Matematica e Automazione

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Summary

Un pò di storia

Due applicazioni di automazione nel campo ferroviario

Conclusioni

La nascita della teoria matematica del controllo

Nel 1840, l'astronomo britannico G.B. Airy sviluppò un sistema retroazionato per il puntamento di un telescopio, che regolava la sua velocità in maniera da compensare la rotazione della terra. Egli fu il primo a discutere l'instabilità di un sistema di controllo e il primo ad usare l'equazioni differenziali, ormai ben conosciute grazie alle scoperte del calcolo infinitesimale di I. Newton, G.B. Leibniz, dei fratelli Bernoulli, di Jacopo Riccati e altri (fine del '600, inizio del '700). L'uso delle equazioni differenziali per l'analisi del moto è soprattutto dovuto a J.L. Lagrange (1736-1813), W.R. Hamilton (1805-1865).

Teoria della stabilita'

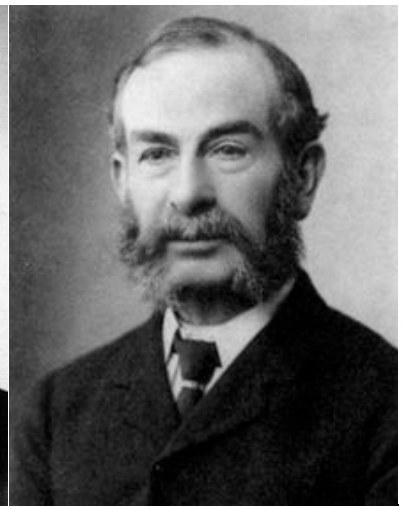
James Clerk Maxwell (1831-1879): Gas kynetic theory, color theory, electromagnetism



Analisi della stabilià del "flyball governor" di Watt (per la regolazione di velocità di una macchina a vapore). Linearizzazione delle equazioni differenziali del moto, effetto dei parametri sulla stabilità tramite analisi delle radici dell'equazione caratteristica nel semipiano sinistro del piuano complesso.

Teoria della stabilita'

Adolf Hurwitz (1859-1919): Riemann surface theory, foundational results on algebraic curves
Eduard John Routh (1831-1907): Mathematical Theory of Mechanics

$$s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n = 0 \quad \longrightarrow \quad Re(s_i) < 0, \forall i?$$

Teoria della stabilita'

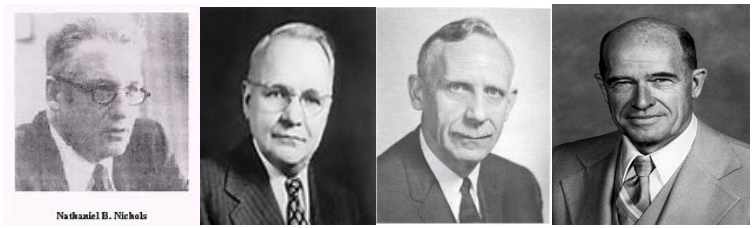
Aleksandr Mikhailovič Lyapunov (1857-1918): Stability theory, probability theory. Stabilità di un sistema di equazioni differenziale nonlineare attraverso l'uso di una nozione generalizzata di "energia". Il suo lavoro rimase sostanzialmente ignoto fino a circa il 1960.

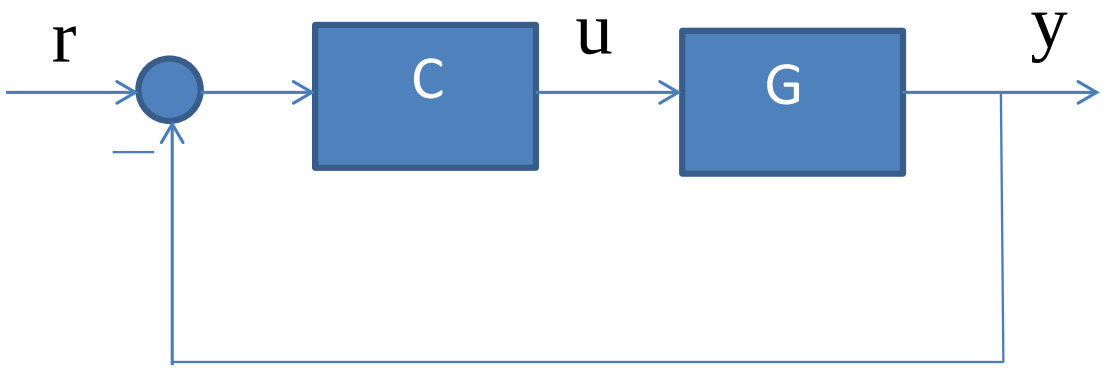


$$\dot{x} = f(x),\ f(0) = 0,\ V(x) > 0,\ \dot{V}(x) \leq 0 \longrightarrow \text{Stabilita}' \text{ di } x = 0$$
$$\dot{x} = Ax,\ A'P + PA < 0 \longrightarrow \text{Stabilita}' \text{ del sistema}$$

Classical theory: Nichols, Nyquist, Bode, Evans, ...

MIT Radiation Laboratory

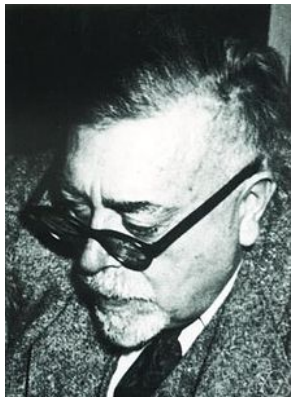


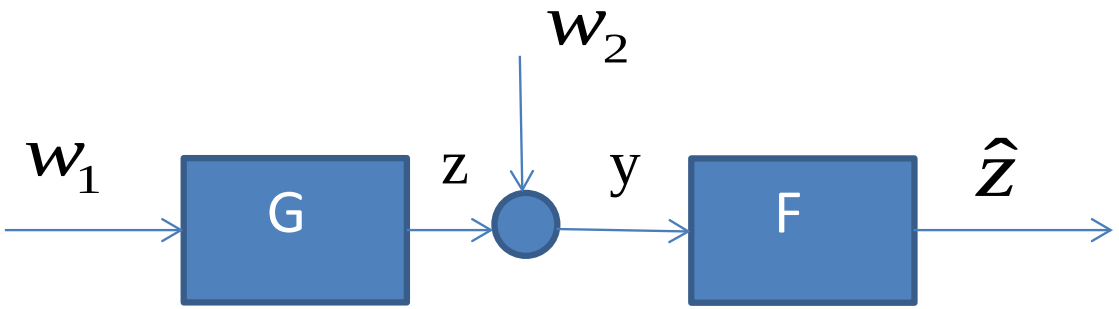


Diagrammi di Bode, Nyquist, carte di Nichols, luogo delle radici di Evans, PID. Strada aperta per la parametrizzazione di Youla-Kucera-Larin (anni 70'):

$$G = N/M, \quad NX + MY = 1, \quad \longrightarrow C = (X + MQ)/(Y - NQ)$$

Filtro di Wiener





$$G(s)G(s)^{\sim} = \bar{G}(s)\bar{G}(s)^{\sim} \quad \longrightarrow \quad F^{opt}(s) = 1 - \frac{1}{\bar{G}(s)}$$

Rudolf E. Kalman, 1930–

Allievo di John R. Ragazzini, insieme a Jury e Zadeh



Filtro di Kalman, proprietà algebriche di sistemi lineari

$$\dot{x} = Ax + Bw_1,\ y = Cx + Dw_2, \quad \dot{\hat{x}} = Ax + L(C\hat{x} - y)$$

L viene da un'equazione di Riccati - fattorizzazione spettrale - Filtro di Wiener

Jacopo Francesco Riccati, 1676-1754

Idrodinamica, Meccanica Newtoniana



$$\frac{d}{dt}\begin{bmatrix}\alpha\\ \beta\end{bmatrix}=\begin{bmatrix}w_1 & w_2\\ w_3 & w_4\end{bmatrix}\begin{bmatrix}\alpha\\ \beta\end{bmatrix},\quad x=\alpha/\beta \rightarrow \dot{x}=ax^2+bx+c$$

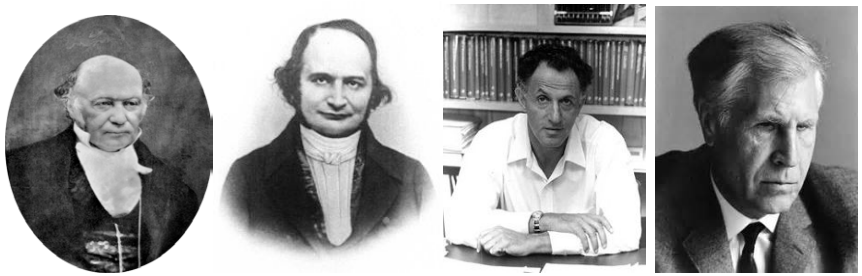
Euler (1760), Liouville (1840), calcolo delle variazioni, Fermat, 1662, brachystochrone problem, Galilei (1638), John Bernoully (1697), second variation problem, Legendre, Lagrange, Jacobi, Hamilton, Ernesto Pascal, Caratheodory, 1780–1940) Controllo e filtraggio ottimo, LQG, Kalman,1960, Norme in spazi di Hardy, Entropia, Model matching

$$\min J=\int_0^\infty x(t)'C'Cx(t)+u(t)'u(t)dt$$

$$A'P+PA-PBB'P+C'C=0 \rightarrow u=-B'Px,\quad J^o=x_0'Px_0$$

Hamilton, Jacobi — Bellman, Pontragyn

Classical mechanics, optimal control of nonlinear systems



$$\dot{x}(t) = f(x(t), u(t))$$
$$0 \le t \le T$$

$$h(x(T)) + \int_0^T g(x(t), u(t))\, dt$$

$$V(T, x) = h(x)$$

$$0 = \min_{u \in U} \{ g(x,u) + \nabla_t V(t,x) + \nabla_x V(t,x)' f(x,u) \}$$

$$H(x,u,\lambda) = \lambda' f(x,u) + g(x,u)$$
$$-\dot{\lambda}' = \lambda' f_x(x,u) + g_x(x,u)$$
$$\lambda(T) = h_x(x(T))$$

Automazione per veicoli ferroviari

Due problemi in campo ferroviario:

- Risparmio energetico

  Modulazione DC link - Ecodrive collaborativo
- Controllo della frenata

  Antipattinamento distribuito

Automazione per veicoli ferroviari

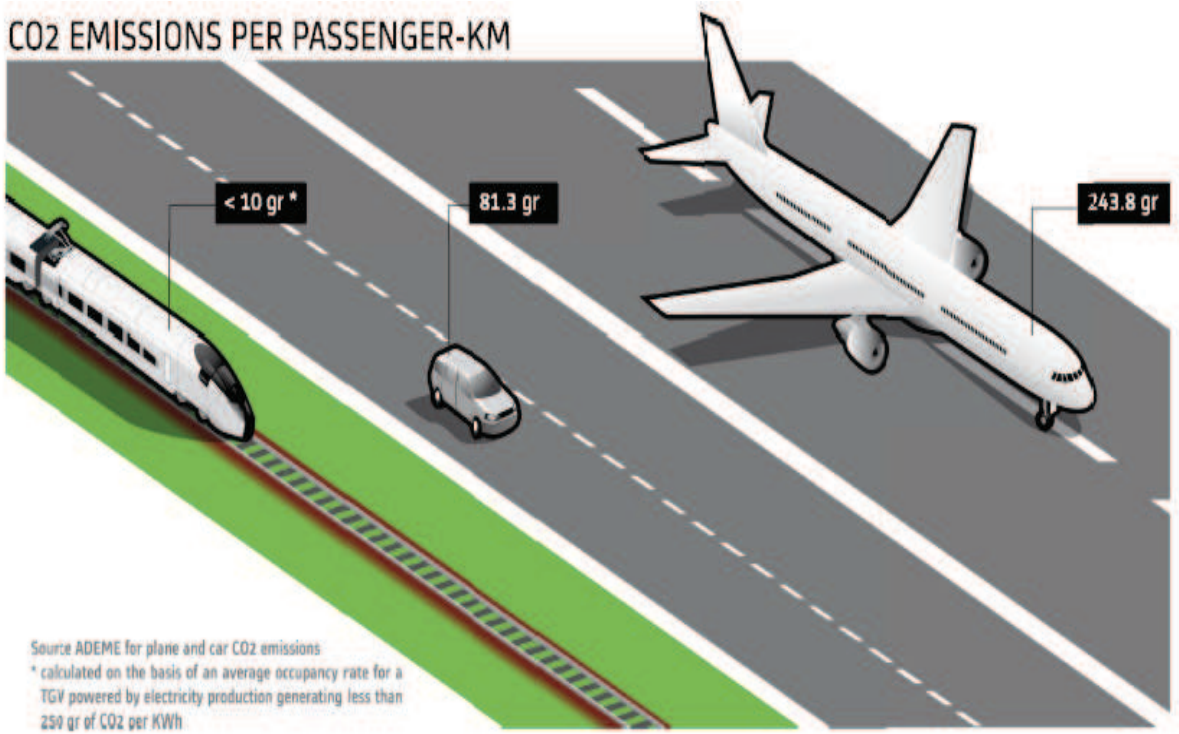


Figure 1: Comparizon

SNCF 2012: Raiway transportation is the most energy-efficient mode of transport, and electrification is the most energy-efficient way to power the trains. -68% w.r. cars, -73% w.r. airplanes.

