisimilarity constructed using these techniques.

Synopsis In Section 2 we recall some definitions about directed bigraphs. In Section 3 we introduce directed bigraphical reactive and transition systems, and show that standard bisimilarity is a congruence. In Section 4, we show that (under some conditions) standard bisimilarity can be characterized by a smaller LTS. As an application, in Section 5 we represent the Fusion Calculus using DBRSs and DBTSs. Conclusions and direction for future work are in Section 6.

2 Directed bigraphs

In this section we recall some definitions about directed bigraphs, as in [2]. Following Milner's approach, we work in *precategories;* see [4, §3] for an introduction to the theory of supported monoidal precategories.

Let $\mathcal{K}$ be a given signature of controls, and $ar : \mathcal{K} \to \omega$ the arity function.

Definition 1. *A* polarized interface X *is a pair of disjoint sets of names* $X = (X^-, X^+)$*; the two elements are called* downward *and* upward *faces, respectively.*

A directed link graph $A : X \to Y$ *is* $A = (V, E, ctrl, link)$ *where* X *and* Y *are the* inner *and* outer *interfaces,* V *is the set of* nodes*,* E *is the set of* edges*,* $ctrl : V \to \mathcal{K}$ *is the* control map*, and* $link : \mathsf{Pnt}(A) \to \mathsf{Lnk}(A)$ *is the* link map*, where the* ports*, the* points *and the* links *of* A *are defined as follows:*

$$\mathsf{Prt}(A) \triangleq \sum_{v \in V} ar(ctrl(v)) \qquad \mathsf{Pnt}(A) \triangleq X^+ \uplus Y^- \uplus \mathsf{Prt}(A) \qquad \mathsf{Lnk}(A) \triangleq X^- \uplus Y^+ \uplus E$$

The link map cannot connect downward and upward names of the same interface, i.e., the following condition must hold: $(link(X^+) \cap X^-) \cup (link(Y^-) \cap Y^+) = \emptyset$.

Directed link graphs are graphically depicted much like ordinary link graphs, with the difference that edges are explicit objects and points and names are associated to edges (or other names) by (simple) directed arcs. This notation makes explicit the "resource request flow": ports and names in the interfaces can be associated either to locally defined resources (i.e., a local edge) or to resources available from outside the system (i.e., via an outer name).

Definition 2 ('DLG). *The precategory of* directed link graphs *has polarized interfaces as objects, and directed link graphs as morphisms.*

Given two directed link graphs $A_i = (V_i, E_i, ctrl_i, link_i) : X_i \to X_{i+1}$ $(i = 0, 1)$*, the composition* $A_1 \circ A_0 : X_0 \to X_2$ *is defined when the two link graphs have disjoint nodes and edges. In this case,* $A_1 \circ A_0 \triangleq (V, E, ctrl, link)$*, where*

$V \triangleq V_0 \uplus V_1$, $ctrl \triangleq ctrl_0 \uplus ctrl_1$, $E \triangleq E_0 \uplus E_1$ and $link : X_0^+ \uplus X_2^- \uplus P \to E \uplus X_0^- \uplus X_2^+$ is defined as follows (where $P = \mathsf{Prt}(A_0) \uplus \mathsf{Prt}(A_1)$):

$$link(p) \triangleq \begin{cases} link_0(p) & \text{if } p \in X_0^+ \uplus \mathsf{Prt}(A_0) \text{ and } link_0(p) \in E_0 \uplus X_0^- \\ link_1(x) & \text{if } p \in X_0^+ \uplus \mathsf{Prt}(A_0) \text{ and } link_0(p) = x \in X_1^+ \\ link_1(p) & \text{if } p \in X_2^- \uplus \mathsf{Prt}(A_1) \text{ and } link_1(p) \in E_1 \uplus X_2^+ \\ link_0(x) & \text{if } p \in X_2^- \uplus \mathsf{Prt}(A_1) \text{ and } link_1(p) = x \in X_1^-. \end{cases}$$

The identity link graph of X is $id_X \triangleq (\emptyset, \emptyset, \emptyset_\mathcal{K}, Id_{X^- \uplus X^+}) : X \to X$.

Definition 3. *The* support *of $A = (V, E, ctrl, link)$ is the set $|A| \triangleq V \oplus E$.*

Definition 4. *Let $A : X \to Y$ be a link graph.*

A link $l \in \mathsf{Lnk}(A)$ is idle *if it is not in the image of the link map (i.e., $l \notin link(\mathsf{Pnt}(A))$). The link graph A is* lean *if there are no idle links.*

A link l is open *if it is an inner downward name or an outer upward name (i.e., $l \in X^- \cup Y^+$); it is* closed *if it is an edge.*

A point p is open *if $link(p)$ is an open link; otherwise it is closed. Two points p_1, p_2 are* peer *if they are mapped to the same link, that is $link(p_1) = link(p_2)$.*

Definition 5. *The* tensor product $\otimes$ *in* 'DLG *is defined as follows. Given two objects X, Y, if these are pairwise disjoint then $X \otimes Y \triangleq (X^- \uplus Y^-, X^+ \uplus Y^+)$. Given two link graphs $A_i = (V_i, E_i, ctrl_i, link_i) : X_i \to Y_i$ $(i = 0, 1)$, if the tensor products of the interfaces are defined and the sets of nodes and edges are pairwise disjoint then the tensor product $A_0 \otimes A_1 : X_0 \otimes X_1 \to Y_0 \otimes Y_1$ is defined as $A_0 \otimes A_1 \triangleq (V_0 \uplus V_1, E_0 \uplus E_1, ctrl_0 \uplus ctrl_1, link_0 \uplus link_1)$.*

Finally, we can define the *directed bigraphs* as the composition of standard place graphs (see [4, §7] for definitions) and directed link graphs.

