University of Udine
Department of Mathematics and Computer Science

Expressiveness, decidability, and undecidability of Interval Temporal Logic

ITL - Beyond the end of the light

Ph.D. Defence

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At the beginning...

At the beginning, it was the darkness...
Then, logicians made the light,
they became curious,
and moved toward the darkness...
... as close as they could

Outline

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Interval-based temporal reasoning: origins and applications

Interval-based temporal reasoning: reasoning about time, where the primary concept is 'time interval', rather than 'time instant'.

Origins:

- Philosophy, in particular philosophy and ontology of time.
- Linguistics: analysis of progressive tenses, semantics of natural languages.
- Artificial intelligence: temporal knowledge representation, temporal planning, theory of events, etc.
- Computer science: specification and design of hardware components, concurrent real-time processes, temporal databases, etc.

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The truth of a formula over an interval does not necessarily depend on its truth over subintervals.

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New issues arise regarding the nature of the intervals:

- Can intervals be unbounded?
- Are intervals with coinciding endpoints admissible or not?

Intervals and interval structures

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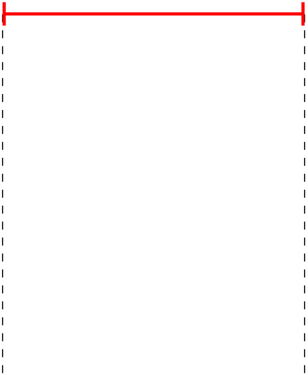
In particular, standard interval structures on $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$, and $\mathbb{R}$ with their usual orders.

Binary interval relations on linear orders



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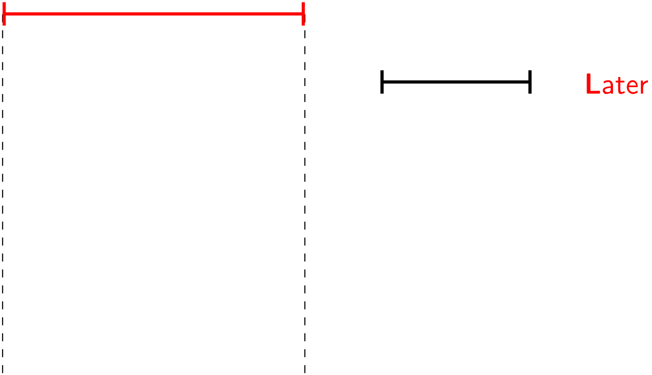

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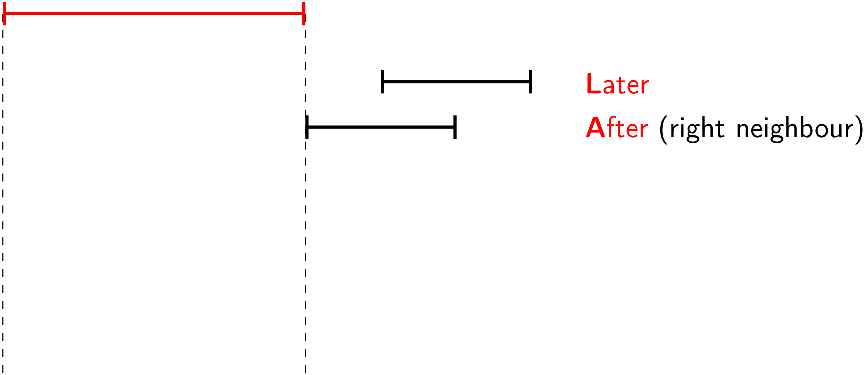

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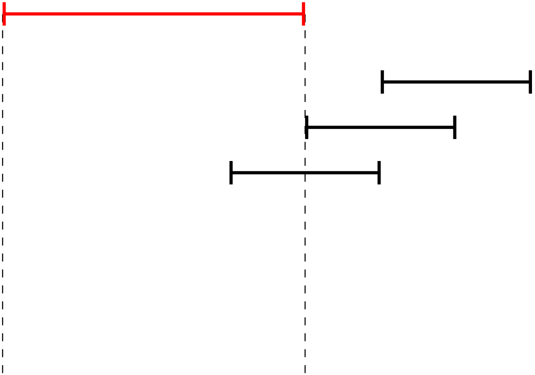

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After (right neighbour)

Overlaps (to right)

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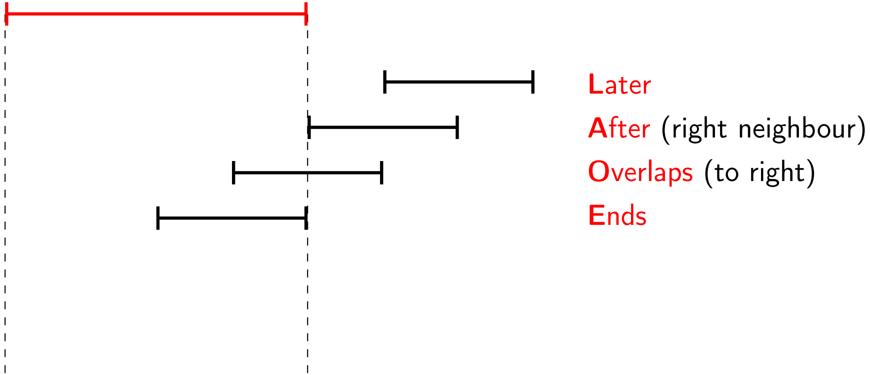

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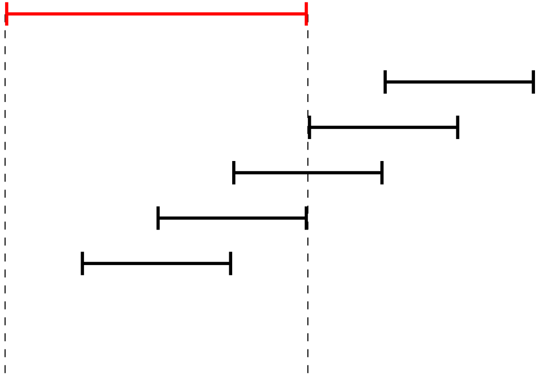

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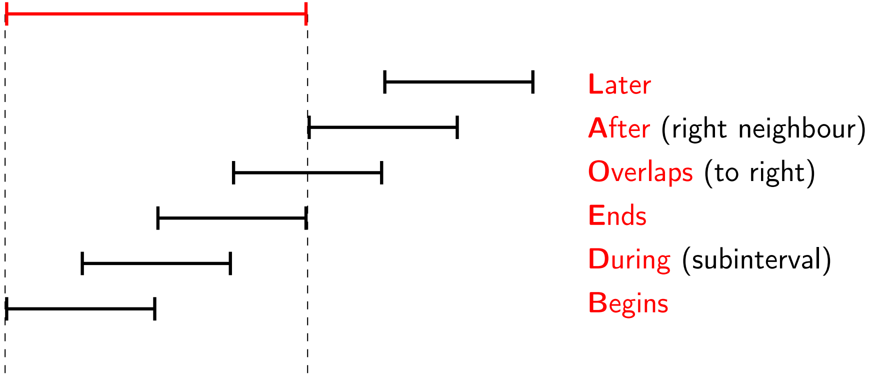