Control strategies for power losses reduction



Figure 2: New pendolino

Reduction of the harmonic losses in motors and transformers through modulation of the DC-link voltage. The method has been implemented in ALSTOM Control for the New Pendolino Cisalpino for SBB (15kVac, 16.7Hz).

Control strategies for power losses reduction

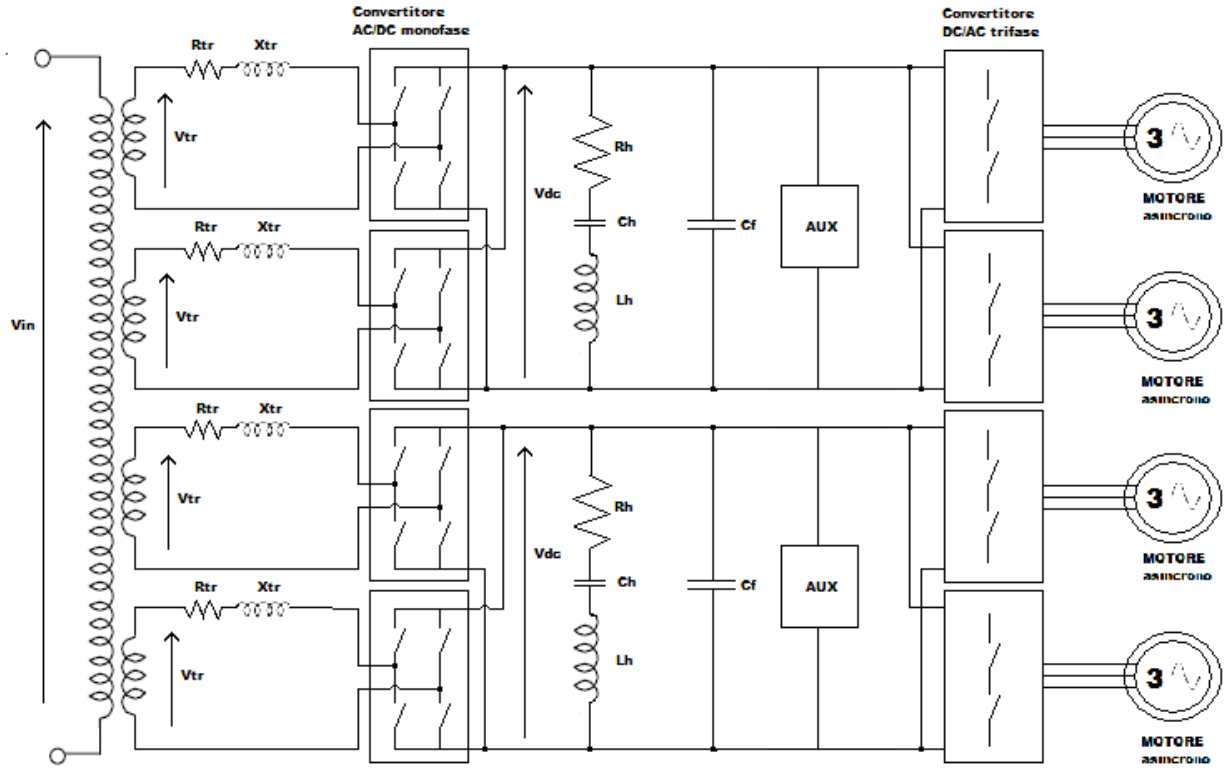


Figure 3: Power scheme

Monophase transformer 15kV/1770 V, four secondary drives, rectifiers AC/DC (called PMCF), DC link, resonant filter, capacitors, inverters, motors.

Control strategies for power losses reduction

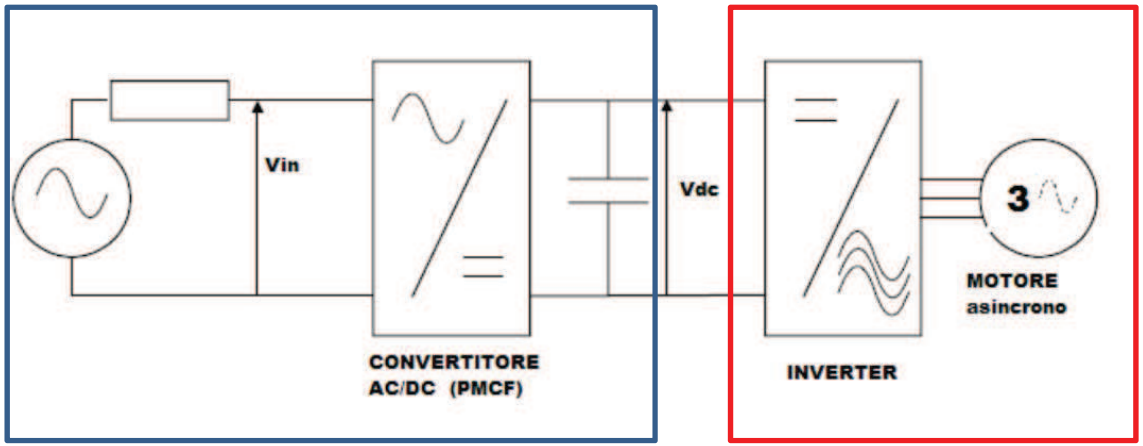


Figure 4: General traction drive

Conduction losses (IGBT), commutation losses (IGBT), thermic losses, Joule losses, harmonic losses, Foucalt losses, iron losses.

$$V_{dc} = g_2(\omega_m, T_{dem}, V_{cat}), \quad V_{min} \leq V_{dc} \leq V_{max}$$
$$P = g_1(x, u), \quad \dot{x} = f(x, u)$$

Control strategies for power losses reduction

Recap:
optimised control law of the DC-link voltage depending on the motor frequency, on the traction torque reference and taking into account also the limit imposed by the catenary voltage. Calculation of the main power losses (MTF, PMCF, VSI, ASM) at different motor frequencies and torque demands (Handle).

$$V_{dc} = g_2(\omega_m, T_{dem}, V_{cat})$$

Control strategies for power losses reduction in high speed trains

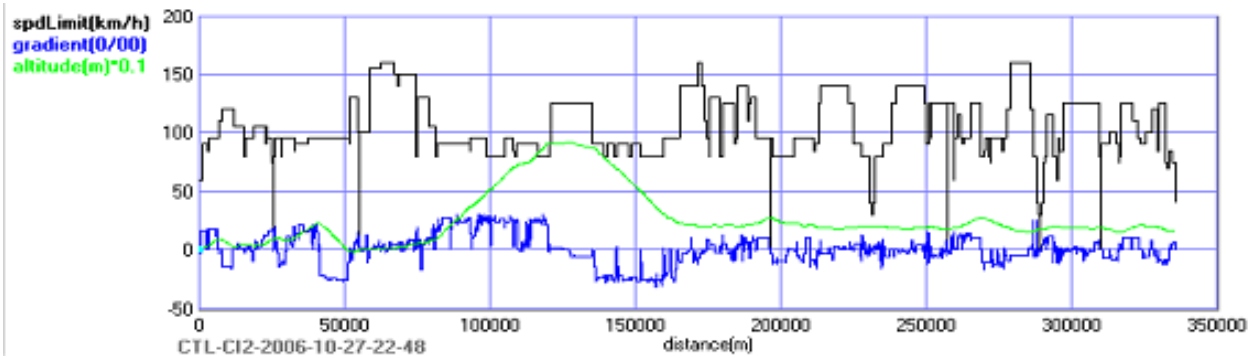
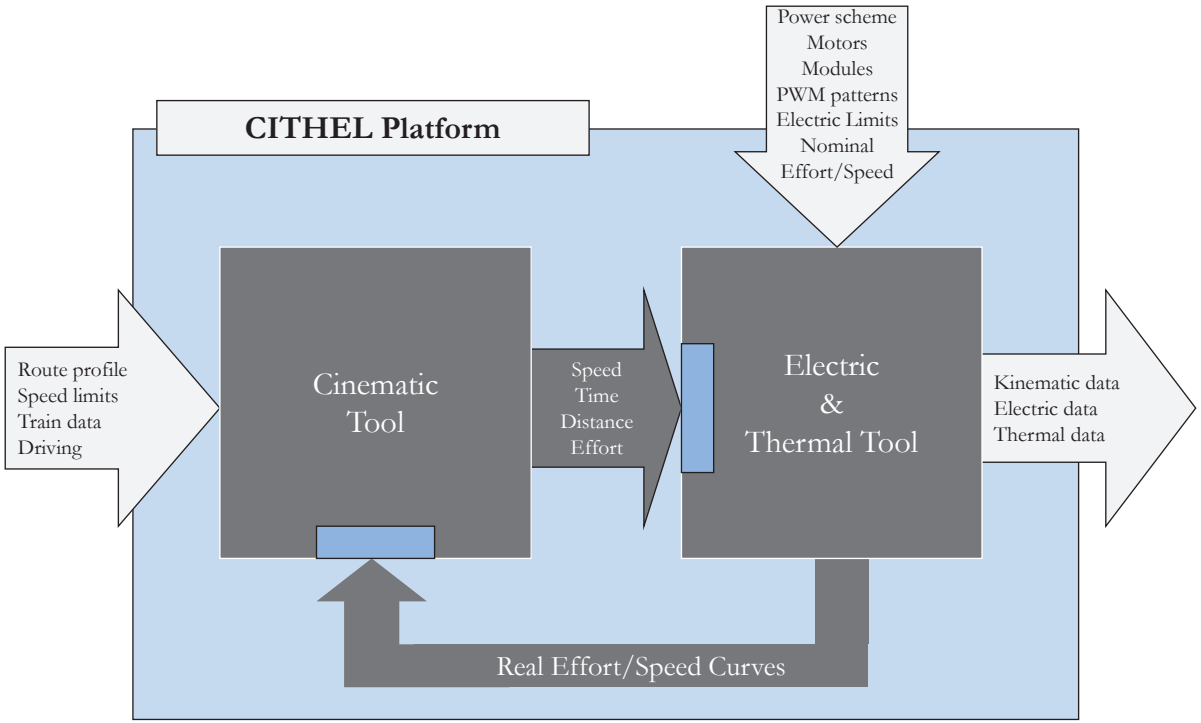


Figure 5: Route Chiasso-Shaffausen

In the next figures the energy saving on a real SBB route Chiasso-Shaffausen is reported: It can be seen that speed limit is quite low so that the required power and torque are often limited.

Control strategies for power losses reduction



The comparizons has been made using CITHEL (ALSTOM): Fast solver for iterative calculation of all steady state electrical and thermal curves and waveforms. Perform cinematic, electrical and thermal simulations on a real route (given the route profile and the driving data in order), to evaluate the energy consumption, the losses and the thermal behaviour.

