Synchronous networks

A synchronous network proceeds in pulses. In one pulse, each process:

1. sends messages
2. receives messages
3. performs internal events

A message is sent and received in the same pulse.

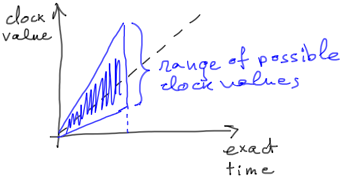
Such synchrony is called lockstep.

E.g. useful for electing a leader (e.g. bully algorithm)

Clock at process p: $C_p : \mathbb{R} \to \mathbb{R}$

can have:

- <u>ρ-bounded drift</u> if $C_p(t_1) - C_p(t_2) \in (t_1 - t_2) \cdot [\frac{1}{\rho}, \rho]$



- <u>precision δ</u> if $C_p(t) - C_q(t) \in [-\delta, +\delta]$

NOTE: achieved by regularly synchronizing clocks across processes

Building a synchronous network

Assume ρ-bounded local clocks with precision δ.

For simplicity, we ignore the network latency.

When a process reads clock value $(i-1)\rho^2\delta$, it starts pulse i.

Key question: Does a process p receive all messages for pulse i before it starts pulse $i+1$? That is, for all q,

$$C_q^{-1}((i-1)\rho^2\delta) \;\leq\; C_p^{-1}(i\rho^2\delta)$$

Because then q starts pulse i no later than p starts pulse $i+1$.

($C_r^{-1}(\tau)$ is the moment in time the clock of r returns τ.)

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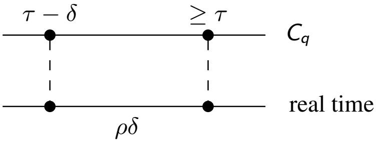
Since the clock of q is ρ-bounded from below,

$$C_q^{-1}(\tau) \;\leq\; C_q^{-1}(\tau - \delta) + \rho\delta$$



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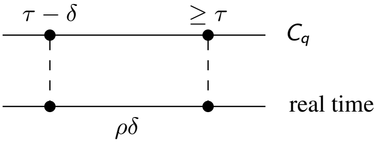

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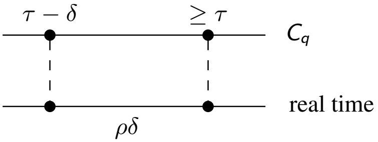

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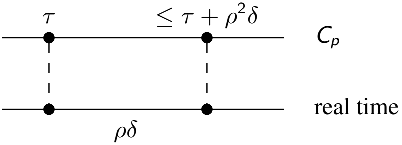

Hence,

$$\begin{aligned} C_q^{-1}((i-1)\rho^2\delta) &\leq C_p^{-1}((i-1)\rho^2\delta) + \rho\delta \\ &\leq C_p^{-1}(i\rho^2\delta) \end{aligned}$$

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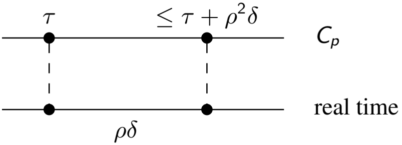

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