Definition 6 (directed bigraphs). *A* (bigraphical) interface *I is composed by a* width *(a finite ordinal, denoted by $width(I)$) and by a polarized interface of link graphs (i.e., a pair of finite sets of names).*

A directed bigraph *with signature $\mathcal{K}$ is $G = (V, E, ctrl, prnt, link) : I \to J$, where $I = \langle m, X \rangle$ and $J = \langle n, Y \rangle$ are its inner and outer interfaces respectively; V and E are the sets of nodes and edges respectively, and $prnt$, $ctrl$ and $link$ are the parent, control and link maps, such that $G^P \triangleq (V, ctrl, prnt) : m \to n$ is a place graph and $G^L \triangleq (V, E, ctrl, link) : X \to Y$ is a directed link graph.*

We denote G as combination of G^P and G^L by $G = \langle G^P, G^L \rangle$. In this notation, a place graph and a (directed) link graph can be put together if and only if they have the same sets of nodes and edges.

Definition 7 ('DBig). *The precategory* 'DBig *of directed bigraph with signature $\mathcal{K}$ has interfaces $I = \langle m, X \rangle$ as objects and directed bigraphs $G = \langle G^P, G^L \rangle : I \to J$ as morphisms. If $H : J \to K$ is another directed bigraph with sets of nodes and edges disjoint from V and E respectively, then their composition is defined by composing their components, i.e.: $H \circ G \triangleq \langle H^P \circ G^P, H^L \circ G^L \rangle : I \to K$.*

The identity directed bigraph of $I = \langle m, X \rangle$ is $\langle id_m, Id_{X^- \uplus X^+} \rangle : I \to I$.

A bigraph is *ground* if its inner interface is $\epsilon = \langle 0, (\emptyset, \emptyset)\rangle$; we denote by $Gr\langle I\rangle$ the set of ground bigraphs with outer interface I, i.e., $Gr\langle I\rangle = {}'\mathrm{DB}\textsc{ig}(\epsilon, I)$.

Definition 8. *The* tensor product $\otimes$ *in* $'\mathrm{DB}\textsc{ig}$ *is defined as follows. Given* $I = \langle m, X\rangle$ *and* $J = \langle n, Y\rangle$*, where* X *and* Y *are pairwise disjoint, then* $\langle m, X\rangle \otimes \langle n, Y\rangle \triangleq \langle m + n, (X^- \uplus Y^-, X^+ \uplus Y^+)\rangle$*.*

The tensor product of $G_i : I_i \to J_i$ *is defined as* $G_0 \otimes G_1 \triangleq \langle G_0^P \otimes G_1^P, G_0^L \otimes G_1^L\rangle : I_0 \otimes I_1 \to J_0 \otimes J_1$*, when the tensor products of the interfaces are defined and the sets of nodes and edges are pairwise disjoint.*

Remarkably, directed link graphs (and bigraphs) have relative pushouts and pullbacks, which can be obtained by a general construction, subsuming both Milner's and Sassone-Sobociński's variants. We refer the reader to [2].

Actually, in many situations we do not want to distinguish bigraphs differing only on the identity of nodes and edges. To this end, we introduce the category DBig of *abstract directed bigraphs*. The category DBig is constructed from $'$DBig forgetting the identity of nodes and edges and any idle edge. More precisely, abstract bigraphs are concrete bigraphs taken up-to an equivalence $\mathrel{\widehat{\simeq}}$.

Definition 9 (abstract directed bigraphs). *Two concrete directed bigraphs* G *and* H *are* lean-support equivalent*, written* $G \mathrel{\widehat{\simeq}} H$*, if they are support equivalent after removing any idle edges.*

The category DBig *of abstract directed bigraphs has the same objects as* $'$DBig*, and its arrows are lean-support equivalence classes of directed bigraphs. We denote by* $\mathcal{A} : {}'\mathrm{DB}\textsc{ig} \to \mathrm{DB}\textsc{ig}$ *the associated quotient functor.*

We remark that DBig is a category (and not only a precategory); moreover, $\mathcal{A}$ enjoys several properties which we omit here due to lack of space; see [4].

3 Directed Bigraphical Reactive and Transition Systems

In this section we introduce wide reactive systems and wide transition systems over directed bigraphs, called *directed bigraphical reactive systems* and *directed bigraphical transition systems* respectively.

3.1 Directed Bigraphical Reactive Systems

We assume the reader familiar with wide reactive systems over premonoidal categories; see [6] for the relevant definitions.

In order to define wide reactive systems over directed bigraphs, we need to define how a parametric rule, that is a "redex-reactum" pair of bigraphs, can be instantiated. Essentially, in the application of the rule, the "holes" in the reactum must be filled with the parameters appearing in the redex. This relation can be expressed by a function mapping each site in width n of the reactum to a site in width m of the redex; notice that this allows to replace, duplicate or forget in the reactum the parameters of the redex. Formally:

Definition 10 (instantiation). *An* instantiation ρ *from (width)* m *to (width)* n, *written* $\rho :: m \to n$, *is determined by a function* $\bar{\rho} : n \to m$. *For any pair* X, *this function defines the map* $\rho : Gr\langle m, X\rangle \to Gr\langle n, X\rangle$ *as follows. Decompose* $g : \langle m, X\rangle$ *into* $g = \omega(d_0 \otimes \cdots \otimes d_{m-1})$, *with* $\omega : (\emptyset, Y) \to X$ *and each* d_i $(i \in m)$ *prime and discrete. Then define:*

$$\rho(g) \triangleq \omega(e_0 \curlywedge \ldots \curlywedge e_{n-1} \curlywedge id_{(\emptyset,Y)})$$

where $e_j \simeq d_{\bar{\rho}(j)}$ *for* $j \in n$. *This map is well defined (up to support equivalence).*

If $\bar{\rho}$ *is injective, surjective, bijective then the instantiation* ρ *is said to be* affine, total *or* linear *respectively.*

If ρ is not affine then it replicates at least one of the factor d_i. Support translation is used to ensure that several copies of replicated factor have disjoint support; also outer sharing product is used because copies will share names.

Note that the names of $e_0 \curlywedge \ldots \curlywedge e_{n-1}$ may be fewer than Y, because ρ may be not total, so we add $id_{(\emptyset,Y)}$ to the previous product to ensure that the composition with ω is defined (in that composition idle names can be generated).