Control strategies for power losses reduction

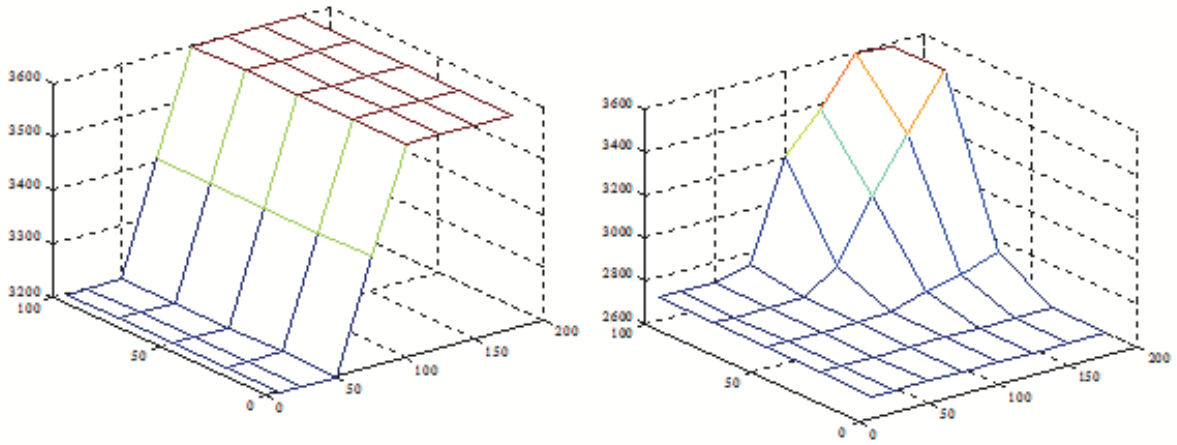


Fig. 1: dc-link voltage before and after the optimisation

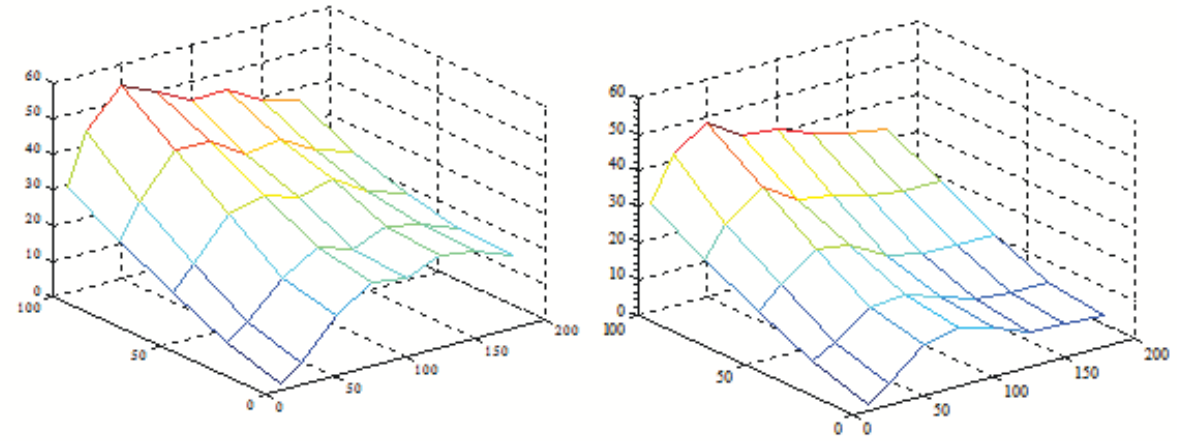


Fig. 2: motor losses before and after the optimisation

Figure 6: DC Voltage and Motor losses

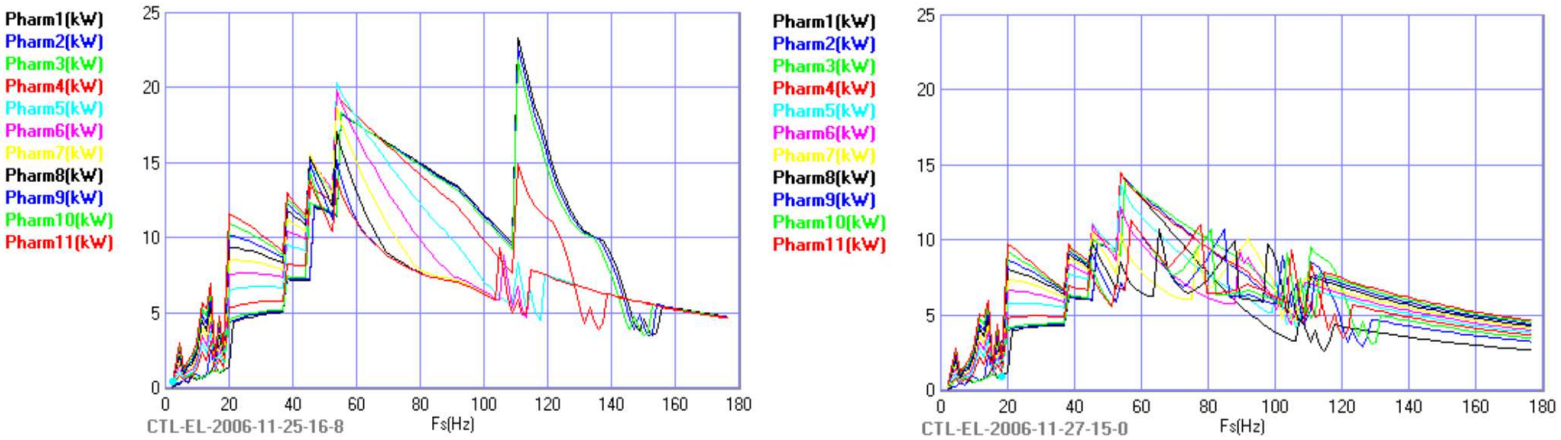


Figure 7: Original and optimized harmonic motor losses

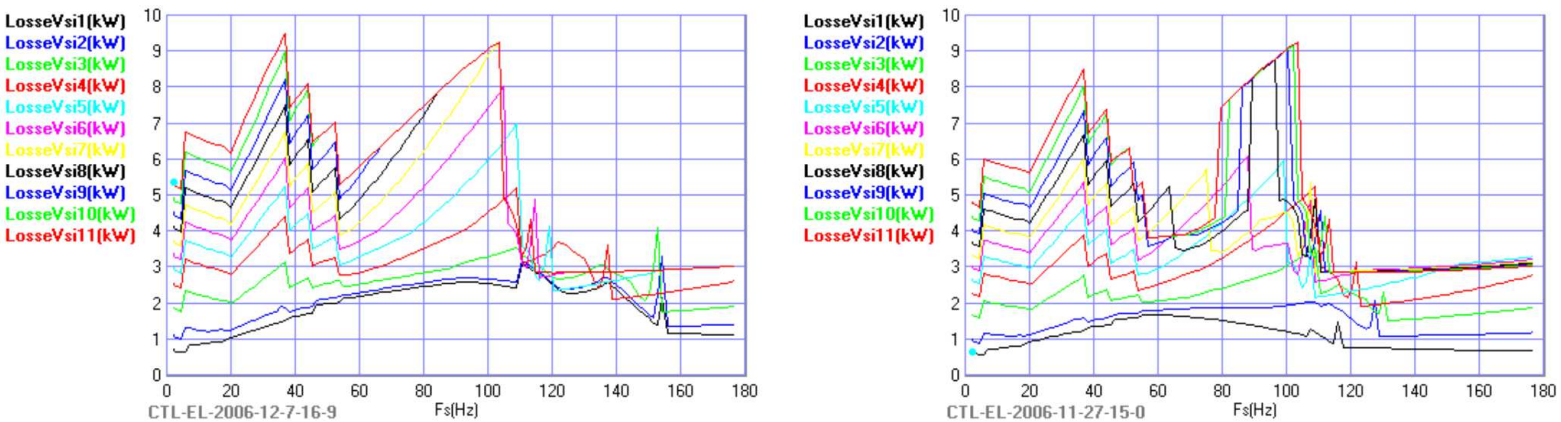


Figure 8: Original and optimized VSI losses

Control strategies for power losses reduction in high speed trains

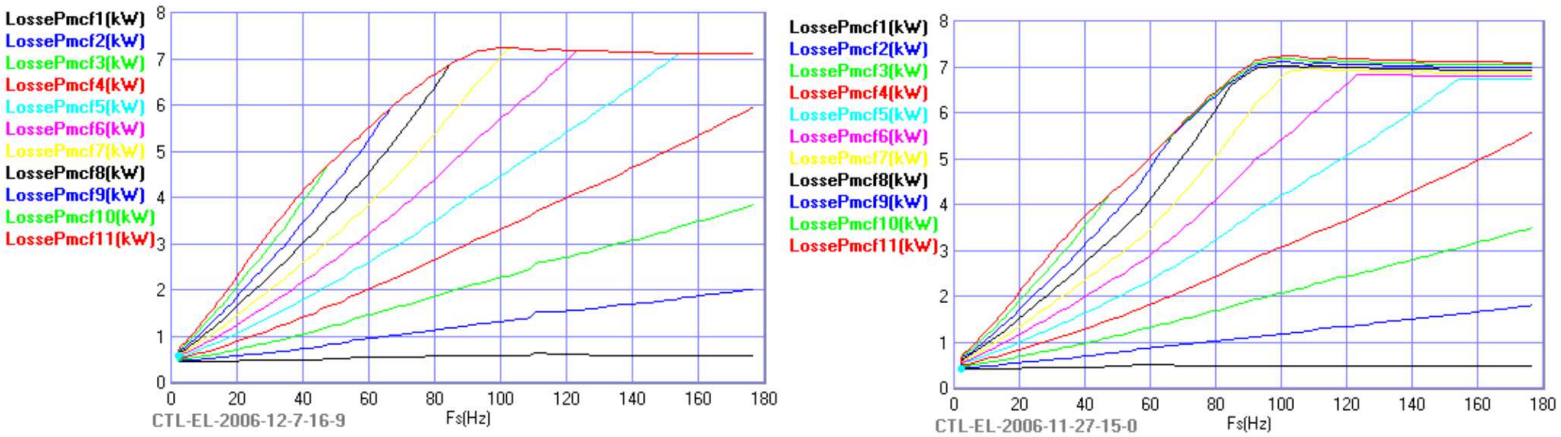
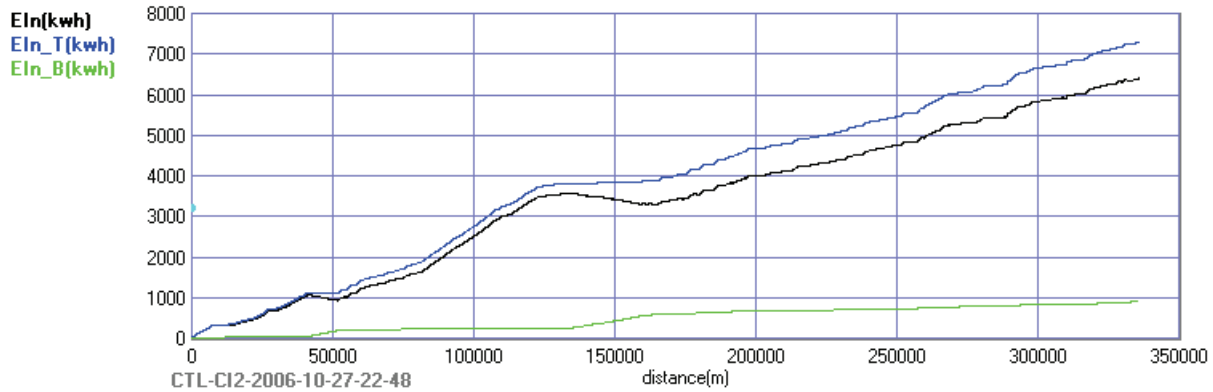


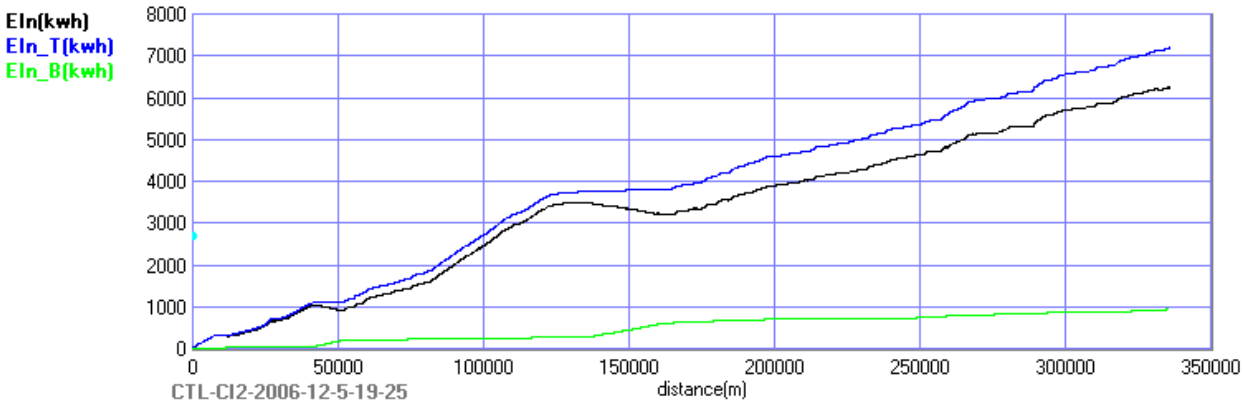
Figure 9: Original and optimized PMCF losses

Control strategies for power losses reduction in high speed trains

$EIn = 6444.41(kwh)$ $EIn_T = 7342(kwh)$ $EIn_B = 897(kwh)$



$EIn = 6295(kwh)$ $EIn_T = 7232(kwh)$ $EIn_B = 937(kwh)$



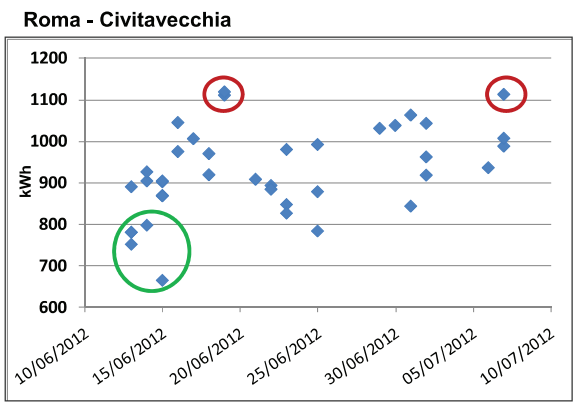
Control strategies for power losses reduction: conclusion

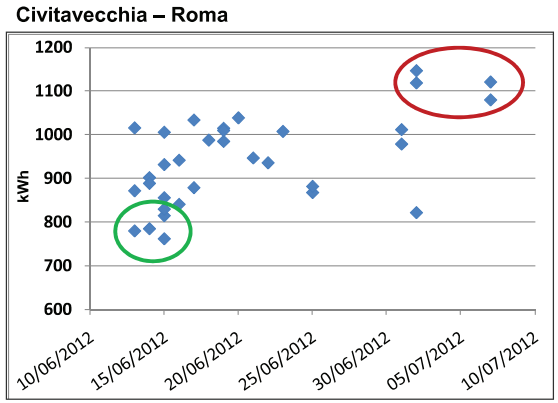
%Handle\frequenza	10 Hz	25 Hz	50 Hz	75 Hz	100 Hz	125 Hz	150 Hz	177 Hz
0%	7,95	8,96	16,13	32,84	36,54	59,86	59,06	54,49
25%	2,48	4,31	10,74	23,01	24,64	41,31	39,00	33,11
50%	1,63	3,94	9,96	13,43	17,28	23,35	16,38	11,16
75%	1,29	3,53	7,59	11,51	6,21	10,60	7,11	4,06
100%	1,07	2,90	5,17	4,64	1,89	4,53	3,05	1,90

The result of optimization is independent of the track. The algorithm is implemented in trains of the Swiss Railway System.