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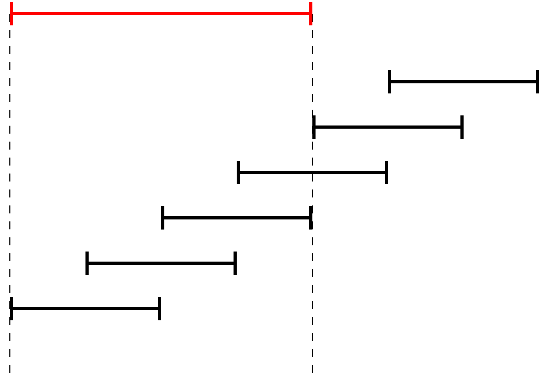

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6 relations + their inverses + equality = 13 Allen's relations.

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non-strict semantics:

All modalities are definable in terms of $\langle \mathrm{B} \rangle, \langle \mathrm{E} \rangle, \langle \overline{\mathrm{B}} \rangle, \langle \overline{\mathrm{E}} \rangle$

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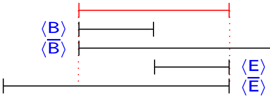

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Also needed additional modalities $\langle A \rangle, \langle \overline{A} \rangle$

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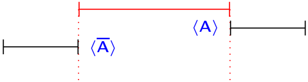

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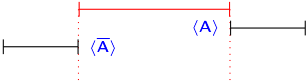

Syntax of Halpern-Shoham's logic, hereafter called HS :

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \psi \mid \langle \mathrm{B} \rangle\phi \mid \langle \mathrm{E} \rangle\phi \mid \langle \overline{\mathrm{B}} \rangle\phi \mid \langle \overline{\mathrm{E}} \rangle\phi \; (\mid \langle \mathrm{A} \rangle\phi \mid \langle \overline{\mathrm{A}} \rangle\phi) \; .$$

Models for propositional interval logics

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Strict interval model:

$$\mathbf{M}^- = \langle \mathbb{I}(\mathbb{D})^-, V \rangle,$$

where $V : \mathcal{AP} \mapsto 2^{\mathbb{I}(\mathbb{D})^-}$.

Formal semantics of HS

$\langle B \rangle$: $\mathbf{M}, [d_0, d_1] \Vdash \langle B \rangle \phi$ iff there exists d_2 such that $d_0 \leq d_2 < d_1$ and $\mathbf{M}, [d_0, d_2] \Vdash \phi$.

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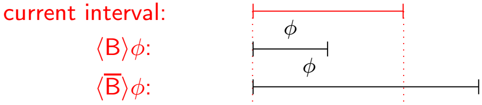

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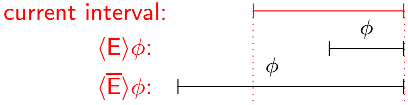

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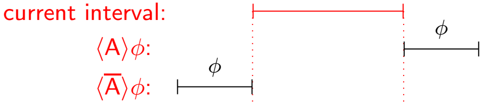

Formal semantics of HS - contd'

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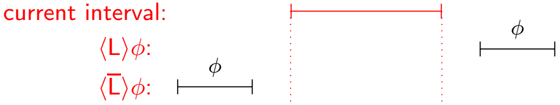

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current interval:

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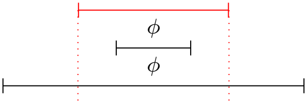

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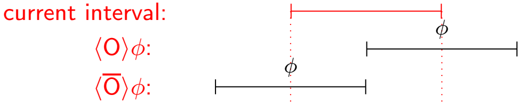

Defining the other interval modalities in HS

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In general, it is possible defining HS modalities in terms of others

The zoo of fragments of HS

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Each of these, considered with respect to some parameters:

1. over special classes of interval structures (all, dense, discrete, finite, etc.)
2. with strict or non-strict semantics
3. including or excluding π operator (whenever it cannot be defined)

Outline

Comparing the expressiveness of fragments of HS

Expressiveness classification problem: classify the fragments of HS with respect to their expressiveness, relative to important classes of interval models.

The problem of comparing expressive power of HS fragments

L_1, L_2 HS-fragments

L_1 L_2

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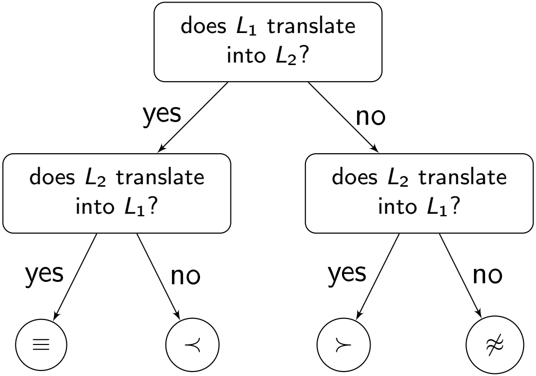

Truth-preserving translation

There exists a truth-preserving translation of L_1 into L_2

iff

L_2 is at least as expressive as L_1

$(L_1 \preceq L_2)$

Truth-preserving translation

There exists a truth-preserving translation of L_1 into L_2
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L_2 is at least as expressive as L_1
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2^{12} fragments... $\frac{2^{12} \cdot (2^{12}-1)}{2}$ comparisons

Inter-definability equations

Notation: $\mathrm{X_1X_2 \ldots X_n}$ will denote the fragment of HS containing the modalities $\langle X_1 \rangle, \langle X_2 \rangle, \ldots, \langle X_n \rangle$

$$\langle L \rangle p \equiv \langle A \rangle \langle A \rangle p$$
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Bisimulation between interval structures

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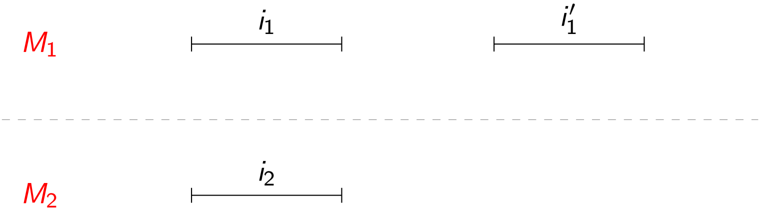