Proposition 1. *Wirings commute with instantiation:* $\omega\rho(a) \simeq \rho(\omega a)$.

Next, we define how to generate ground reaction rules from parametric rules.

Definition 11 (reaction rules for directed bigraphs). *A ground reaction rule is a pair* (r, r'), *where* r *and* r' *are ground with the same outer interface. Given a set of ground rules, the reaction relation* $\longrightarrow$ *over agents is the least, closed under support equivalence* ($\simeq$), *such that* $Dr \longrightarrow Dr'$ *for each active context* D *and each ground rule* (r, r').

A parametric reaction rule has a redex R *and reactum* R', *and takes the form:*

$$(R : I \to J, R' : I' \to J, \rho)$$

where the inner interface I *and* I' *with widths* m *and* m'. *The third component* $\rho :: m \to m'$ *is an instantiation. For any* X *and discrete* $d : I \otimes (\emptyset, X)$ *the parametric rule generate the ground reaction rule:*

$$((R \otimes id_{(\emptyset,X)}) \circ d, (R' \otimes id_{(\emptyset,X)}) \circ \rho(d)).$$

Definition 12 (directed bigraphical reactive system). *A* directed bigraphical reactive system *(DBRS) over* $\mathcal{K}$ *is the precategory* $'\mathrm{DB{\scriptsize IG}}(\mathcal{K})$ *equipped with a set*[1] $'\mathcal{R}$ *of reaction rules closed under support equivalence* ($\simeq$). *We denote it by* $'\mathcal{D}(\mathcal{K}, '\mathcal{R})$, *or simply* $'\mathcal{D}$ *when clear from context.*

3.2 Directed Bigraphical Transition Systems

We can prove that DBRSs are a particular instance of the generic wide reactive systems definable on a wide monoidal precategory [6, Definition 3.1]:

[1] Here we use the "tick" to indicate that elements are objects of a precategory.

Proposition 2. *Directed bigraphical reactive systems are wide reactive systems.*

This result ensures that DBRSs inherit from the theory of WRSs ([6]) the definition of transition system based on IPOs.

Definition 13 (directed bigraphical transition system). *A* transition *for a DBRS $'\mathcal{D}(\mathcal{K}, '\mathcal{R})$ is a quadruple (a, L, λ, a'), written as $a \xrightarrow{L}_{\lambda} a'$, where a and a' are ground bigraphs and there exist a ground reaction rule $(r, r') \in {}'\mathcal{R}$ and an active context D such that $La = Dr$, and $\lambda = width(D)(width(cod(r)))$ and $a' \simeq Dr'$. A transition is* minimal *if the (L, D) is an IPO for (a, r).*

A directed bigraphical transition system *(DBTS) $\mathcal{L}$ for $'\mathcal{D}$ is a pair $(\mathcal{I}, \mathcal{T})$:*

- *$\mathcal{I}$ is a set of bigraphical interfaces; the agents of $\mathcal{L}$ are the ground bigraphs with outer interface I, for $I \in \mathcal{I}$;*
- *$\mathcal{T}$ is a set of transitions whose sources and targets are agents of $\mathcal{L}$.*

The full *(resp.* standard*) transition* FT *(resp.* ST*) system consists of all interfaces, together with all (resp. all minimal) transitions.*

Definition 14 (bisimilarity). *Let $'\mathcal{D}$ be a DBRS equipped with a DBTS $\mathcal{L}$. A* simulation *(on $\mathcal{L}$) is a binary relation $\mathcal{S}$ between agents with equal interface such that if $a\mathcal{S}b$ and $a \xrightarrow{L}_{\lambda} a'$ in $\mathcal{L}$, then whenever Lb is defined there exists b' such that $b \xrightarrow{L}_{\lambda} b'$ and $a'\mathcal{S}b'$.*

A bisimulation *is a symmetric simulation. Then* bisimilarity *(on $\mathcal{L}$), denoted by $\sim_{\mathcal{L}}$, is the largest bisimulation.*

From [6, Theorem 4.6] we have the following property:

Proposition 3 (congruence of wide bisimilarity). *In any concrete DBRS equipped with the standard transition system, wide bisimilarity is a congruence.*

Now we want to transfer ST, with its congruence property, to the abstract DBRS $\mathcal{D}(\mathcal{K}, \mathcal{R})$, where DBIG($\mathcal{K}$) is the category defined by the quotient functor $\mathcal{A}$, and $\mathcal{R}$ is obtained from $'\mathcal{R}$ also by $\mathcal{A}$.

Recall that the functor $\mathcal{A}$ forgets idle edges. For this purpose, as in [6], we impose a constrain upon the reaction rules in $'\mathcal{R}$: every redex R must be lean. Then we can prove that transitions respect lean-support equivalence:

Proposition 4. *In any concrete DBRS with all redex lean, equipped with* ST*:*

1. *in every transition label L, both components are lean;*
2. *transitions respect lean-support equivalence ($\risingdotseq$). For every transition $a \xrightarrow{L}_{\lambda} a'$, if $a \risingdotseq b$ and $L \risingdotseq M$ where M is another label with $M \circ b$ defined, then there exists a transition $b \xrightarrow{M}_{\lambda} b'$ for some b' such that $a' \risingdotseq b'$.*

Now we want to transfer the congruence result of Proposition 3 to abstract DBTSs. The following result is immediate by the [6, Theorem 4.8].