Ecodrive: speed tuning for energy saving in railway vehicles

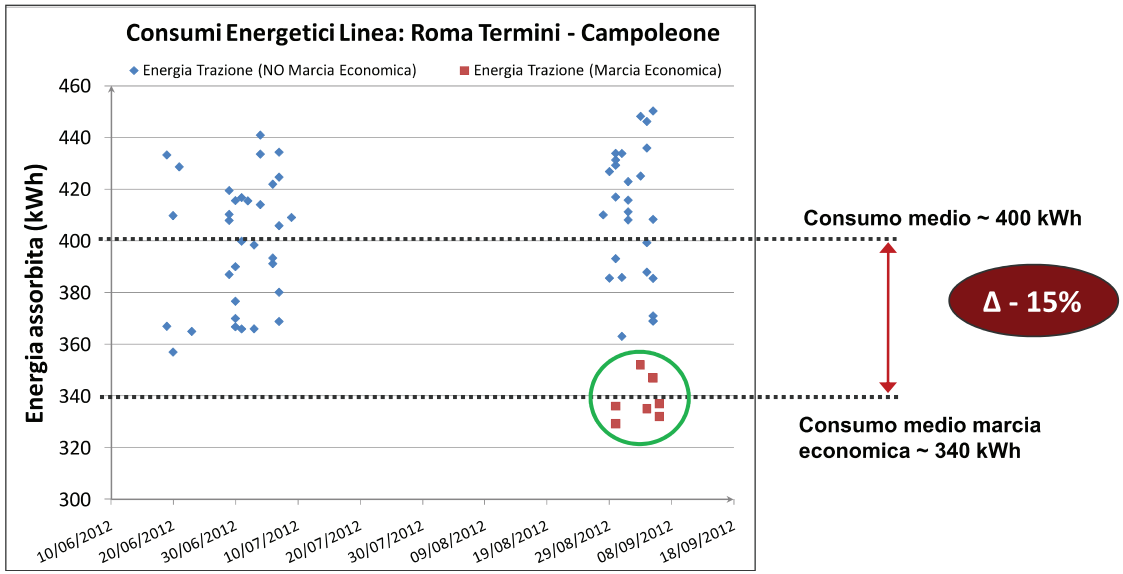
EXPO in Turin, 2014: report presenting measures in the route Rome-Civitavecchia





Impact of the driving style of different drivers on the energy consumption. The difference can reach 50%!

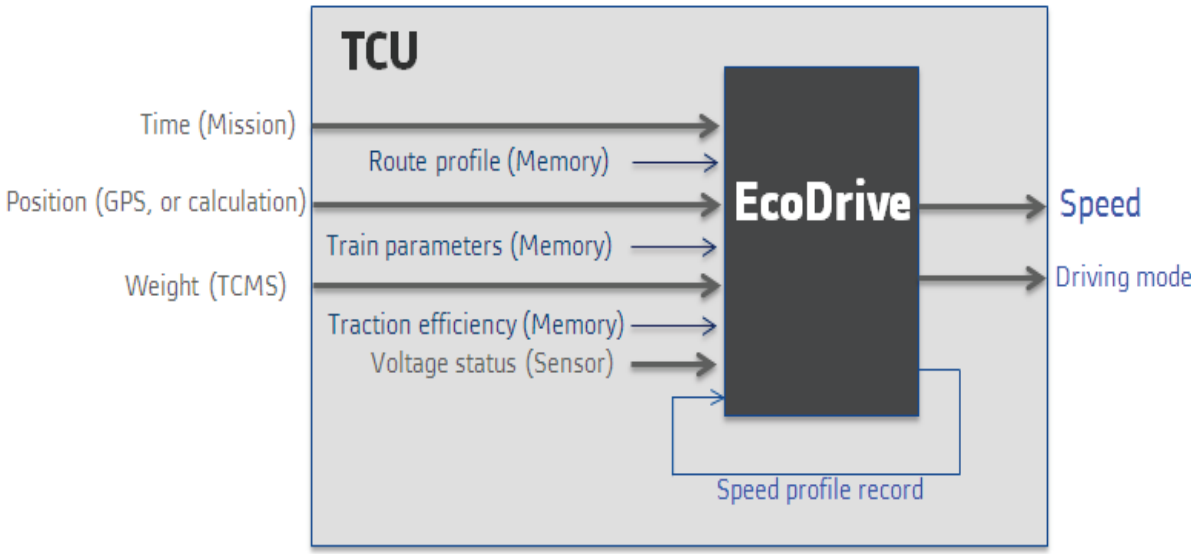
Drive assistance (economy driving)



Average energy reduction of 15%. No "free" driver reached the results obtained with the assistance of an "economy driver"

Ecodrive: speed tuning for energy saving in railway vehicles

Data of the train, Line profile (slopes, speed limits, curves, etc...), Travel time, Position



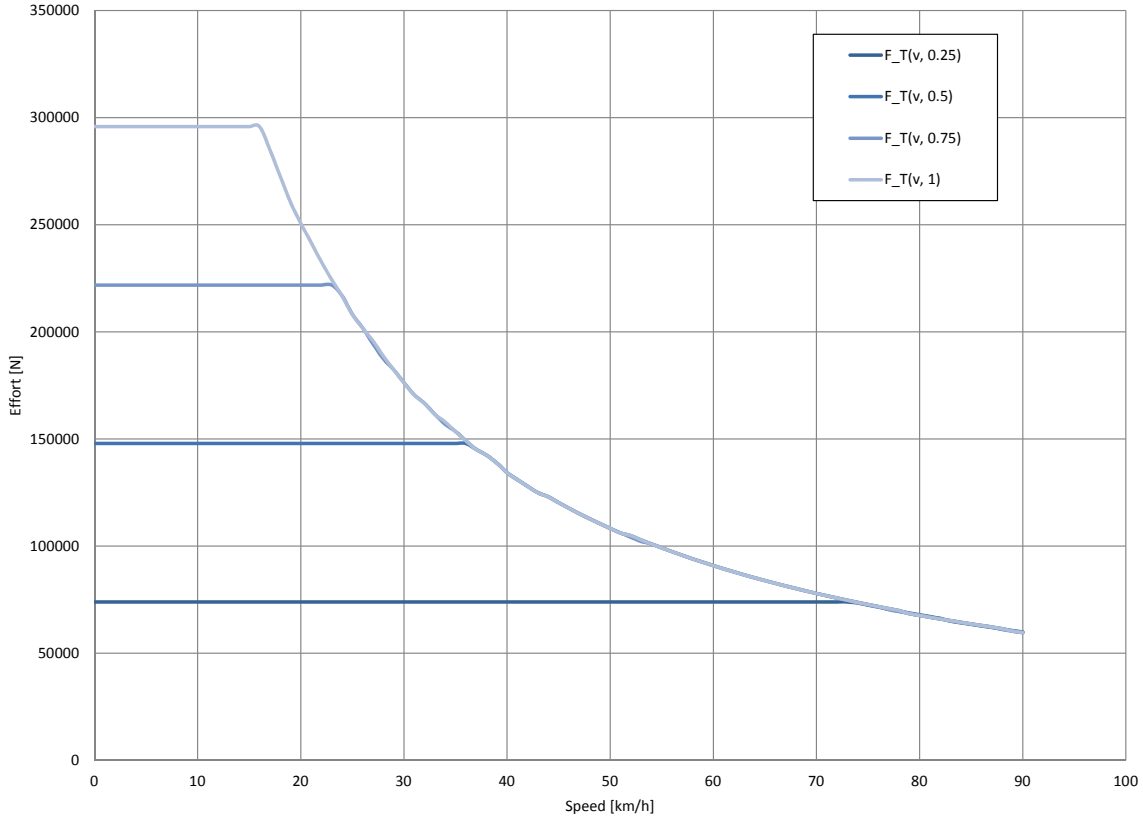
Speed profile (and driving style) optimizing the energy and satisfying the constraints

Dynamic equation

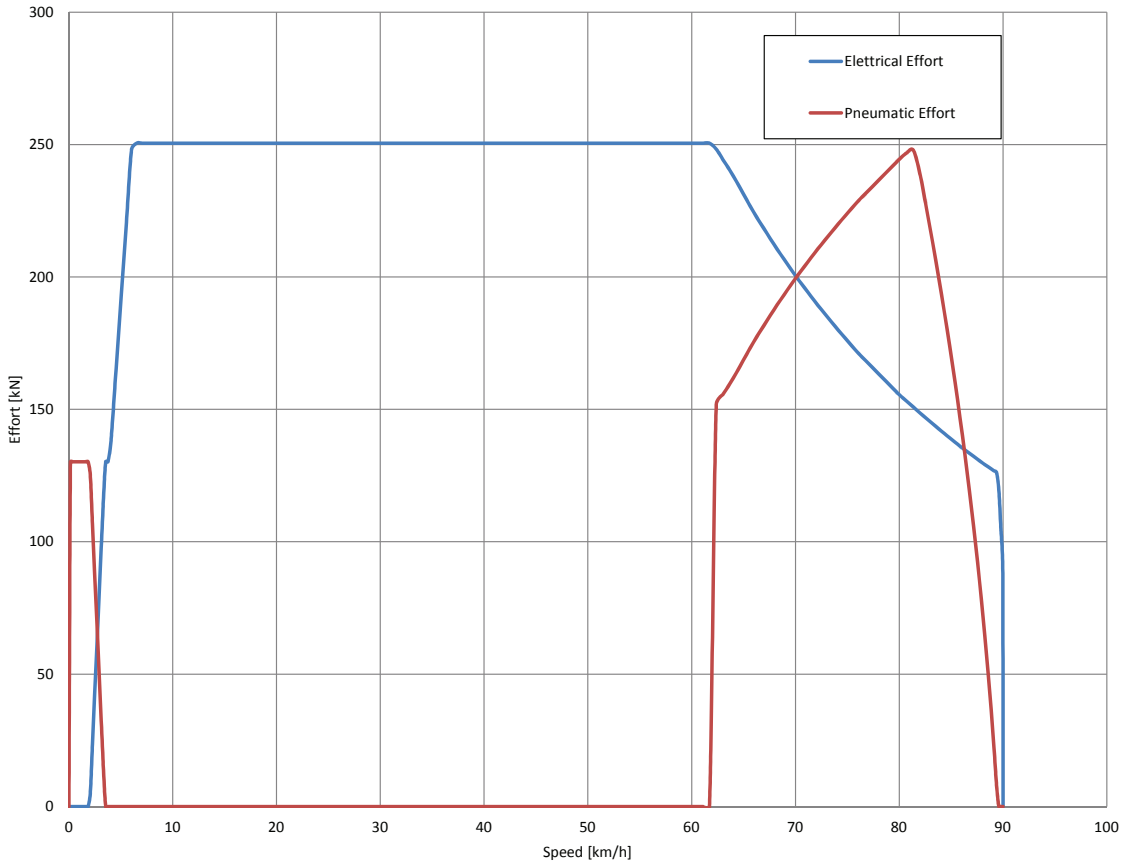
$$
\begin{aligned}
(M_s + M_i)\frac{dv}{ds} &= F_T(v, h(s)) - F_B(v, h(s)) - F_R(v, s) \\
F_R(v, s) &= A + Bv + Cv^2 + M_s g tan(\alpha) + \frac{G}{r_c}
\end{aligned}
$$

where M_s=static mass
M_i=rotating mass
s=position
v=speed
$h(s)$=handle
F_B=braking force
r_c=curve radius
A, B, C= constants for friction: rolling, aerodynamic, damber hysteresis
G=track gauge coefficient