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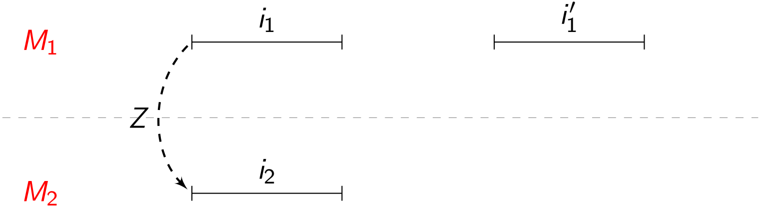

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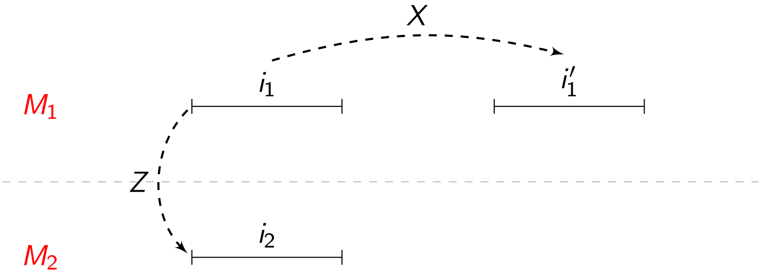

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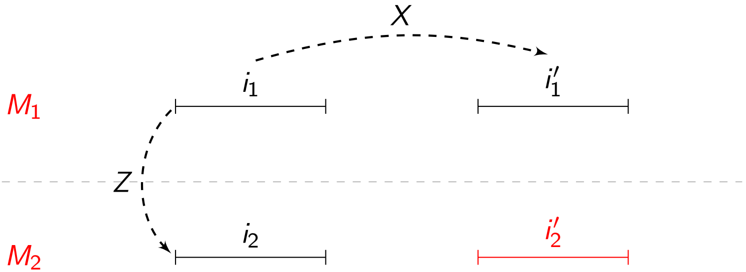

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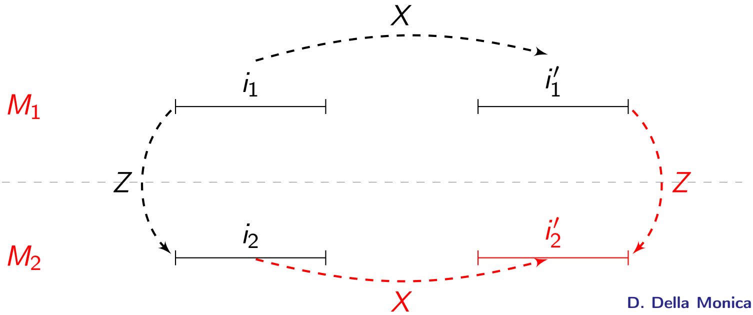

Bisimulation between interval structures - cont'd

Theorem Let Z be a bisimulation between M_1 and M_2 for the language $\mathcal{L}$ and let i_1 and i_2 be intervals in M_1 and M_2, respectively. Then, truth of $\mathcal{L}$-formulae is preserved by Z, i.e.,

If $(i_1, i_2) \in Z$, then for every formula φ of $\mathcal{L}$:

$$M_1, i_1 \Vdash \varphi \text{ iff } M_2, i_2 \Vdash \varphi$$

How to use bisimulations to disprove definability

Suppose that we want to prove:

$\langle X \rangle$ is not definable in terms of $\mathcal{L}$

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By contradiction

If $\langle X \rangle$ is definable in terms of $\mathcal{L}$, then $\langle X \rangle p$ is.

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Truth of $\langle X \rangle p$ should have been preserved by Z, but $\langle X \rangle p$ is true in i_1 (in M_1) and false in i_2 (in M_2)

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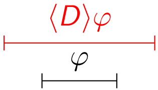
An example: the operator $\langle D \rangle$

Semantics:

$$M, [a, b] \Vdash \langle D \rangle \varphi \overset{def}{\Leftrightarrow} \exists c, d \text{ such that } a < c < d < b \text{ and } M, [c, d] \Vdash \varphi$$



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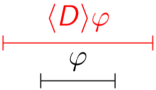

Operator $\langle D \rangle$ is definable in terms of BE $\qquad \langle D \rangle \varphi \equiv \langle B \rangle \langle E \rangle \varphi$

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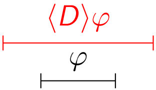


Operator $\langle D \rangle$ is definable in terms of BE $\quad \langle D \rangle \varphi \equiv \langle B \rangle \langle E \rangle \varphi$

To prove that $\langle D \rangle$ is not definable in terms of any other fragment, we must prove that:

1) $\langle D \rangle$ is not definable in terms of ALBO$\overline{\text{ALBEDO}}$

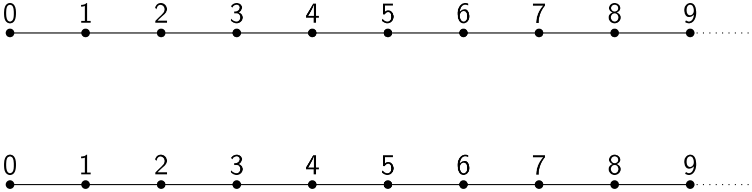
2) $\langle D \rangle$ is not definable in terms of ALEO$\overline{\text{ALBEDO}}$

$\langle D \rangle$ is not definable in terms of A

A bisimulation wrt fragment A but not D

Bisimulation wrt A ($\mathcal{AP} = \{p\}$):

- models: $M_1 = \langle \mathbb{I}(\mathbb{N}), V_1 \rangle, M_2 = \langle \mathbb{I}(\mathbb{N}), V_2 \rangle$



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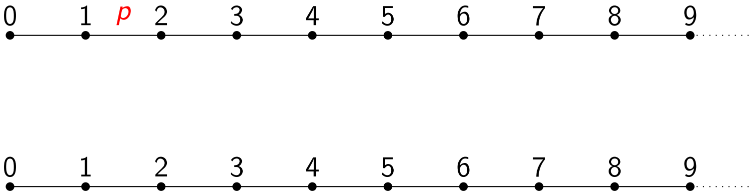

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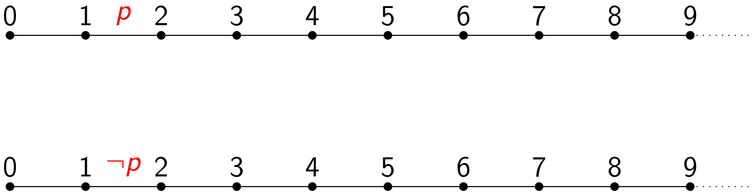

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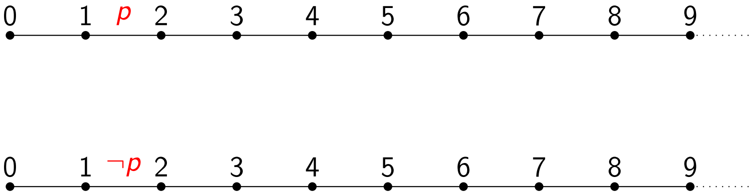