Proposition 5. *Let $'\mathcal{D}(\mathcal{K}, '\mathcal{R})$ be a concrete DBRS with all redexes lean, with* ST*. Let $\mathcal{A} : {}'\text{DBIG}(\mathcal{K}) \to \text{DBIG}(\mathcal{K})$ be the lean-support equivalence functor. Then*

1. *$a \sim_{\text{ST}} b$ in $'\mathcal{D}(\mathcal{K}, '\mathcal{R})$ if and only if $\mathcal{A}(a)\mathcal{A}(\sim_{\text{ST}})\mathcal{A}(b)$ in $\mathcal{D}(\mathcal{K}, \mathcal{R})$;*
2. *bisimilarity $\mathcal{A}(\sim_{\text{ST}})$ is a congruence in $\mathcal{D}(\mathcal{K}, \mathcal{R})$.*

4 Reducing directed bigraphical transition systems

The standard DBTS ST is already smaller than the full one FT, since we restrict to labels which form an IPO. Still, a lot of observations generated by the IPOs are useless. Actually, if the agent a and the redex R share nothing (i.e. $|a| \cap |R| = \emptyset$) or the redex R does not access to any resources of a, the observation L gives no information about what a can do.

In this section we discuss how, and when, the standard DBTS can be reduced further, but whose bisimilarity coincides with the standard one.

For the rest of the paper we work only with *hard* DBRSs, i.e., DBRSs over $'\mathrm{DBIG}_h(\mathcal{K})$ and $\mathrm{DBIG}_h(\mathcal{K})$ where the place graphs are hard, that is, have no barren roots (see [4, Definition 7.13]).

4.1 Engaged transition system

A possible way for reducing the transitions in the standard transition system is to consider only transitions where the labels carry some information about the agent. This can be expressed by considering only transitions in whose underlying IPO diagram the redex shares something with the agent, or the label accesses some of agent's resources (i.e. edges).

Definition 15 (engaged transitions). *In $'\mathrm{DBIG}_h$ a standard transition of an agent a is said to be* engaged *if it can be based on a reaction rule with redex R such that $|a| \cap |R| \neq \emptyset$ or some nodes of R access to resources of a (via the IPO-bound). We denote by* ET *the transition system of engaged transitions.*

Notice that this definition extends smoothly the one given by Milner for pure (i.e., output linear) bigraphs [6, Definition 9.10]; in fact, on output-linear bigraphs these definition coincide.

Definition 16 (relative bisimilarity). *A* relative bisimulation *for* ET *(on* ST*) is a symmetric relation $\mathcal{S}$ such that if $a\mathcal{S}b$, then for every engaged transition $a \xrightarrow{L}_\lambda a'$ and Lb is defined, there exists b' such that $b \xrightarrow{L}_\lambda b'$ in* ST*, and $a'\mathcal{S}b'$.*

The relative bisimilarity *for* ET *(on* ST*), denoted by $\sim^{\mathrm{ET}}_{\mathrm{ST}}$, is the largest relative bisimulation for* ET *(on* ST*).*

Our aim is to prove that ET is *adequate* for ST, that is, $\sim^{\mathrm{ET}}_{\mathrm{ST}} = \sim_{\mathrm{ST}}$.

To this end, we need to recall and introduce some technical definitions:

Definition 17. *1. A bigraph is* open *if every link is open;*

2. *it is* inner accessible *if every edge and upward outer name is connected to an upward inner name;*
3. *it is* outer accessible *if every edge and downward inner name is connected to a downward outer name;*
4. *it is* accessible *if it is inner and outer accessible;*
5. *it is* inner guarding *if it has no upward inner names and has no site has a root as parent;*

6. *it is* outer guarding *if it has no downward outer names;*
7. *it is* guarding *if it is inner and outer guarding;*
8. *it is* simple *if it has no idle names, no barren roots and no downward inner names, and it is prime, guarding, inner-injective and open.*
9. *it is* pinning *if it has no upward outer names, no barren roots, no two downward outer names are peers, and it is prime, ground and outer accessible.*

Intuitively, a simple bigraph has no edges (i.e., no resources), and the ports of its controls are separately linked to names in the outer interface. Instead, in a pinning bigraph ports are connected only to local edges, each of them is also accessible from exactly one name of the outer interface. Notice that in simple (resp. pinning) bigraphs, the downward (resp. upward) outer interface is empty.

Definition 18. *A DBRS is* simple *if all redexes are simple; it is* pinning *if all redexes are pinning; it is* orthogonal *if all redexes are either simple or pinning.*

Note that all these DBRS have mono and epi redexes.

We recall and give some properties of openness and accessibility.

Proposition 6 (openness properties).

1. *A composition $F \circ G$ is open if and only if both F and G are open.*
2. *Every open bigraph is lean (i.e. has no idle edges).*
3. *If $\vec{B}$ is an IPO for $\vec{A}$ and A_1 is open, then B_0 is open.*

Proposition 7 (accessibility properties).

1. *A composition $F \circ G$ is outer (resp. inner) accessible if and only if both F and G are outer (resp. inner) accessible.*
2. *Every inner or outer accessible bigraph is lean.*
3. *If $\vec{B}$ is an IPO for $\vec{A}$ and A_1 is outer (resp. inner) accessible, then B_0 is outer (resp. inner) accessible.*

We can now state and prove the main result of the section.

Theorem 1 (adequacy of engaged transitions). *In an orthogonal and linear DBRS equipped with* ST*, the engaged transitions are adequate; that is, relative engaged bisimilarity $\sim_{\mathrm{ST}}^{\mathrm{ET}}$ coincides with $\sim_{\mathrm{ST}}$.*

Proof. The fact that $\sim_{\mathrm{ST}} \subseteq \sim_{\mathrm{ST}}^{\mathrm{ET}}$ is trivial. For the vice versa we shall show that

$$\mathcal{S} = \{(Ca_0, Ca_1) \mid a_0 \sim_{\mathrm{ST}}^{\mathrm{ET}} a_1\} \cup \risingdotseq$$

is a standard bisimulation up to support equivalence for $\sim_{\mathrm{ST}}$, for details see [6, Proposition 4.5]. This more general result ensures $\sim_{\mathrm{ST}}^{\mathrm{ET}} \subseteq \sim_{\mathrm{ST}}$ by taking $C = id$.