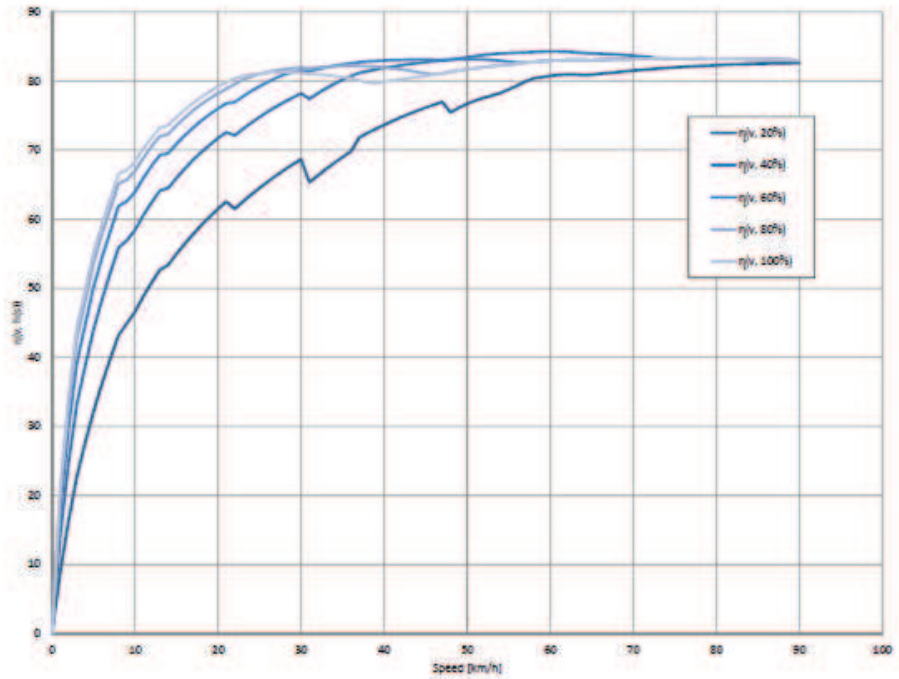
Traction effort at various values of the handle



Braking effort at $h(s) = -1$



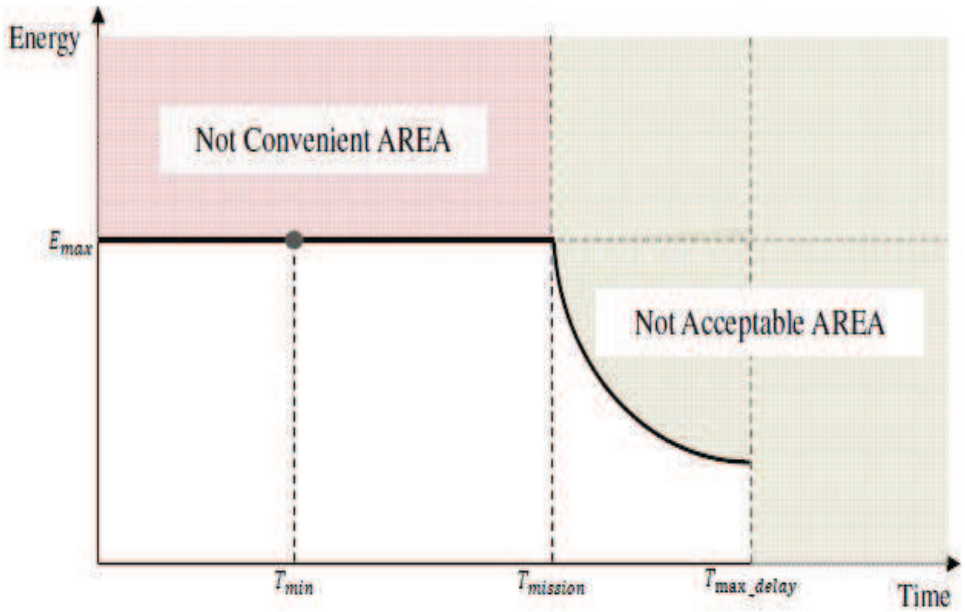
Performance coefficient of the traction chain $\eta(v, h(s))$.



$$E_L = \int_0^S \frac{F_T(v(s), h(s))}{\eta(v(s), h(s))} ds, \quad T = \int_0^S \frac{1}{v(s)} ds < T_{max}, \quad v(s) < v_L(s)$$

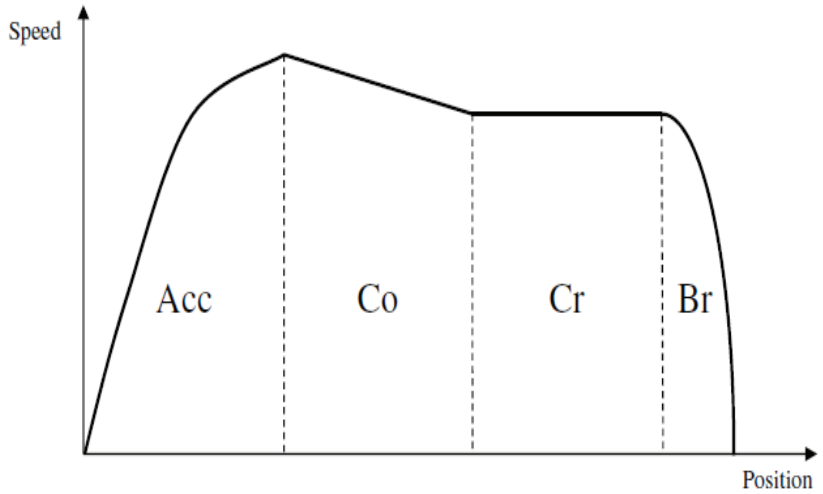
Ecodrive: speed tuning for energy saving in railway vehicles

$$\min E_L,\ h_1(s) \leq v(s) \leq h_2(s),\ -1 \leq h(s) \leq 1,\ E_L, T \in \mathscr{A}$$



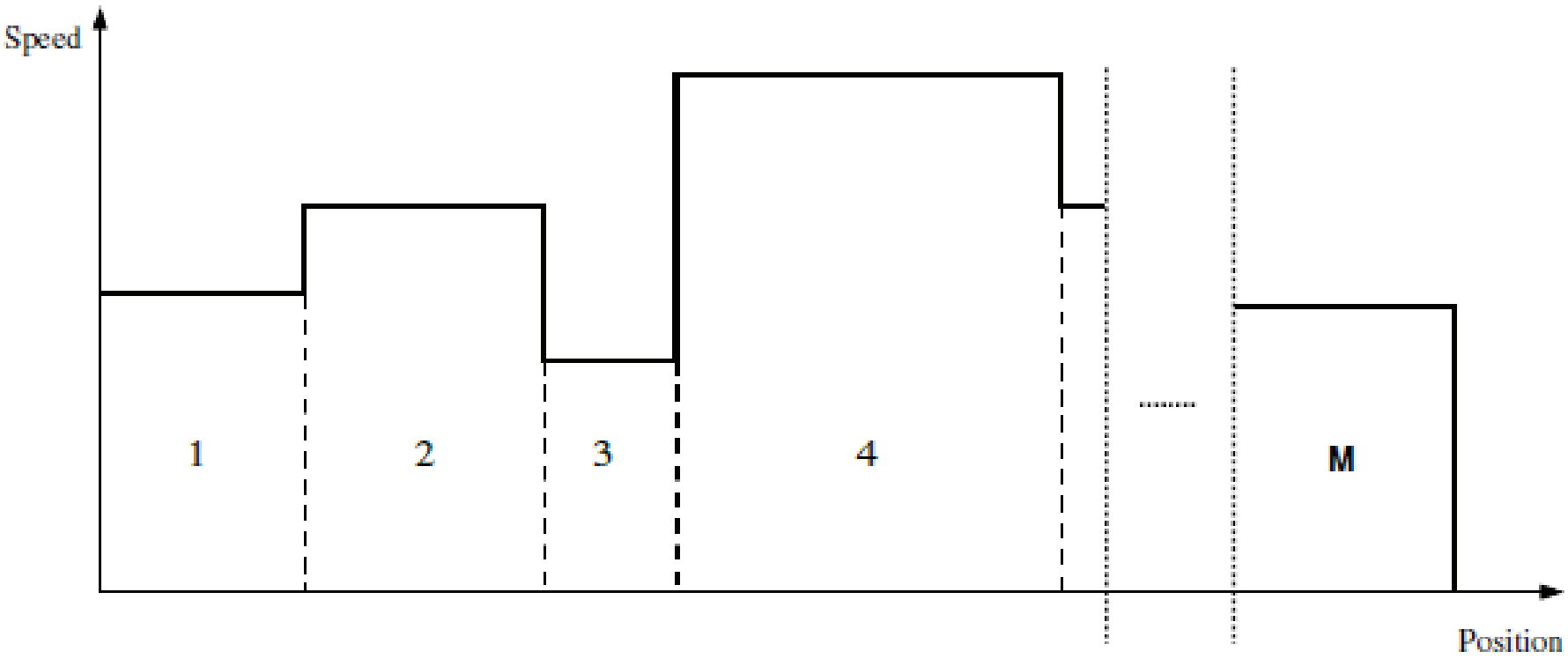
Ecodrive: discretization of the handle

$h(s) = 0,\ h(s) = 0.5,\ h(s) = 0.75,\ h(s) = 1,\ h(s) = -1.$

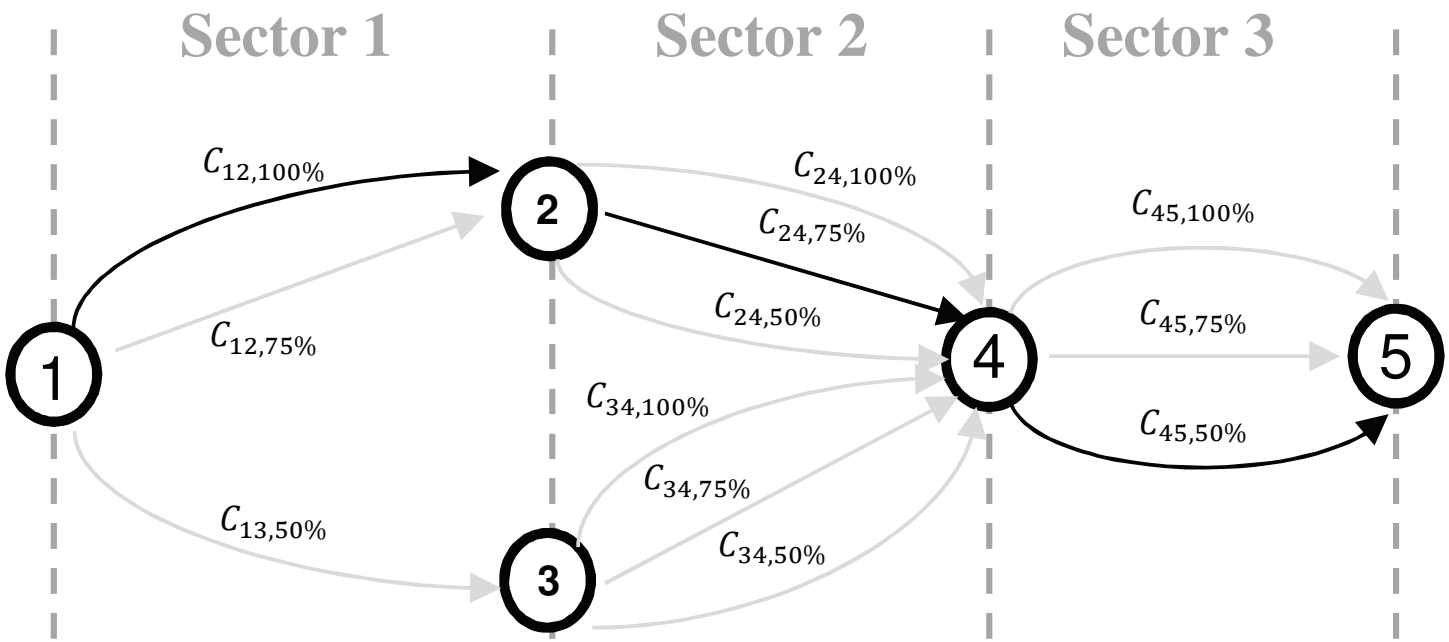


Acceleration $F_T(v, h(s)) > F_R(v, h(s)),\ F_B(v, h(s)) = 0,$
$h(s) \in \{0.5, 0.75, 1\}$
Cruising $F_R(v, h(s)) > 0 \longrightarrow F_T(v, h(s) = F_R(v, h(s)),$
$F_R(v, h(s)) < 0 \longrightarrow,\ F_B(v, h(s)) = F_R(v, h(s))$
Coasting $F_T(v, h(s)) = 0.$
Braking $h(s) = -1, F_T(v, h(s)) = 0.$