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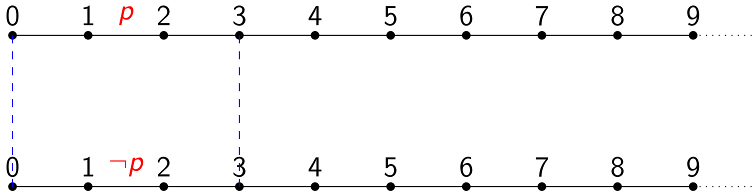

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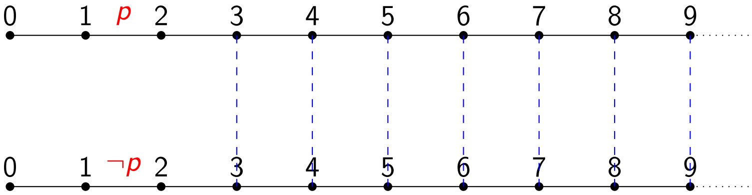

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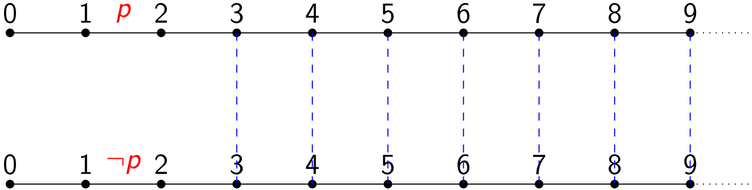

$\boxed{M_1, [0, 3] \Vdash \langle D \rangle p \text{ and } M_2, [0, 3] \Vdash \neg \langle D \rangle p}$

$\langle D \rangle$ is not definable in terms of A

A bisimulation wrt fragment A but not D

Bisimulation wrt A ($\mathcal{AP} = \{p\}$):

- models: $M_1 = \langle \mathbb{I}(\mathbb{N}), V_1 \rangle, M_2 = \langle \mathbb{I}(\mathbb{N}), V_2 \rangle$
  - $V_1(p) = \{[1,2]\}$
  - $V_2(p) = \emptyset$
- bisimulation relation Z: $([x,y],[w,z]) \in Z$ iff
  1. $[x,y] = [w,z] = [0,3]$
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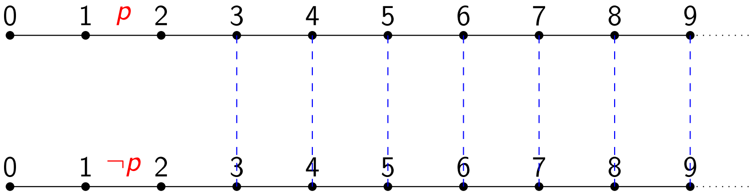

$\boxed{M_1, [0,3] \Vdash \langle D \rangle p \text{ and } M_2, [0,3] \Vdash \neg \langle D \rangle p}$ $\Rightarrow$ the thesis

Outline

The satisfiability problem for HS

Satisfiability problem for a logic $\mathcal{L}$ Given an $\mathcal{L}$-formula φ, is φ satisfiable, i.e., there exists a model and an interval in which φ is true?

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Classification of all HS fragments wrt (un)decidability

Maximal decidable HS fragments

- PNL ($\equiv \mathsf{A}\overline{\mathsf{A}}$) in general case

D. Bresolin, V. Goranko, A. Montanari, G. Sciavicco

Propositional Interval Neighborhood Logic: Decidability, Expressiveness, and Undecidable Extensions.

Annals of Pure and Applied Logics, 2009, 161, 289-304.

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D. Bresolin, A. Montanari, P. Sala, G. Sciavicco

What's decidable about Halpern and Shoham's interval logic? The maximal fragment ABBL.

LICS 2011, 2011.

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A. Montanari, G. Puppis, P. Sala

Maximal Decidable Fragments of Halpern and Shoham's Modal Logic of Intervals.

ICALP 2010, 2010, LNCS 6199, 2010, 345-356.

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- $\mathsf{D}\overline{\mathsf{D}}\mathsf{B}\overline{\mathsf{B}}\mathsf{L}\overline{\mathsf{L}}$ over $\mathbb{Q}$

P. Sala

PhD thesis

2010

Weakest undecidable HS fragments

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D (and $\overline{\mathsf{D}}$) over discrete [Michaliszyn, Marcinkowski]
The Ultimate Undecidability Result for the Halpern-Shoham Logic
LICS 2011

Outline

The Octant Tiling Problem

This is the problem of establishing whether a given finite set of tile types $\mathcal{T} = \{t_1, \ldots, t_k\}$ can tile the 2nd octant of the integer plane:

$$\mathcal{O} = \{(i, j) : i, j \in \mathbb{N} \wedge 0 \leq i \leq j\},$$

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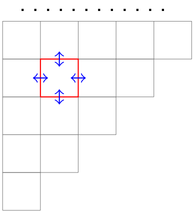

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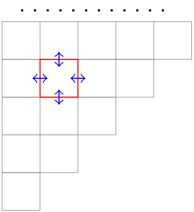

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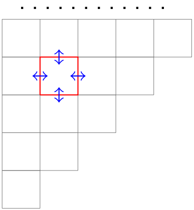

Proposition The Octant Tiling Problem is undecidable.

Proof: by reduction from the tiling problem for $\mathbb{N} \times \mathbb{N}$, using König's Lemma.

Undecidability of the interval logics via tiling: generic construction

1. Encoding of the octant
2. Encoding of the neighborhood relations
   - Right-neighborhood relation SIMPLE
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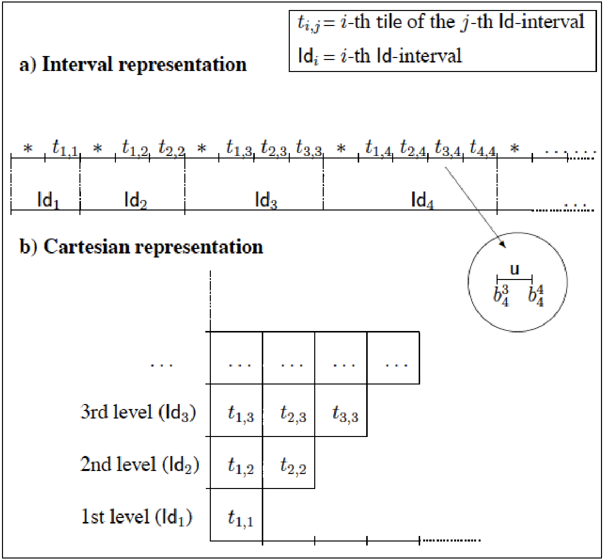
- ID-intervals are then introduced to represent the layers of tiles. (propositional letter Id)

Undecidability of the interval logics via tiling: generic construction cont'd



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For that, auxiliary propositional letter up_rel can be used to connecting (endpoints of) two intervals representing tiles that are above connected in the octant

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Eventually, we encode the given Octant tiling problem by specifying the matching conditions between intervals that are right-connected or above-connected.