Suppose that $a_0 \sim_{\mathrm{ST}}^{\mathrm{ET}} a_1$. Let $Ca_0 \xrightarrow{M}_{\mu} b_0'$ be a standard transition, with MCa_1 defined. We must find b_1' such that $Ca_1 \xrightarrow{M}_{\mu} b_1'$ and $(b_0', b_1') \in \mathcal{S}^{\risingdotseq}$, where $\mathcal{S}^{\risingdotseq} \triangleq \risingdotseq \mathcal{S} \risingdotseq$ is the closure of $\mathcal{S}$ under $\risingdotseq$.

There exists a ground reaction rule (r_0, r_0') and an IPO (the large square in diagram (a) below) underlying the previous transition of Ca_0. Moreover E_0 is active and $width(E_0)(width(cod(r_0))) = \mu$ and $b_0' \risingdotseq E_0 r_0'$. By taking an RPO for (a_0, r_0) relative to (MC, E_0), we get two IPOs as shown in diagram (a).

(a)
$$\begin{tikzcd}
{} \arrow[r, "M"] & {} \\
{} \arrow[u, "C"] \arrow[r, "L"] & {} \arrow[u, "E"'] \\
{} \arrow[u, "a_0"] \arrow[r, "r_0"] & {} \arrow[u, "D_0"'] \arrow[uu, bend right, "E_0"']
\end{tikzcd}$$
(b)
$$\begin{tikzcd}
{} \arrow[r, "L"] & {} \\
{} \arrow[u, "a_1"] \arrow[r, "r_1"] & {} \arrow[u, "D_1"']
\end{tikzcd}$$
(c)
$$\begin{tikzcd}
{} \arrow[r, "M"] & {} \\
{} \arrow[u, "C"] \arrow[r, "L"] & {} \arrow[u, "E"'] \\
{} \arrow[u, "a_1"] \arrow[r, "r_1"] & {} \arrow[u, "D_1"']
\end{tikzcd}$$

Now D_0 is active, so the lower IPO in diagram (a) underlies a transition $a_0 \xrightarrow{L}_\lambda a'_0 \triangleq D_0 r'_0$, where $\lambda = width(D_0)(width(cod(r_0)))$. Also E is active at λ and $b'_0 \simeq E a'_0$. Since MCa_1 is defined we deduce that La_1 is defined; we have to show the existence of a transition $a_1 \xrightarrow{L}_\lambda a'_1$, with underlying IPO as shown in diagram (b). Substituting this IPO for the lower square in diagram (a) we get a transition $Ca_1 \xrightarrow{M}_\mu b'_1 \triangleq Ea'_1$ as shown in diagram (c).

Now, to complete the proof we have to show that $a_1 \xrightarrow{L}_\lambda a'_1$ exists and that $(b'_0, b'_1) \in \mathcal{S}^{\simeq}$. This can be done by cases, on how the transition is engaged. □

Notice that in DBTSs, differently from pure bigraphical transition systems, ET restricted to prime agents is not adequate for ST; that is, in general the bisimilarity defined using ET restricted to prime agents does not coincide with $\sim_{\mathrm{ST}}$ on prime agents.

4.2 Definite engaged transition system

From the DBTS ET defined in the previous subsection, we want to obtain an abstract DBTS for the corresponding abstract DBRS, obtained by the quotient functor $\mathcal{A} : {}'\mathrm{DBIG}_h(\mathcal{K}) \to \mathrm{DBIG}_h(\mathcal{K})$. To do this, we need to prove that ET is *definite* for ST, that is, that we can infer whether a transition $a \xrightarrow{L}_\lambda a'$ in ST is engaged just by looking at the observation (L, λ) [6, Definition 4.11].

Definition 19 (split, connected). *A split of $F : I \to K$ takes the form:*

$$F = F_1 \circ (F_2 \otimes id_{I_1}) \circ \iota$$

where $F_0 : I_0 \to J$ and $F_1 : J \otimes I_1 \to K$ each have at least one node and $\iota : I \to I_0 \otimes I_1$ is an iso. The split is connected if some port in F_0 is linked to some port in F_1. It is prime if I_0 is prime. F is (prime-)connected *if every (prime) split of F is connected.*

Now we are ready to prove that if the simple redexes of the DBRS satisfies connected condition, then ET is definite.

Now we can prove the main result of this section.

Definition 20. *A label (L, λ) of a transition system is* ambiguous *if it occurs both in an engaged and a not engaged transition. A transition system is* ambiguous *if it has a ambiguous label.*

Proposition 8. *In an orthogonal and linear DBRS, where all the simple redexes are connected, then a label* (L, λ) *is unambiguous.*

Then by [6, Proposition 4.12] we can conclude that $\sim^{\text{ET}}_{\text{ST}}$ is equal to the absolute one $\sim_{\text{ET}}$ (defined as per Definition 14).

Note that this property applies equally to concrete and abstract DBRSs. Now applying [6, Corollary 4.13] and Theorem 1, we obtain

Proposition 9. *In an orthogonal linear DBRS where all simple redexes are connected:*

1. *The engaged transition system* ET *is definite for* ST.
2. *Absolute engaged bisimilarity* $\sim_{\text{ET}}$ *coincides with* $\sim_{\text{ST}}$.

We can finally transfer engaged transitions and bisimilarity to the abstract bigraphs. The "engagedness" can be defined only for concrete bigraphs; we say that an abstract transition is *engaged* if it is the image of an engaged transition under $\mathcal{A}$ and we still call engaged bisimilarity the induced bisimilarity under $\mathcal{A}$.

We prove a result for ET similar to Proposition 5 for ST.

Proposition 10. *Let* $'\mathcal{D}$ *be an orthogonal linear DBRS where all simple redexes are connected, and let* $\mathcal{D}$ *be its lean-support quotient. Then*

1. $a \sim_{\text{ET}} b$ *if and only if* $\mathcal{A}(a)\ \mathcal{A}(\sim_{\text{ET}})\ \mathcal{A}(b)$.
2. *In* $\mathcal{D}$, $\mathcal{A}(\sim_{\text{ET}})$ *is a congruence.*

4.3 Extending to non-hard abstract bigraphs

The DBTS above is obtained using the quotient functor $\mathcal{A} : {'\text{DBIG}}_h(\mathcal{K}) \to \text{DBIG}_h(\mathcal{K})$, which considers only hard place graphs. As a consequence of this restriction, we cannot use the unit 1 of the sharing products because 1 is not hard. In some cases this is too restrictive; for example, we need to use 1 for encoding the null agent of many process calculi.