Ecodrive: scomposition of the track and piecewise constant speed limits



Optimization on graph



Minimum path with time constraint

Subway, 59.7 Km, 120 minutes, 43 stops, 215 ton, $v_{max} = 90 Km/h$, Line $750 V$

Solution	Duration		Energy	
AllOut	5437		1056	
Cithel	7120	32%	548	-48%
Ecodrive	7101	30.06%	389	-63%

Table 1: Results: Subway

Tram, 28.4 Km, 85 minutes, 53 stops, 43.5 ton, $v_{max} = 70 Km/h$, Line $750 V$

Solution	Duration		Energy	
AllOut	4395		272	
Cithel	5112	16%	188	-30%
Ecodrive	5179	18%	164	-40%

Table 2: Results: Tram

Regional train, 33K m, 30 minutes, 10 stops, 92.7 ton, $v_{max} = 120 Km/h$, Line $3kV$

Solution	Duration		Energy	
AllOut	1730		285	
Cithel	1796	4%	210	-26%
Ecodrive	1800	4%	180	-37%

Table 3: Results: Regional Train

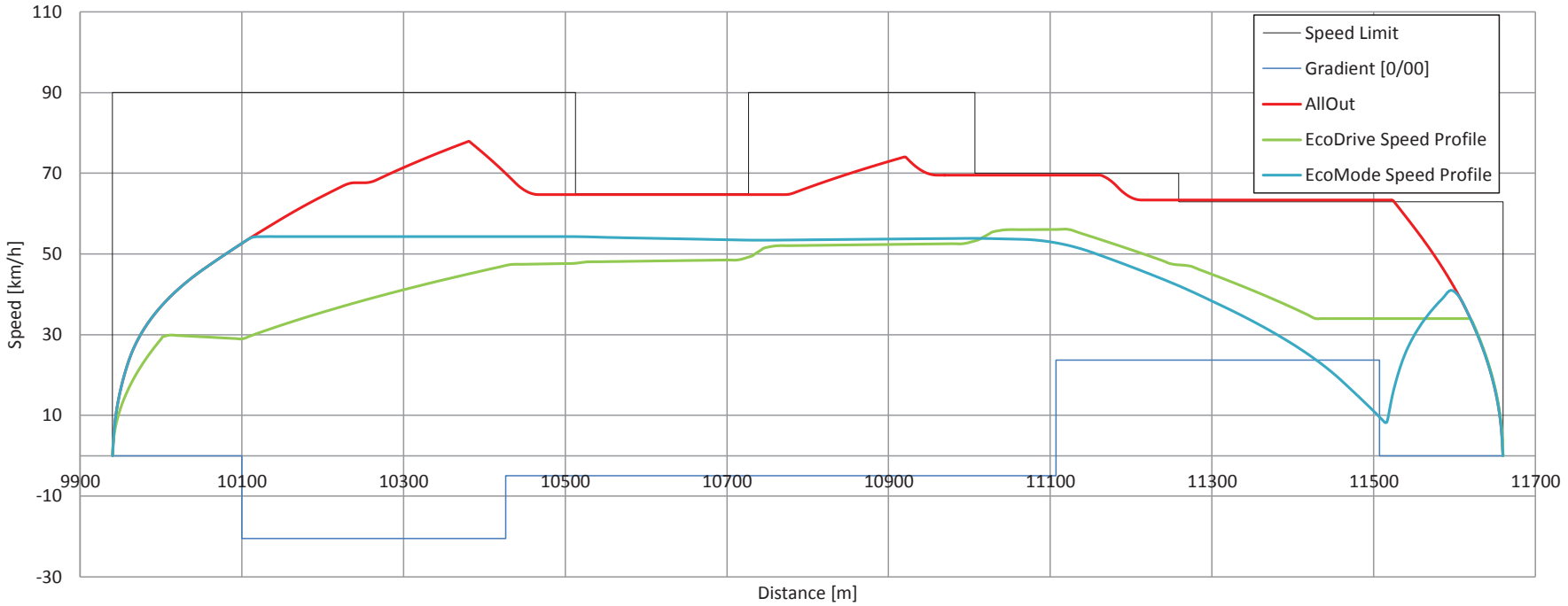


Figure 10: Subway: Comparizon between speed profiles

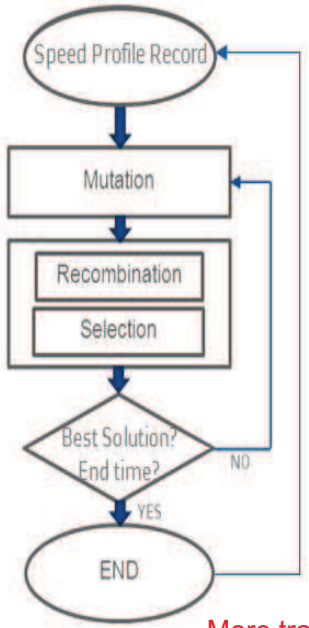
Collaborative ecodrive

Ecodrive

With new mission time received by TMS, Train calculates driving style to consume less as possible

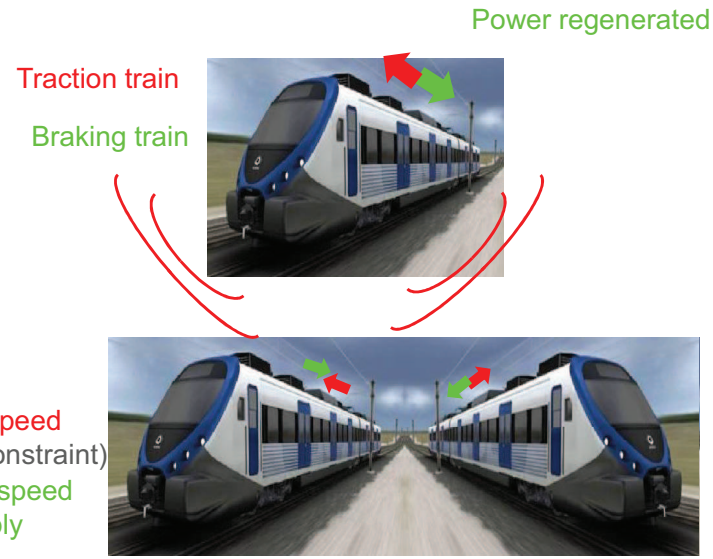
POLITECNICO DI MILANO

ALSTOM Sesto



New degree of freedom:
System Energy not more Train Energy

Trains communicate among themselves to find a best solution at **System level**



More trains increase speed
(according to safety constraint)
More trains decrease speed
to support power supply

Each train receives new target and collaborate with near trains to find best solution

Collaborative EcoDrive

Engineering of sharing

$$\begin{aligned} x_i(t) &= \beta_i x_i(t_k) + \alpha_i (t - t_k) \\ t &\in (t_k,\ t_{k+1}] \\ 0 &\leq \beta_i \leq 1,\ \alpha_i \geq 0 \\ 1 &= \sum_{i=1}^{N} x_i(t_k) \end{aligned}$$

Trigger instants

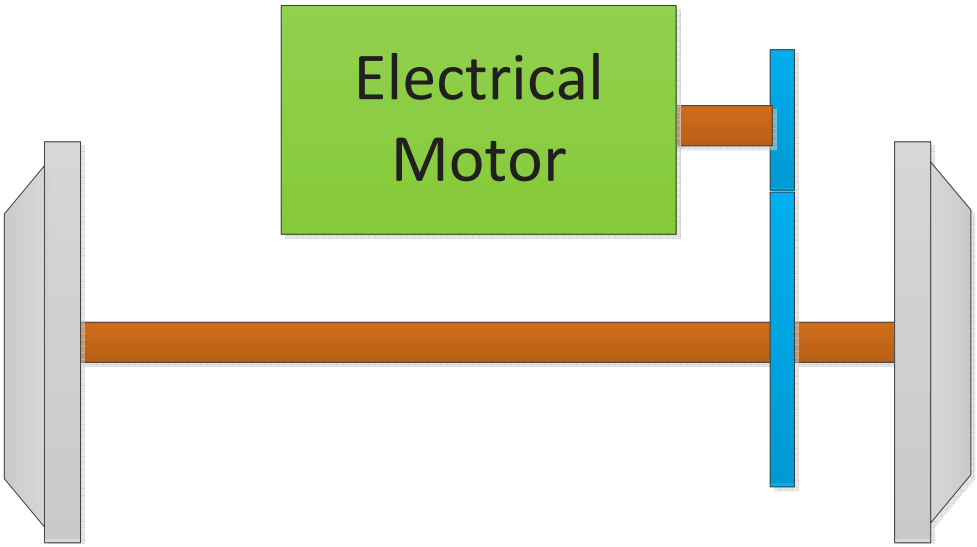
$$t_{k+1} = t_k + \frac{\sum_{i=1}^{N} (1 - \beta_i) x_i(t_k)}{\sum_{i=1}^{N} \alpha_i}$$

Sistema "stocastico" a tempo discreto

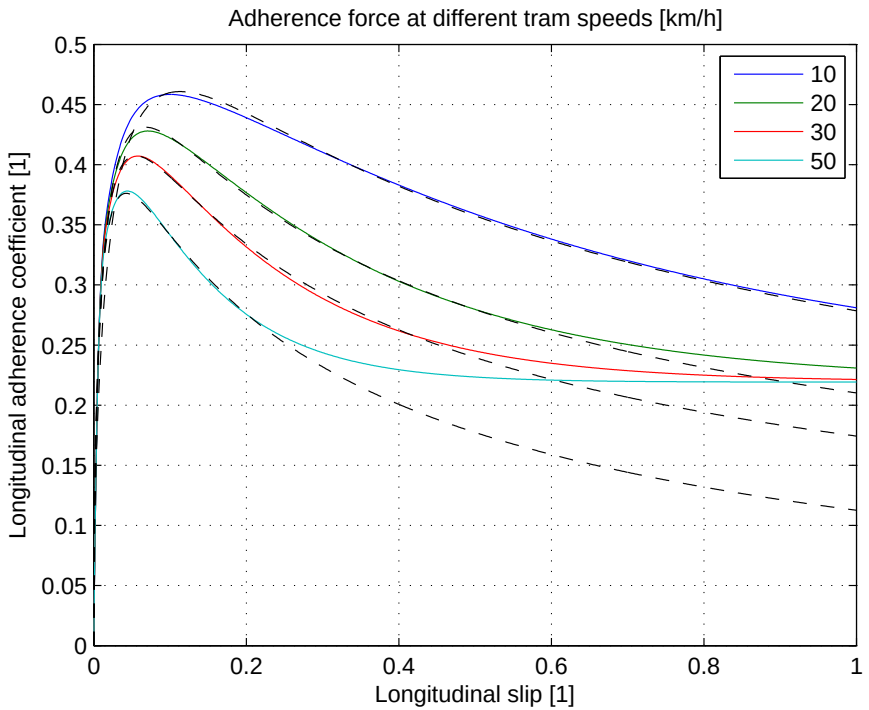
$$x(t_{k+1}) = \left(D(\beta) + (\mathbf{1}^\top \alpha)^{-1} D(\alpha) \mathbf{1}\mathbf{1}^\top D(\mathbf{1} - \beta) \right) x(t_k)$$

Braking

$$J\dot{\omega} = -b_\omega \omega + RF\mu(\sigma) - gT$$
$$M\dot{v} = -b_{v_1} V - b_{v_2} v^2 - F\mu(\sigma)$$
$$\sigma(t) = \frac{v(t) - R\omega(t)}{v(t)}$$



Braking



$$\mu(\sigma,\theta) = \frac{\sqrt{\sigma}}{a + b\sigma + c\sigma^2} = \frac{\sqrt{\sigma}}{\Phi(\sigma)'\theta}, \quad \theta = [a\, b\, c]'$$

Braking

Neglecting the friction forces:

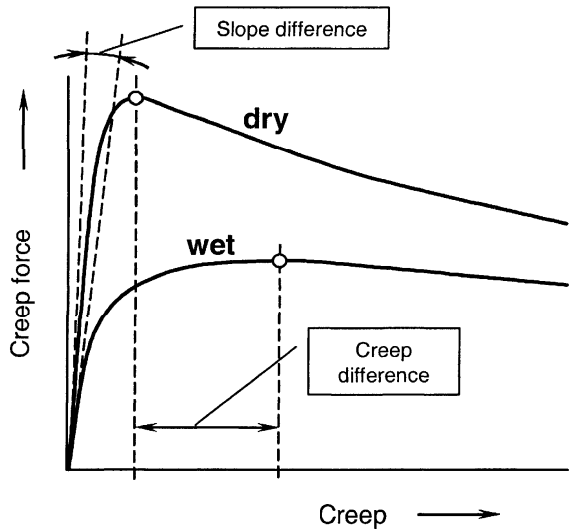
$$\begin{cases} \dot{v}(t) &= -g\mu(\sigma(t),\theta) \\ \dot{\sigma}(t) &= \frac{g}{v}\left[(\sigma(t)-1-v)\mu(\sigma(t),\theta)+\Gamma T(t)\right] \end{cases}$$

- The parameters $v = \frac{MR^2}{J}$ and $\Gamma = \frac{R}{Jg}$ are constant and known.
- T is the control variable. It is a torque and there is a known, constant gear ratio not shown here.
- θ is a vector of unknown parameters.
- In the second equation $\mu(\sigma(t),\theta)$ can be seen as an **unknown matched disturbance**

Braking

Stop the vehicle it in the shortest distance possible.

