THE BDC-DECOMPOSITION

\[ \dot{x}(t) = Sg(x(t)) + g_0, \quad g \text{ monotonous functions} \]

**Local BDC-decomposition**

The Jacobian can be decomposed as:

\[ J(x) = \frac{\partial Sg(x)}{\partial x} = B \Delta(x) C, \quad \Delta(x) = \text{diag}(\{ \frac{\partial g_i}{\partial x_k} \}) > 0. \]

The decomposition is unique (up to permutations).

**Global BDC-decomposition**

Given the equilibrium \( x^* \) (0 = Sg(\( x^* \)) + g_0), z = x - x^*.

The system can be rewritten as:

\[ z(t) = Sg(z(t) + x) - Sg(x) = [BD(x)] C z(t), \quad D(x) > 0. \]

**BDC-based computation of Polyhedral Lyapunov Functions for structural stability**

Monotonic functions \( g \) and dissipative reactions \( \dot{g}_0 < 0 \) exploit the BDC-decomposition! Structurally \( \Leftrightarrow \) for any \( D_i > 0 \),

**Idea: D(x(t)) \rightarrow D(t)**

Absorb the nonlinear system in a Linear Differential Inclusion

\[ \dot{x}(t) = [BD(t)] C \ x(t), \quad D(t) > 0. \quad (LDI) \]

Any trajectory of the original system is also a trajectory of (LDI).

To analyse stability we can assume \( 0 \leq D_i \leq 1 \).

Polyhedral... why? The only structural Lyapunov function is polyhedral!

**Structural Stability Inference:**

For systems satisfying a BDC decomposition

**Structural Influence Matrix**

\[ \Sigma_{ij} = H_{BDC}^{-1} E_j \]

\( H \) output matrix, \( E \) input matrix \( \rightarrow \) efficient vertex algorithm

**References**