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The specific part of the construction is to use the given fragment of HS to set the chain of unit intervals and to express all necessary properties of IDs, the propositional letters for correspondence intervals, and the tile matching conditions.

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- More statistics are available on the web page: https://itl.dimi.uniud.it/content/logic-hs

Outline

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PNL: syntax and semantics

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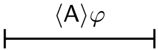

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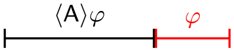

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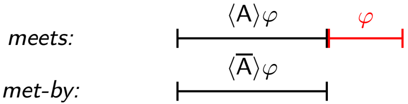

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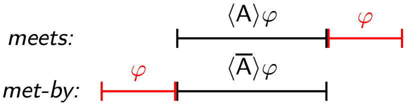

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MPNL: PNL extended with integer constraints for interval lengths.

Decidability of metric interval logic

Theorem Satisfiability in MPNL on $\mathbb{N}$ is decidable. It is NEXPTIME-complete if the metric constraints are represented in unary, and in between EXPSPACE and 2NEXPTIME if they are represented in binary.

D. Bresolin, D. Della Monica, V. Goranko, A. Montanari, G. Sciavicco
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Exact complexity is an open problem

Relative expressive power of logics in MPNL

$$\begin{tikzcd}
\mathsf{MPNL}_{\boldsymbol{l}}^{=} & & \mathsf{MPNL}_{\boldsymbol{l}}^{()} \\
\mathsf{MPNL}^{=} \arrow[u] & & \mathsf{MPNL}^{()} \arrow[u] \\
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\end{tikzcd}$$

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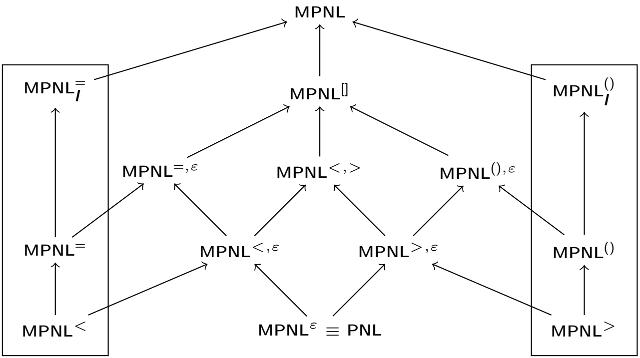

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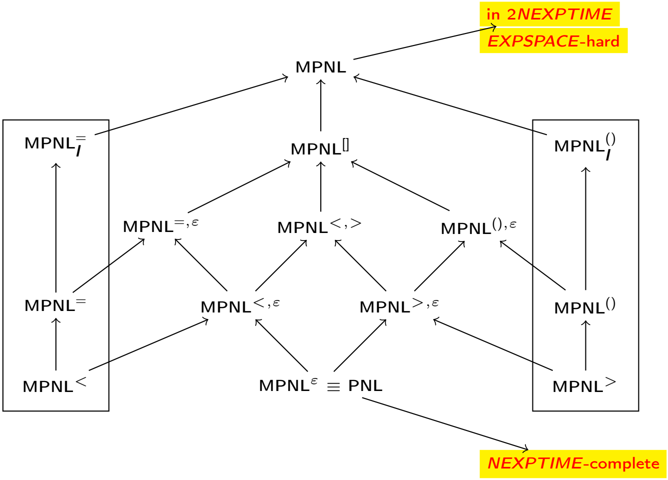

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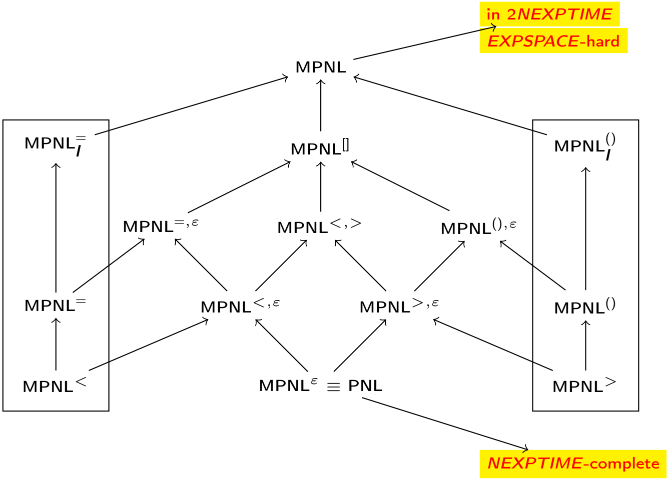

Decidability of *MPNL*: by small model property

Comparing expressiveness of metric fragments: by bisimulations

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PNL

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NEXPTIME-co

PNL

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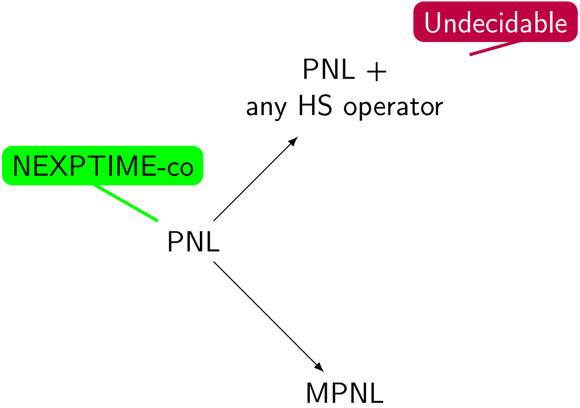

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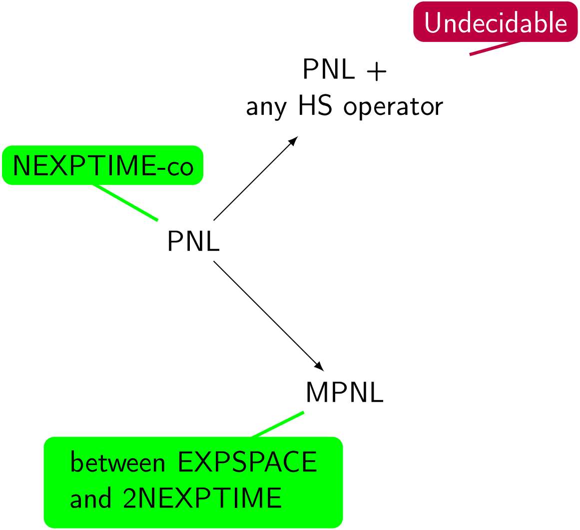