A possibility is to introduce a specific zero-arity node $\square$ (called *place node*) which can be used to fill the barren roots; but in this way the structural congruence $\mathbf{0}|P \equiv P$ does not hold; we can only prove that $\mathbf{0}|P \sim P$. In fact, as in [4], we want to quotient the bigraphs by "place equivalence" ($\equiv_\square$), that is, two bigraphs are equal if they differ only by $\square$ nodes.

Let $\risingdotseq_\square$ be the smallest equivalence including $\risingdotseq$ and $\equiv_\square$. Then, similarly to $\risingdotseq$, we obtain the $\risingdotseq_\square$-quotient functor:

$$\mathcal{A}^\square : {'\text{DBIG}}_h(\mathcal{K}^\square) \to \text{DBIG}(\mathcal{K})$$

where $\mathcal{K}^\square$ is the signature $\mathcal{K}$ extended with the place node $\square$.

We want to obtain an abstract (possibly not hard) DBTSs on $\mathcal{K}$ from an hard concrete DBTS on $\mathcal{K}^\square$, using the functor $\mathcal{A}^\square$. As for $\mathcal{A}$, we must prove $\risingdotseq_\square$ respects ET transitions. We know that $\risingdotseq$ does so, it remains to show that $\equiv_\square$ respects ET. For this we require that all redexes of the DBRS are *flat*.

Definition 21. *A bigraph is* flat *if no node has a node as parent.*

Now we can prove the same result shown in Proposition 10:

Proposition 11. *Let $'\mathcal{D}(\mathcal{K}^{\square}, '\mathcal{R})$ be an orthogonal linear hard DBRS (that is, definite and whose redexes are flat), and let $\mathcal{D}(\mathcal{K}, \mathcal{R})$ be its $\bumpeq_{\square}$-quotient.*

1. *$a \sim_{\mathrm{ET}} b$ if and only if $\mathcal{A}^{\square}(a)\ \mathcal{A}^{\square}(\sim_{\mathrm{ET}})\ \mathcal{A}^{\square}(b)$.*
2. *In $\mathcal{D}$, $\mathcal{A}^{\square}(\sim_{\mathrm{ET}})$ is a congruence.*

5 An Application: the Fusion Calculus

In this section we apply the theory developed in the previous sections to the Fusion calculus. Recall that the processes of the (monadic) Fusion calculus (without replication) are generated by the following grammar:[2]

$$P, Q ::= \mathbf{0} \mid zx.P \mid \bar{z}x.P \mid P|Q \mid (x)P$$

where the processes are taken up to the structural congruence ($\equiv$), that is the least congruence satisfying the abelian monoid laws for composition and the scope laws and scope extension law:

$$(x)\mathbf{0} \equiv \mathbf{0} \qquad (x)(y)P \equiv (y)(x)P \qquad P|(x)Q \equiv (x)(P|Q) \text{ where } x \notin fn(P).$$

The semantics is defined by the following set of rules (which is a monadic version of that given in [7]), closed under the structural congruence $\equiv$.

$$Pref\ \frac{-}{\alpha.P \xrightarrow{\alpha} P} \qquad Par\ \frac{P \xrightarrow{\alpha} P'}{P|Q \xrightarrow{\alpha} P'|Q} \qquad Open\ \frac{P \xrightarrow{uz} P',\ u \notin \{z, \bar{z}\}}{(z)P \xrightarrow{(z)uz} P'}$$

$$Scope\ \frac{P \xrightarrow{\{x=y\}} P'}{(y)P \xrightarrow{\mathbf{1}} P'\{x/y\}} \qquad Pass\ \frac{P \xrightarrow{\alpha} P',\ x \notin n(\alpha)}{(x)P \xrightarrow{\alpha} (x)P'} \qquad Com\ \frac{P \xrightarrow{ux} P',\ Q \xrightarrow{\bar{u}y} Q'}{P|Q \xrightarrow{\{x=y\}} P'|Q'}$$

We write (P, φ) to mean that a process has reached the configuration P with associated fusion relation φ. We define $(P, \varphi) \rightarrow (P', \varphi')$ iff $P \xrightarrow{\psi} P'$ and φ' is the transitive closure of $\varphi \cup \psi$.

Finally, we recall the notions of *fusion bisimilarity* and *hyperequivalence.*

Definition 22. *A* fusion bisimulation *is a symmetric relation $\mathcal{S}$ between processes such that whenever $(P, Q) \in \mathcal{S}$, if $P \xrightarrow{\alpha} P'$ with $bn(\alpha) \cap fn(Q) = \emptyset$, then $Q \xrightarrow{\alpha} Q'$ and $(P'\sigma_\alpha, Q'\sigma_\alpha) \in \mathcal{S}$ (where σ_α denotes the substitutive effect of α).*

P and Q are fusion bisimilar *if $(P, Q) \in \mathcal{S}$ for some fusion bisimulation $\mathcal{S}$.*

A hyperbisimulation *is a substitution closed fusion bisimulation, i.e., an $\mathcal{S}$ such that $(P, Q) \in \mathcal{S}$ implies $(P\sigma, Q\sigma) \in \mathcal{S}$ for any substitution σ. P and Q are* hyperequivalent*, written $P \sim_F Q$, if they are related by a hyperbisimulation.*

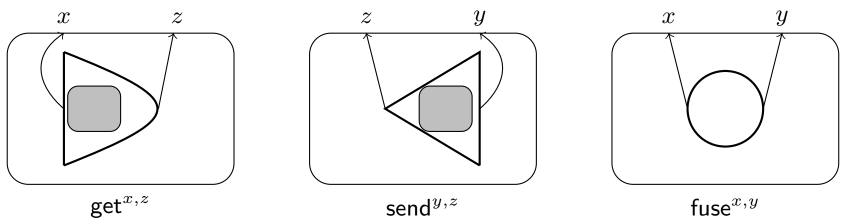


Fig. 1. The controls of the signature for the Fusion calculus.