This is achieved when the slip (or creep) is regulated where the peak of the adherence curve lies.



$\mu(\sigma)$ in dry and wet conditions

Braking

$$z = \hat{\theta} - \theta + \beta(\sigma), \quad [1\ \sigma\ \sigma^2]z \to 0, \quad \hat{\mu} \to \mu$$

$$\zeta = \hat{\alpha} - \theta + \gamma(\dot{v}, \sigma, \delta), \quad [1 - \sigma - 3\sigma^2]\zeta \to 0, \quad \frac{\partial \hat{\mu}}{\partial \sigma} \to \frac{\partial \mu}{\partial \sigma}$$

$$T = -\frac{1}{\Gamma}(\sigma - 1 - v)(\hat{\mu} + \frac{v}{g}\rho)$$

$$\rho = -k\delta\sigma(\sigma - 1) + n, \quad n \to 0$$

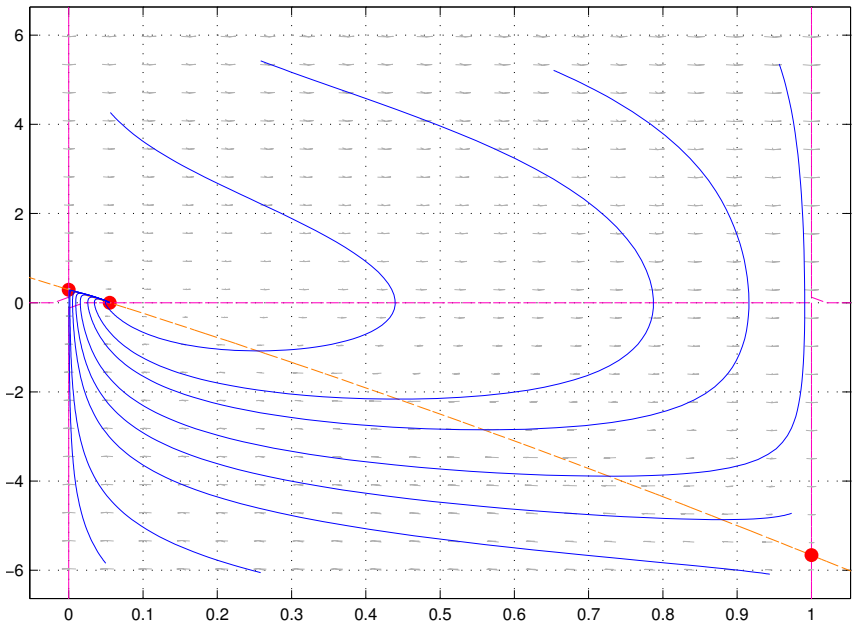
The proof relies on the LF

$$V = k\int_0^{\sigma^\star} \left(\frac{\theta_1 - r\theta_2 - 3r^2\theta_3}{r(1-r)}\right) dr + \delta^2 + z'z + \zeta'\zeta$$

Braking

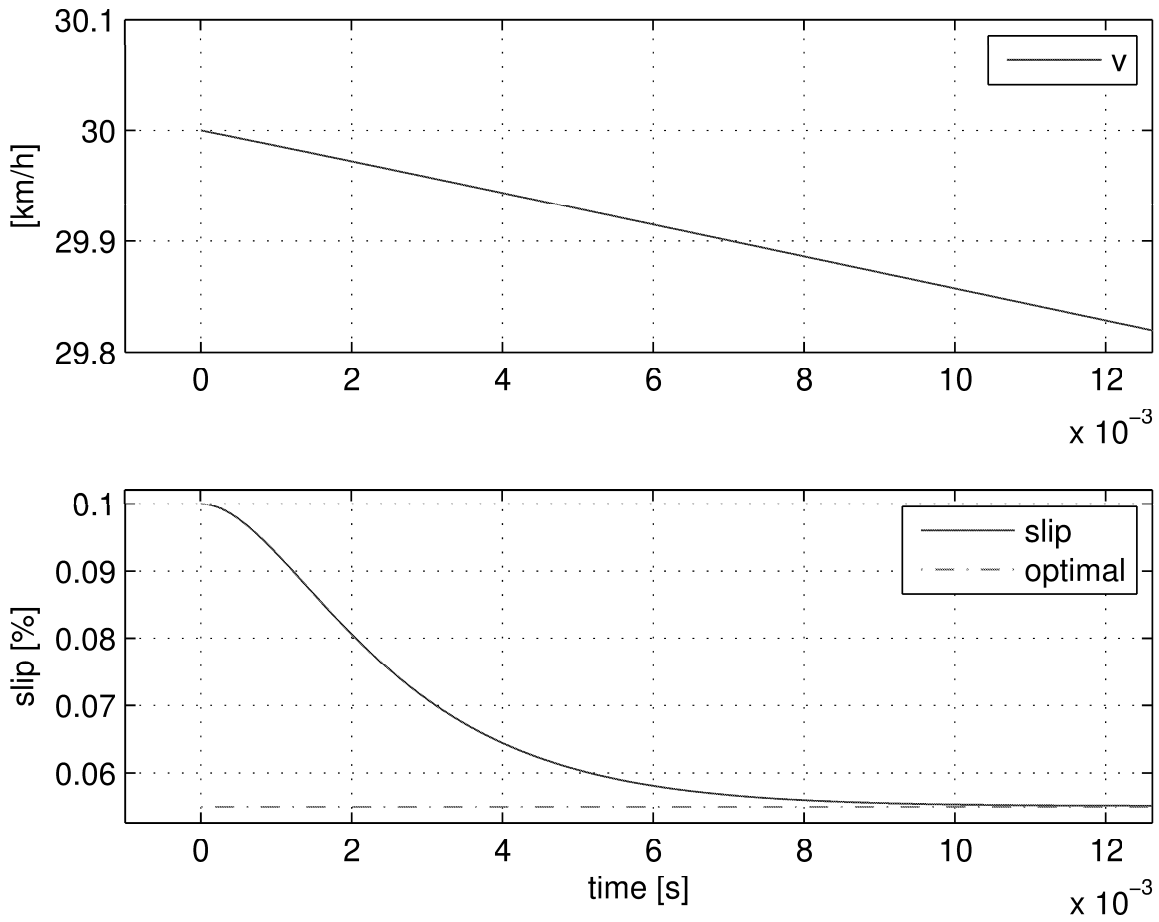
The overall system is then:

$$\begin{aligned} \dot{\sigma} &= -k_\sigma \delta \sigma(\sigma - 1) \\ \dot{\delta} &= k_{d1} \Phi(\sigma)' H[\hat{\alpha} + \gamma(\dot{v}, \sigma, \delta)] - k_{d2}\delta \end{aligned} \tag{1}$$



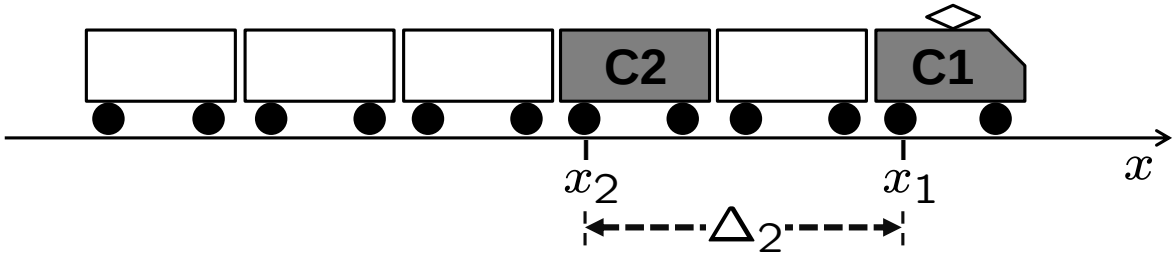
Phase portrait

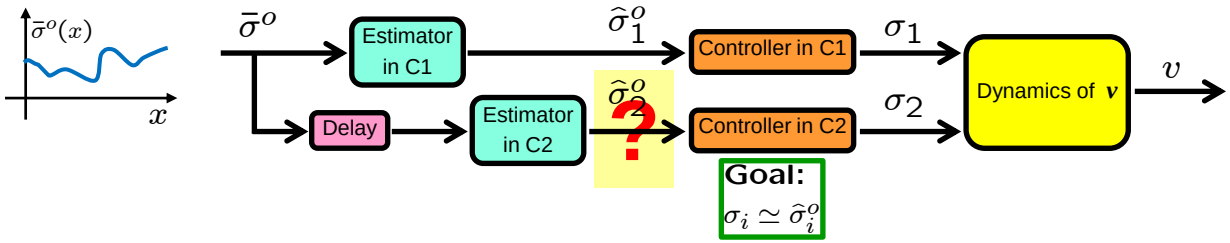
Braking



Alstom/Politecnico patent (Nonlinear filter replaced by adaptive Kalman filter)

Distributed braking

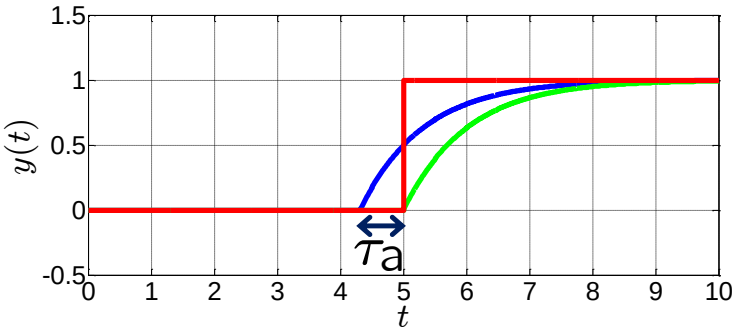


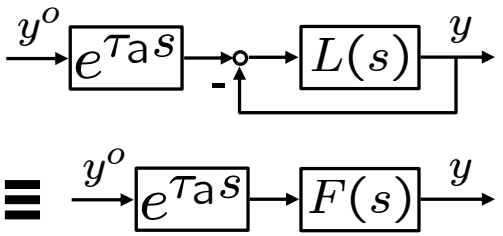


- LF: the second coach mimics what the first coach is doing
- BD: the second coach ignores the first one and tracks its own optimum
- BP: the second coach takes advantage of information available from coach 1 so as to anticipate its actions and better track its own optimum

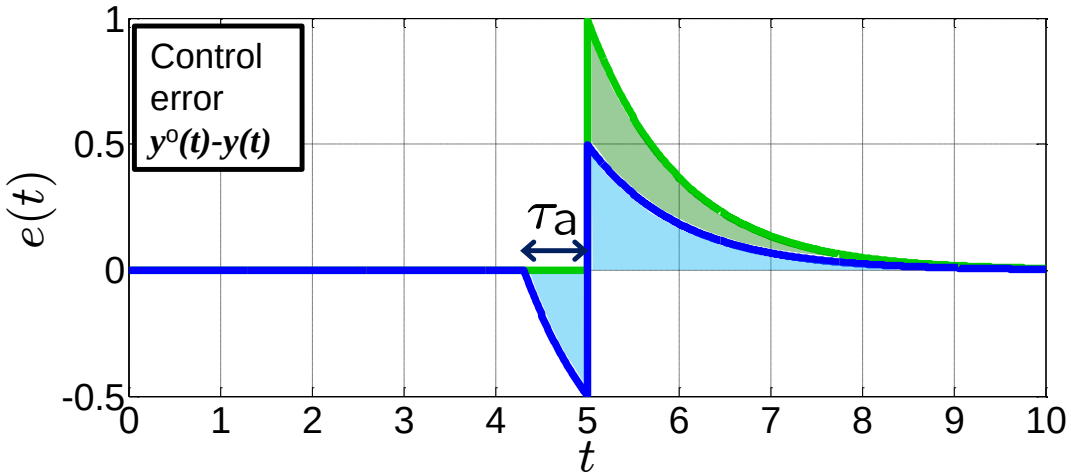
Distributed braking

Is there an advantage with preview? (YES) Which is the best τ_a ?