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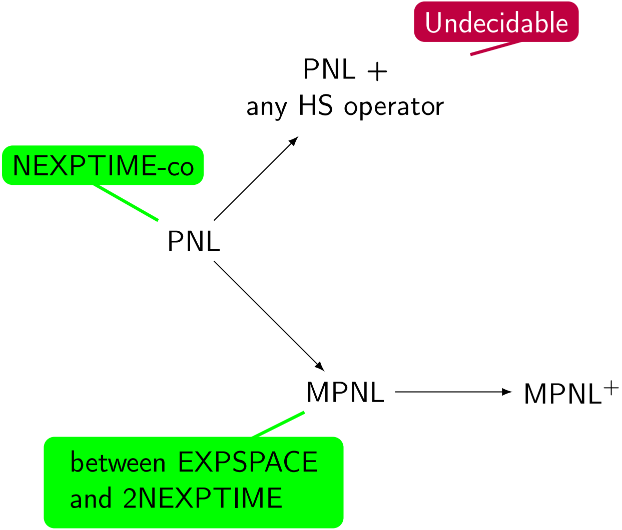

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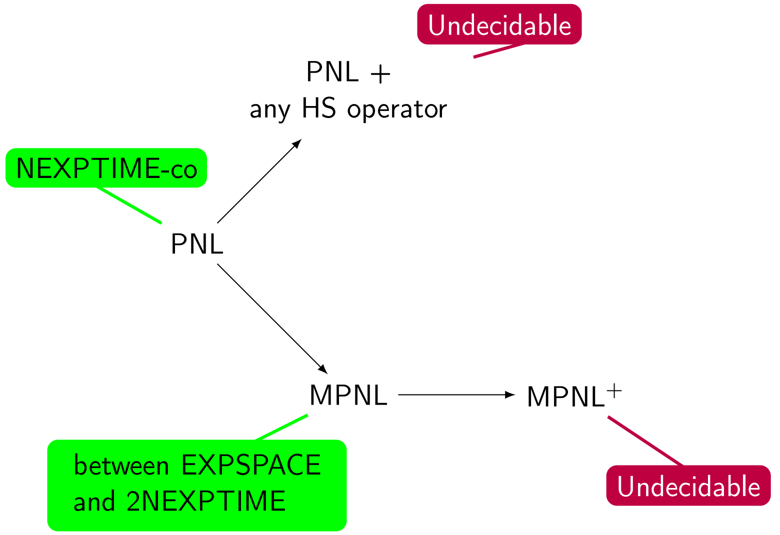

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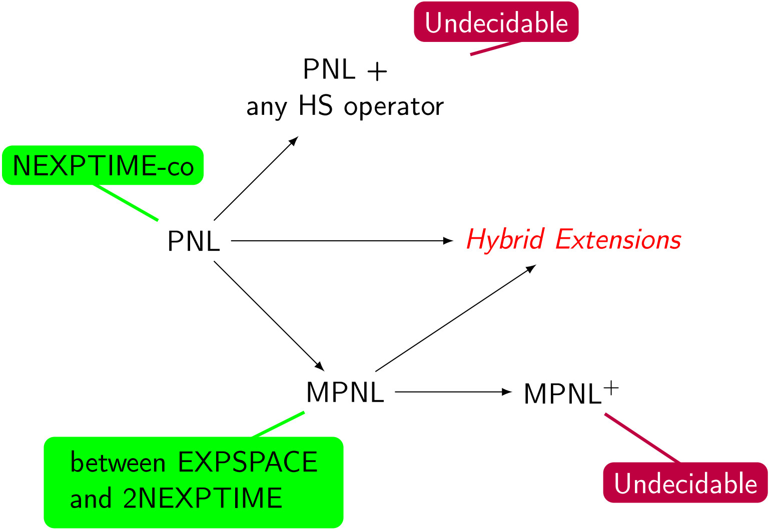

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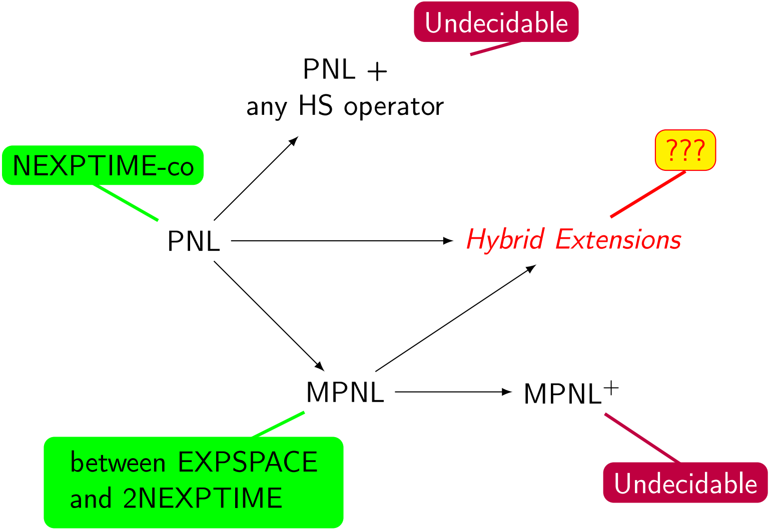

Possible hybrid extension of PNL and MPNL

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Binders over length of intervals
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Weakly Hybrid MPNL (WHMPNL)

Metric constraints of MPNL use constants

$$\mathsf{len}_{=5}, \mathsf{len}_{>2}, \ldots$$

WHMPNL allows one to store the length of the current interval and to refer to it in sub-formulae

$$\downarrow_x (\ldots |{=}| x), \downarrow_x (\ldots |{\leq}| x), \ldots$$

WHMPNL fragments

Remark

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	set of hybrid constraints	constant constraints	# of length variables
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$WHPNL(<, =)$	$\{<, =\}$	NO	unbounded
$WHPNL(<)_1$	$\{<\}$	NO	1

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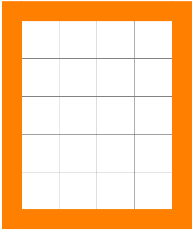
	set of hybrid constraints	constant constraints	# of length variables
$WHMPNL(<, \leq, =, \geq, >)$	$\{<, \leq, =, \geq, >\}$	YES	unbounded
$WHPNL(<, =)$	$\{<, =\}$	NO	unbounded
$WHPNL(<)_1$	$\{<\}$	NO	1

The fragment $WHPNL(=)_1$

Reduction from the Finite Tiling Problem

This is the problem of establishing whether, for a given finite set of tile types $\mathcal{T} = \{t_1, \ldots, t_k\}$, there exists a finite rectangle $\mathcal{R}$ having the border colored with a fixed color such that $\mathcal{T}$ can tile $\mathcal{R}$ respecting the color constraints.