Notice that hyperequivalence only is a congruence, while bisimilarity is not [7].

The signature for representing Fusion processes in directed bigraphs is $\mathcal{K}_F \triangleq \{\mathsf{get}{:}2, \mathsf{send}{:}2, \mathsf{fuse}{:}2\}$, where get, send are passive and fuse is atomic (Figure 1).

A process P is translated to a bigraph of $\mathrm{DBig}(\mathcal{K}_F)$ in two steps, using some algebraic operators of directed bigraphs [1]. First, for X a set of names such that $fn(P) \subseteq X$, we define a bigraph $[\![P]\!]_X : \epsilon \to \langle 1, (\emptyset, X)\rangle$:

$$[\![\mathbf{0}]\!]_X = 1 \curlywedge X \quad [\![P|Q]\!]_X = [\![P]\!]_X \curlywedge [\![Q]\!]_X \quad [\![(x)P]\!]_X = \blacktriangle^x \circ [\![P]\!]_{X \uplus \{x\}}$$
$$[\![zx.P]\!]_X = \mathsf{get}^{x,z} \circ [\![P]\!]_X \quad [\![\bar{z}x.P]\!]_X = \mathsf{send}^{x,z} \circ [\![P]\!]_X \quad \textit{where } x, z \in X$$

Notice that names in X are represented as outer upward names. In this translation bound names are represented by local (not accessible) edges.

Then, the encoding of a process P under a fusion φ takes the bigraph $[\![P]\!]_{fn(P)}$ and associates to each name in $fn(P)$ an outer accessible edge, according to φ:

$$[\![P]\!]_\varphi = \left(\sum_{[n]_\varphi \in \varphi} \triangledown_n^{[n]_\varphi} \circ \blacktriangledown_n^n \circ \vartriangle_{[n]_\varphi}^n \right) \circ \left([\![P]\!]_{fn(P)} \otimes \sum_{m \in Y \setminus fn(P)} \vartriangle^m \right)$$

Fusions are represented by linking the fused names (in the outer interface) to the same edge. An example of encoding is given in Figure 2.

Proposition 12. *Let P and Q be two processes; then $P \equiv Q$ if and only if $[\![P]\!]_\varphi = [\![Q]\!]_\varphi$, for every fusion φ.*

The set of reaction rules ($\mathcal{R}_F$) are shown in Figure 3. Notice that *Com* is simple, instead *Fuse* and *Disp* are pinning; hence this system is orthogonal. Moreover each rule is flat. We denote this DBRS as $\mathcal{D}_F \triangleq \mathcal{D}(\mathcal{K}_F, \mathcal{R}_F)$.

Proposition 13 (Adequacy of the encoding).

1. *if $(P, \varphi) \to (P', \varphi')$ then $[\![P]\!]_\varphi \longrightarrow^* [\![P']\!]_{\varphi'}$;*
2. *if $[\![P]\!]_\varphi \longrightarrow^* [\![P']\!]_{\varphi'}$ then $(P, \varphi) \to^* (P', \varphi')$.*

[2] Sum and fusion prefix can be easily encoded in this syntax.

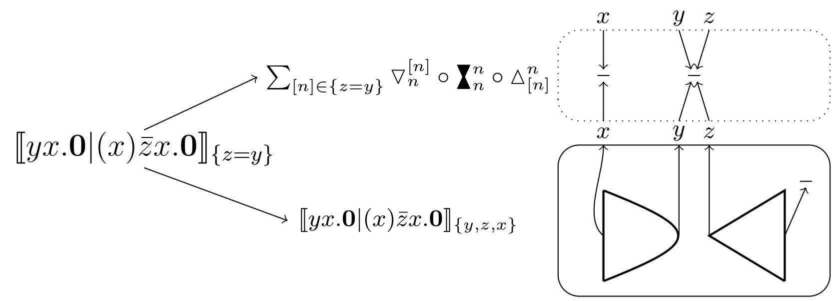


Fig. 2. An example of encoding a fusion process in directed bigraphs.

Proof. By induction on the length of the traces. Point 1. is easy. For point 2, first of all note that, by definition of $[\![\cdot]\!]_\varphi$, $[\![P']\!]_{\varphi'}$ has no fuse controls. If $[\![P]\!]_\varphi \longrightarrow^* [\![P']\!]_{\varphi'}$, in the trace there are one or more applications of the *Com* rule in $\mathcal{D}_F$, so we use the *Com* rule of the Fusion on the corresponding P sub-process. We can ignore the *Fuse* and *Disp* rules, because the fusions are performed immediately in the Fusion calculus. $\square$

Working with the abstract bigraphs we obtain the exact match between the Fusion reactions and bigraphic one. Now we want to define the ET bisimilarity for the Fusion calculus. We define ${}'\mathcal{D}_F \triangleq {}'\mathcal{D}(\mathcal{K}_F^{\Box}, {}'\mathcal{R}_F)$ to be the concrete DBRS whose precategory of bigraphs is defined on the signature $\mathcal{K}_F^{\Box}$, and ${}'\mathcal{R}_F$ are all the reaction rules that are in the preimage of the abstract rules of $\mathcal{D}_F$ via $\mathcal{A}^{\Box}$ (see Figure 3).

First notice that engaged transitions of ${}'\mathcal{D}_F$ yield a congruential bisimilarity.

Corollary 1. *The bisimilarity $\sim_{\mathrm{ET}}$ is a congruence in ${}'\mathcal{D}_F$.*

Proof. Since ${}'\mathcal{R}_F$ is orthogonal and linear, by Theorem 1, ET is adequate for ST in ${}'\mathcal{D}_F$. Moreover there are no subsumption, then it follows by Proposition 9 that ET is definite for ST and hence $\sim_{\mathrm{ET}}=\sim_{\mathrm{ST}}$. $\square$

Now by Proposition 11, we can derive a congruential bisimilarity in our bigraphical representation of the Fusion.