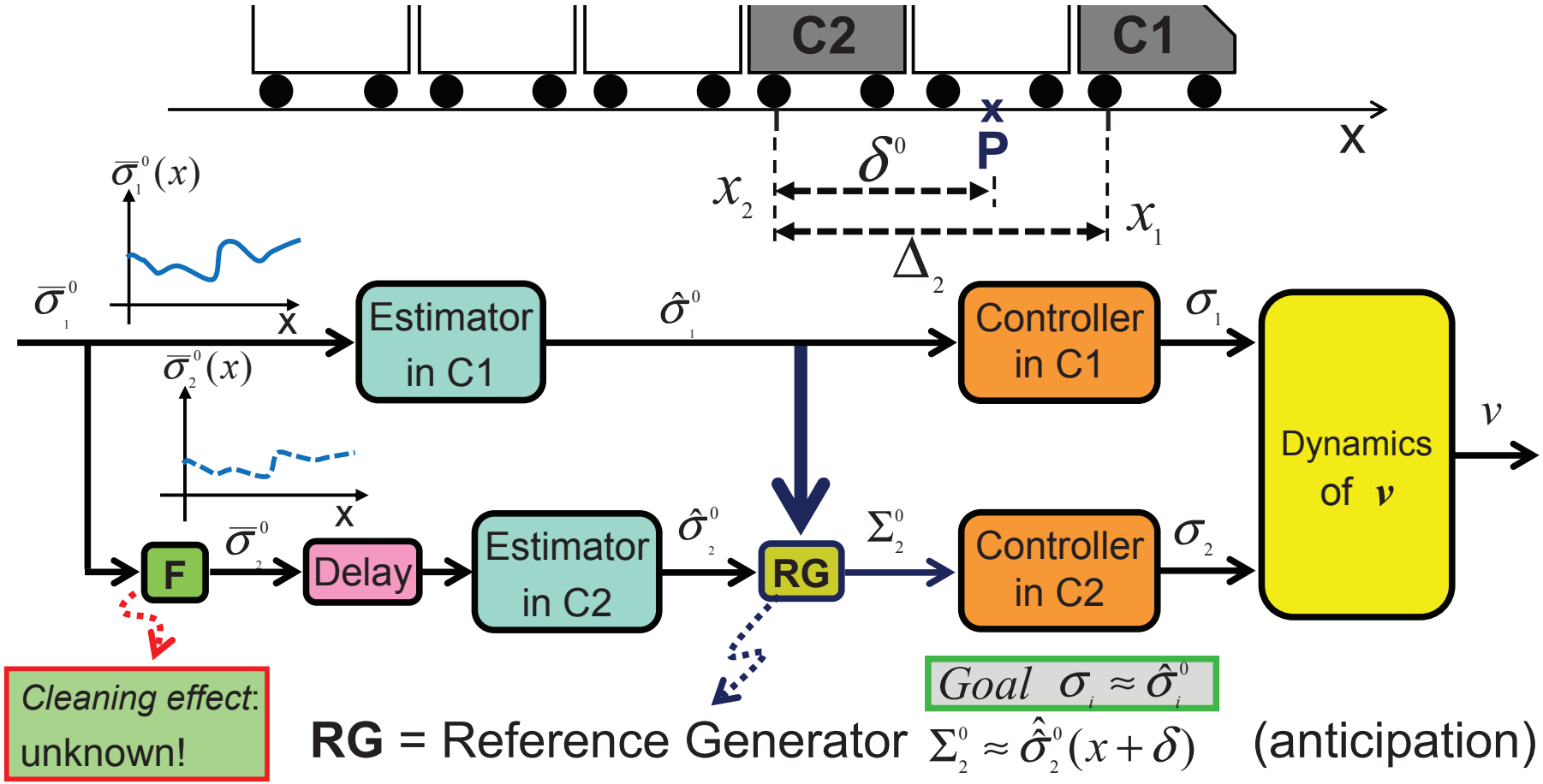
where $F(s) = \frac{\alpha}{s+\alpha}$



$$\|e\|_2^2(\tau_a) = \int_0^{+\infty} e^2(t)dt = \tau_a + \frac{2}{\alpha}e^{-\alpha\tau_a} - \frac{3}{2\alpha}$$

$$\tau_a^o = \frac{log(2)}{\alpha}$$

Distributed braking

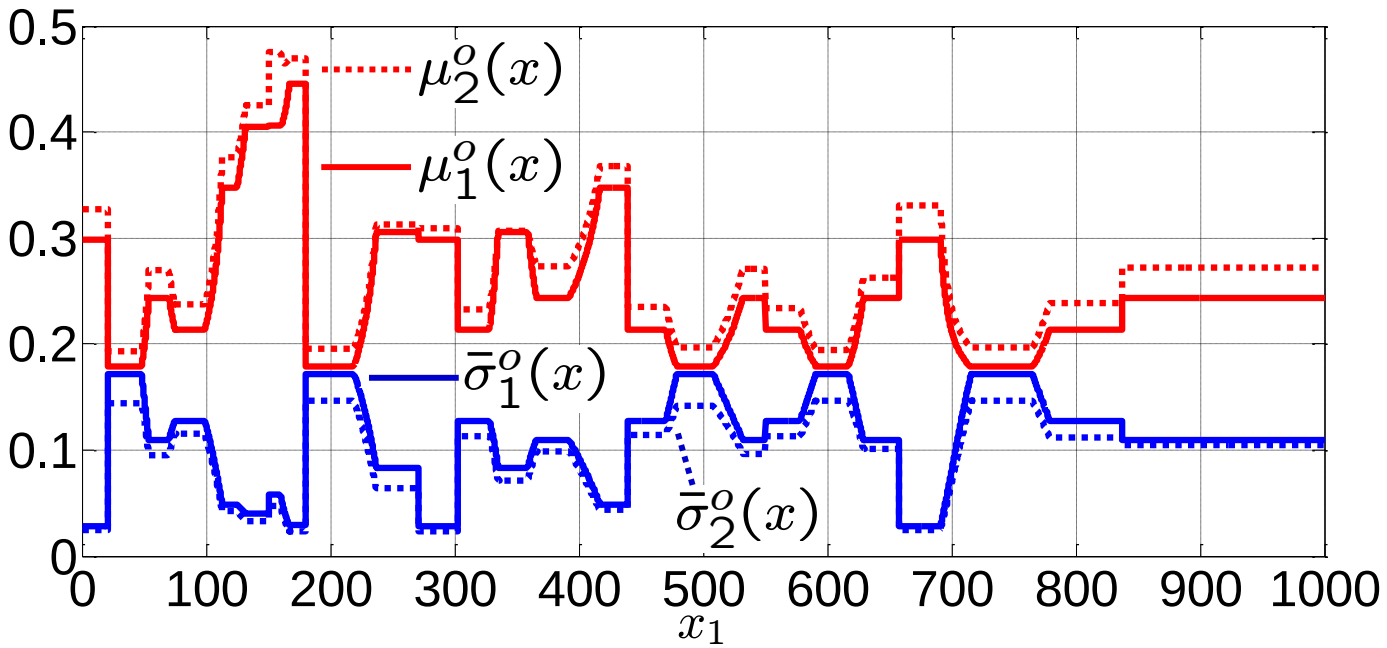


Distributed braking

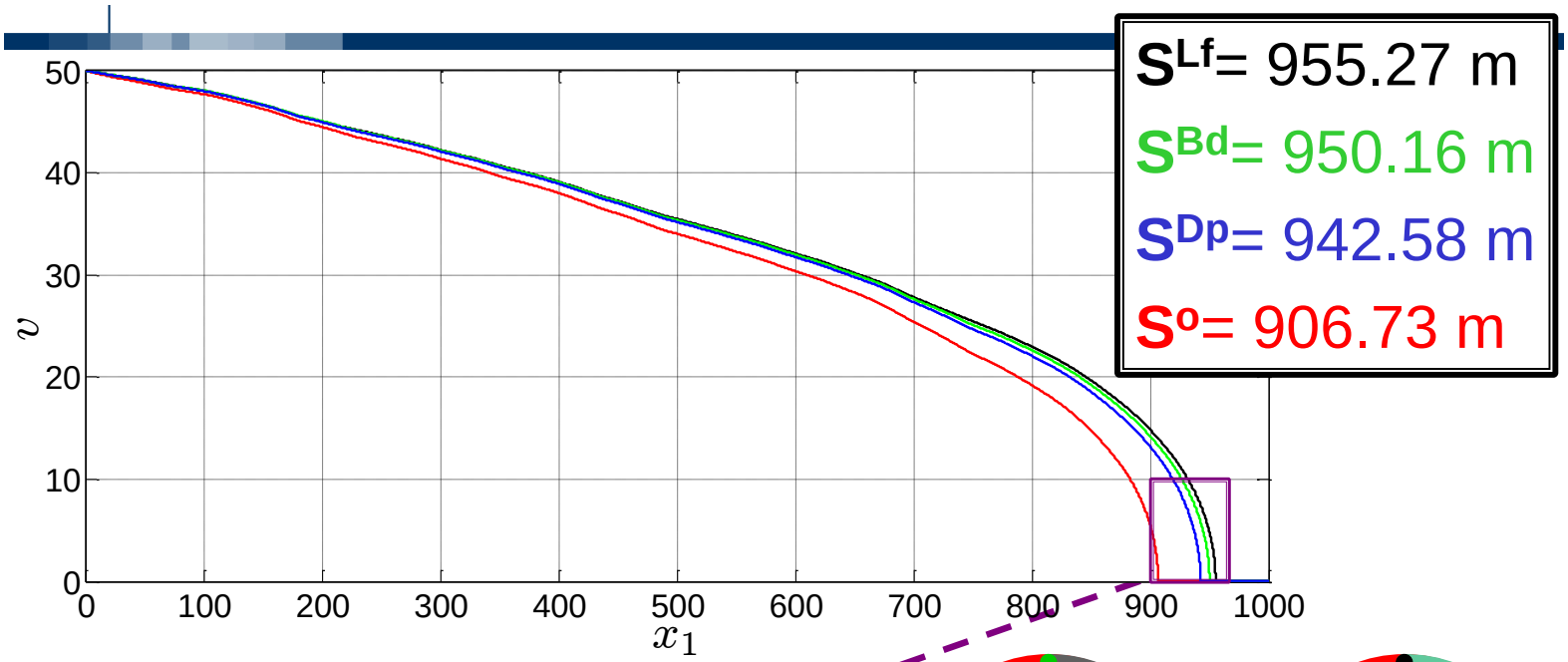
$\Delta_2 = 50$ m

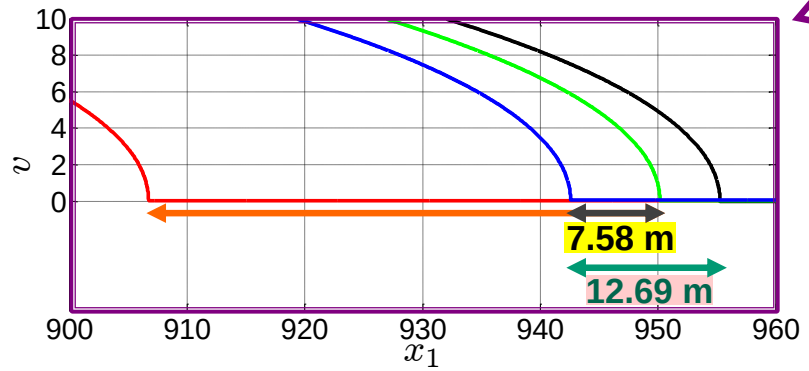
$v(0) = 50$ m/s $\equiv 180$ km/h

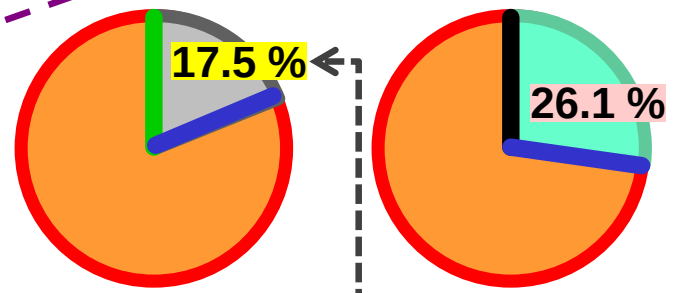
$\left.\begin{array}{l}\lambda = 10 \\ \alpha = 1\end{array}\right\}$ (estimation is faster than control)



Distributed braking







Thanks to communication and preview !

Conclusions

Giuseppe Evangelisti (1903-1981)

IFAC
INTERNATIONAL FEDERATION
OF AUTOMATIC CONTROL



Processo (fisico, chimico, meccanico, sociale), Descrizione (equazioni differenziali - derivate ordinarie o parziali - alle differenze, ibride, grafo), Variabili (logiche, "semantiche", continue, discrete, vincoli), Proprietà (equilibrio, stabilità, robustezza), filtraggio (stima parametri e variabili, adattativo, identificazione, learning), Controllo (robusto, adattativo, iterativo, ottimizzazione).

Vibration reduction in helicopters, Semiactive damping control for car suspensions, Disturbance rejection in Diesel engine test bench, Stabilization of high voltage transmission network in emergency conditions, Mold level control of a continuous casting plant, Virus load mitigation in HIV, Traffic light scheduling