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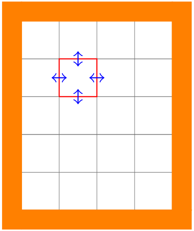
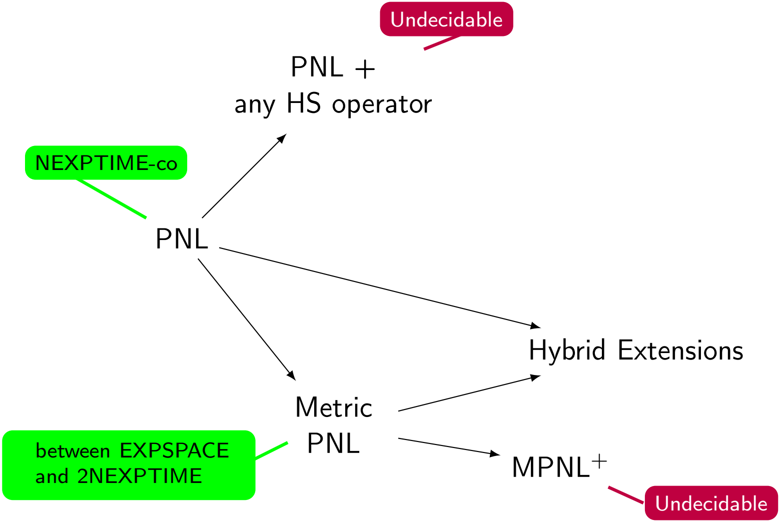


Outline

Extending PNL



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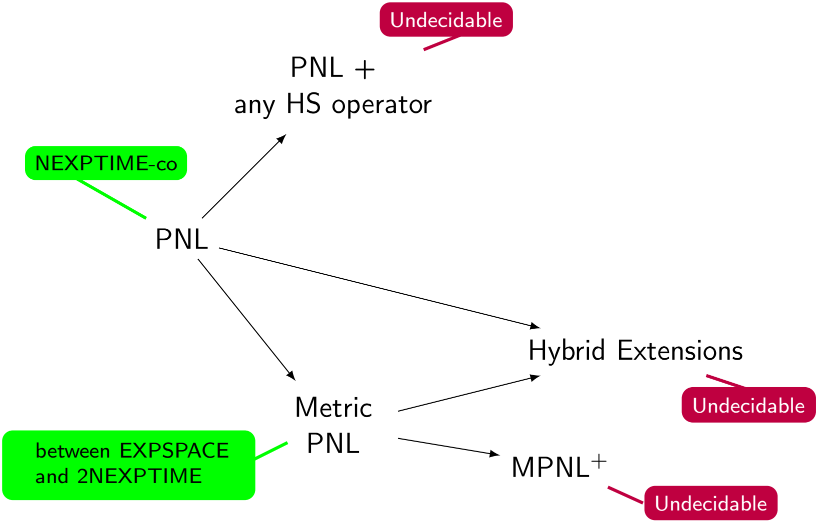

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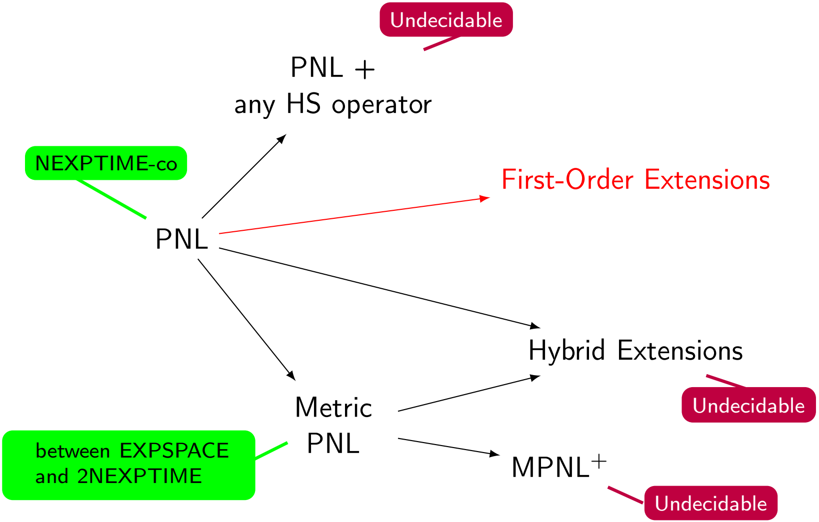

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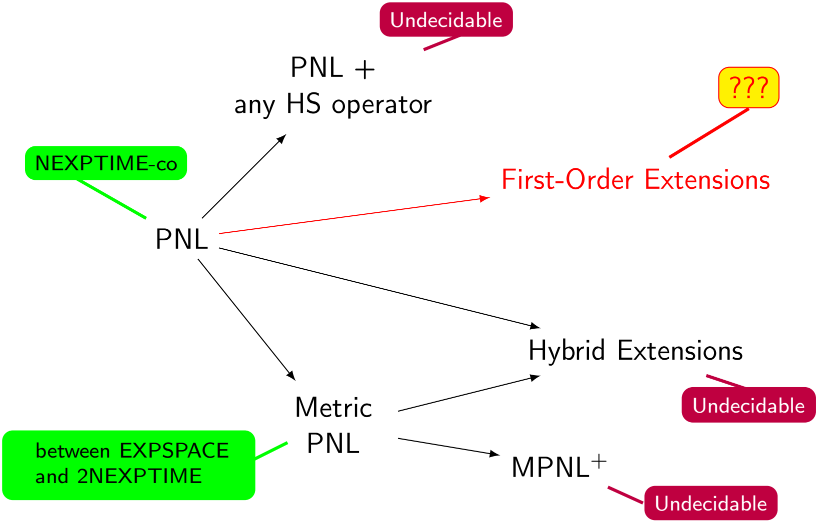

First-Order together with Propositional

FORPNL

First-Order Right Propositional Neighborhood Logic

First-Order together with Propositional

F O R P N L

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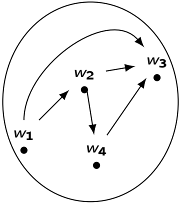
First-Order Right Propositional Neighborhood Logic

1. Propositional (modal) setting

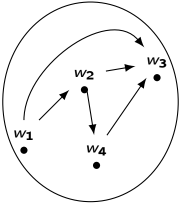


First-Order together with Propositional

F O R P N L

First-Order Right Propositional Neighborhood Logic

1. Propositional (modal) setting
2. First-Order setting
   - predicates over elements
   - existential and universal quantifications



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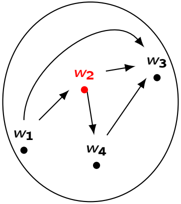

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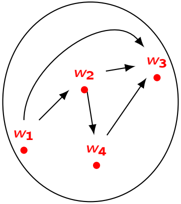

