# Esempio

**Problema**. Scrivere un programma *FINDPOS* per trovare un elemento positivo in un array di interi.

Il programma dovrà soddisfare

$\{\textbf{true}\}\ FINDPOS\ \{1 \leq k \leq N+1 \wedge\ \forall(0 < l < k).\ a[l] \leq 0\ \wedge\ (k \leq N \rightarrow a[k] > 0)\}$

rispetto alla correttezza totale.

# Il programma FINDPOS

$$
\begin{array}{lcl}
FINDPOS & \equiv & i := 1; j := 2; oddtop := N + 1; eventop := N + 1; \\
 & & [S_1 | S_2] \\
 & & k := min(oddtop, eventop);
\end{array}
$$

dove

$$
\begin{array}{lcl}
S_1 & \equiv & \textbf{while } i < min(mathitoddtop, eventop) \textbf{ do} \\
 & & \quad \textbf{if } a[i] > 0 \textbf{ then } oddtop := i \\
 & & \qquad\qquad \textbf{else } i := i + 2 \quad \textbf{fi} \\
 & & \textbf{od}
\end{array}
$$

$$
\begin{array}{lcl}
S_2 & \equiv & \textbf{while } j < min(mathitoddtop, eventop) \textbf{ do} \\
 & & \quad \textbf{if } a[j] > 0 \textbf{ then } eventop := j \\
 & & \qquad\qquad \textbf{else } j := j + 2 \quad \textbf{fi} \\
 & & \textbf{od}
\end{array}
$$

# Invarianti e funzioni bound

$$p_1 \equiv 1 \leq oddtop \leq N+1 \ \wedge \ odd(i) \ \wedge \ 1 \leq i \leq oddtop+1 \\ \wedge \forall l \ (odd(l) \ \wedge \ 1 \leq l < i \ \rightarrow \ a[l] \leq 0) \\ \wedge \ (oddtop \leq N \ \rightarrow \ a[oddtop] > 0)$$

$$p_2 \equiv 2 \leq eventop \leq N+1 \ \wedge \ even(j) \ \wedge \ j \leq eventop+1 \\ \wedge \forall l \ (even(l) \ \wedge \ 1 \leq l < j \ \rightarrow \ a[l] \leq 0) \\ \wedge \ (eventop \leq N \ \rightarrow \ a[eventop] > 0)$$

$$t_1 \equiv oddtop + 1 - i$$

$$t_2 \equiv eventop + 1 - j$$

# Proof outline per $S_1$

$\{\textbf{inv} : p_1\}\{\textbf{bd} : t_1\}$
**while** $i < min(oddtop, eventop)$ **do**
  $\{p_1 \ \wedge \ i < oddtop\}$
  **if** $a[i] > 0$ **then** $\{p_1 \ \wedge \ i < oddtop \ \wedge \ a[i] > 0\}$
    $oddtop := i$
    **else** $\{p_1 \ \wedge \ i < oddtop \ \wedge \ a[i] \leq 0\}$
    $i := i + 2$
  **fi**
**od**
$\{p_1 \ \wedge \ i \geq min(oddtop, eventop)\}$