Corollary 2. *In $\mathcal{D}_F$, the following two sentences are verified:*

1. $a \sim_{\mathrm{ET}} b$ *if and only if* $\mathcal{A}^{\Box}(a)\ \mathcal{A}^{\Box}(\sim_{\mathrm{ET}})\ \mathcal{A}^{\Box}(b)$;
2. $\mathcal{A}^{\Box}(\sim_{\mathrm{ET}})$ *is a congruence.*

Clearly, $\sim_{\mathrm{ET}}$ induces a congruence on processes of Fusion calculus. In fact this is the first congruence for Fusion calculus defined only by coinduction (differently from hyperequivalence, which needs a closure under substitutions). However, comparing this congruence with hyperbisimulation or hyperequivalence turns out to be problematic, because the DBTS ET involves also non-prime transitions

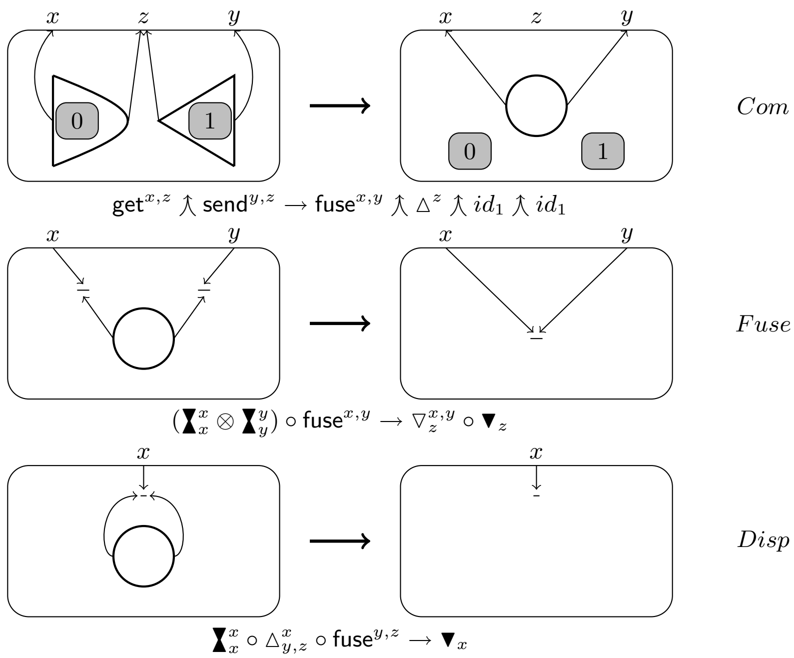


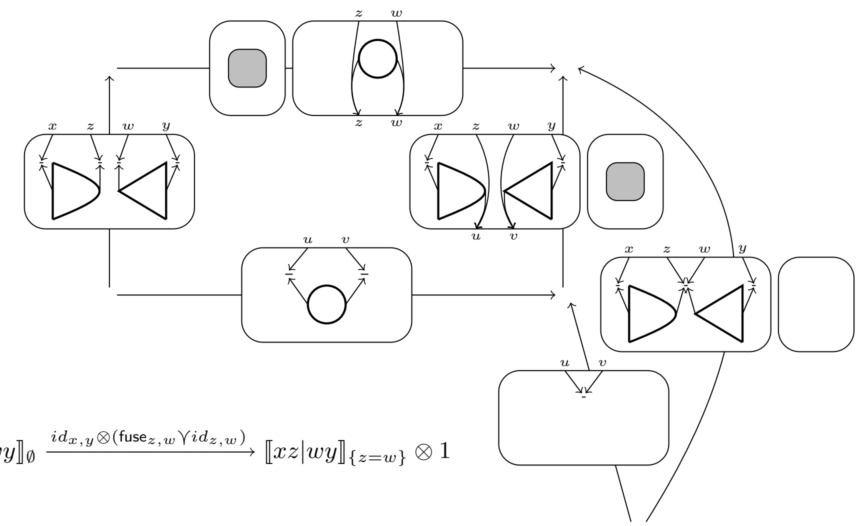
Fig. 3. Reaction rules $\mathcal{R}_F$ for the Fusion calculus.

which are essential to make ET adequate with respect to ST (see Figure 4 for an example of a useful but not prime transition). Thus, when comparing two processes P, Q using $\sim_{\mathrm{ET}}$, we have to consider also non-prime transitions; in these cases, the resulting agents are non-prime, and their connection with the descendants of P and Q in the original semantics is still unclear.

6 Conclusions

In this paper, we have presented *directed bigraphical reactive systems* and *directed bigraphical transition systems*, that is wide reactive and transition systems built over directed bigraphs. We have shown that the bisimilarity induced by the IPO construction is always a congruence; moreover, under a mild condition, this bisimilarity can be characterized by a smaller LTS whose transitions (called "engaged") are only those really relevant for the agents. As an application, we have presented the first encoding of the Fusion calculus as a DBRS; then, using the general constructions given in this paper, we have defined a bisimilarity for Fusion which is also a congruence, without the need of a closure under substitutions.

The exact relation between this equivalence and those defined in the literature (i.e., hyperbisimilarity and hyperequivalence) is still under investigation. The issue is that for DBRSs, engaged transitions of prime agents may include also



$$[\![xz|wy]\!]_\emptyset \xrightarrow{id_{x,y}\otimes(\mathsf{fuse}_{z,w}\curlyvee id_{z,w})} [\![xz|wy]\!]_{\{z=w\}} \otimes 1$$

Fig. 4. An example of a non prime engaged transition in ${}'\mathcal{D}_F$.

non-prime transitions, yielding non-prime agents as results. This happens in the case of Fusion calculus, and these transitions are not easily interpreted in terms of the labelled transition systems by which hyperbisimilarity is defined.

Another possible future work concerns the application of the theory developed in this paper for verification of properties of systems represented as DBRSs. Beside using standard bisimilarity for checking behavioural equivalence, the compact DBTS can be used also for model checking purposes.

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