First-Order together with Propositional

FORPNL

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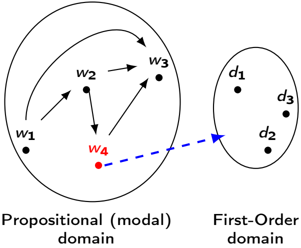
2. First-Order setting
   - predicates over elements
   - existential and universal quantifications
3. Propositional (modal) + First-Order setting



Parameters of the logic

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  - function $f(\ldots), g(\ldots), \ldots$
  - quantifiers
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for tight undecidability only 1 variable (no free variables)

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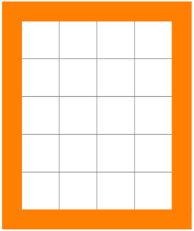

Undecidability of FORPNL

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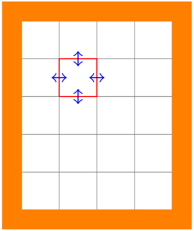

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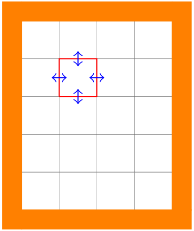
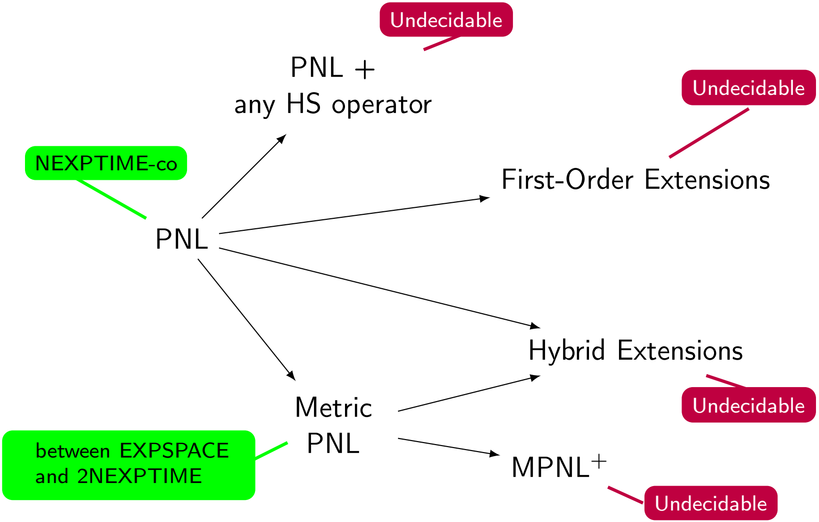


It is possible to *simulate* HS operators ⟨B⟩ ⟨E⟩ ⟨D⟩

Extending PNL: the final picture



Outline

Summary and perspectives

This talk outlined several major topics in the area of interval logics:

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Not discussed, and not yet explored, but important:

- Model checking of Interval logics
- Automata-based techniques for interval logics

Exams and attended courses

- Exams
  - International Lipari Summer School 2008 on "Algorithms: Science and Engineering" 14 - 25 July 2008, Lipari; 1.5 Credits
  - "Constraint Programming and NMR Constraints for Determining Protein Structure", A. Dovier
  - GAMES Spring School 2009
  - "Systems Biology", A. Policriti/M. Miculan
  - "Computational Complexity (Complessità computazionale)", R. Rizzi
  - "Introduction to Software Configuration Management", L. Bendix
- Other courses
  - "(Meta-)Modeling with UML and OCL", M. Gogolla
  - "Data Mining and Mathematical Programming", P. Serafini
  - "Sistemi Reattivi: automi, logica, algoritmi" (Master Course), A. Montanari
  - English course for academic purposes (CLAV)

Other activities

- Summer school
  - International Lipari Summer School 2008 on "Algorithms: Science and Engineering"
  - GAMES Spring School 2009 (Bertinoro)
- Visiting
  - Oct - Dec 2009: University of Murcia - Murcia, Spain (G. Sciavicco)
  - Sept - Nov 2010: Technical University of Denmark (DTU) - Lyngby, Copenhagen, Denmark (V. Goranko)
- Events organization
  - Annual Workshop of the ESF Networking Programme on Games for Design and Verification (GAMES 2009)
  - First International Symposium on Games, Automata, Logics and Formal Verification (GandALF 2010)
  - Second International Symposium on Games, Automata, Logics and Formal Verification (GandALF 2011)

Publications

1. D. Bresolin, D. Della Monica, V. Goranko, A. Montanari, and G. Sciavicco. "Decidable and Undecidable Fragments of Halpern and Shohams Interval Temporal Logic: Towards a Complete Classification". In *Proc. of 15th International Conference on Logic for Programming, Artificial Intelligence, and Reasoning (LPAR 2008)*, 2008.
2. D. Bresolin, D. Della Monica, V. Goranko, A. Montanari, and G. Sciavicco. "Undecidability of Interval Temporal Logics with the Overlap Modality". In *Proc. of 16th International Symposium on Temporal Representation and Reasoning (TIME 2009)*, 2009.
3. D. Bresolin, D. Della Monica, V. Goranko, A. Montanari, and G. Sciavicco. "Undecidability of the Logic of Overlap Relation over Discrete Linear Orderings". Electronic Notes in Theoretical Computer Science (*Proc. of the 6th Workshop on Methods for Modalities (M4M-6 2009)*, 2010.

Publications - contd'

4. D. Della Monica, V. Goranko, and G. Sciavicco. "Hybrid Metric Propositional Neighborhood Logics with Interval Length Binders". In *Proc. of International Workshop on Hybrid Logic and Applications (HyLo 2010)*, 2010. To appear on *ENTCS*.
5. D. Della Monica and G. Sciavicco. "On First-Order Propositional Neighborhood Logics: a First Attempt". In *Proc. of ECAI 2010 Workshop on Spatio-Temporal Dynamics (STeDY 2010)*, 2010.
6. D. Bresolin, D. Della Monica, V. Goranko, A. Montanari, and G. Sciavicco. "Metric Propositional Neighborhood Logics: Expressiveness, Decidability, and Undecidability". In *Proc. of 19th European Conference on Artificial Intelligence (ECAI 2010)*, 2010.
7. D. Bresolin, D. Della Monica, A. Montanari, P. Sala, and G. Sciavicco. "A decidable spatial generalization of Metric Interval Temporal Logic". In *Proc. of 17th International Symposium on Temporal Representation and Reasoning (TIME 2010)*, 2010.

Publications - contd'

8. D. Bresolin, D. Della Monica, V. Goranko, A. Montanari, G. Sciavicco. "Metric propositional neighborhood logics on natural numbers". *Journal of Software & Systems Modeling* (doi: 10.1007/s10270-011-0195-y, online since February 2011).
9. D. Della Monica, V. Goranko, A. Montanari, G. Sciavicco. "Expressiveness of the Interval Logics of Allen's Relations on the Class of all Linear Orders: Complete Classification". *accepted to IJCAI 2011*.

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The end.