Pushing runtime verification to the limit

May process semantics be with us

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Outline

A quick introduction to runtime verification (monitoring)

Monitoring *HML*

Extending runtime verification applicability
- A failed attempt
- A promising road using process semantics

Conclusions

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Runtime verification

monitoring a single partial execution and try to give a verdict

eventually p

Runtime verification

monitoring a single partial execution and try to give a verdict

eventually p $\neg p$

Runtime verification

monitoring a single partial execution and try to give a verdict

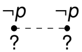
eventually p

$\neg p$
$\bullet$
?

Runtime verification

monitoring a single partial execution and try to give a verdict

eventually p



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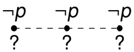

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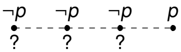

Runtime verification

monitoring a single partial execution and try to give a verdict

	$\neg p$	$\neg p$	$\neg p$	p
eventually p	?	?	?	yes

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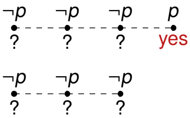

Runtime verification

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?	?	?	yes

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?	?	?	?

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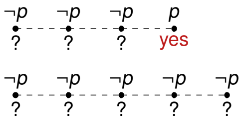

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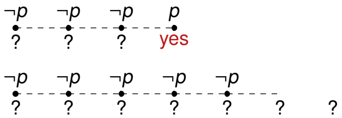

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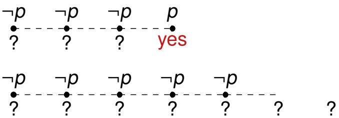

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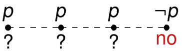
always p



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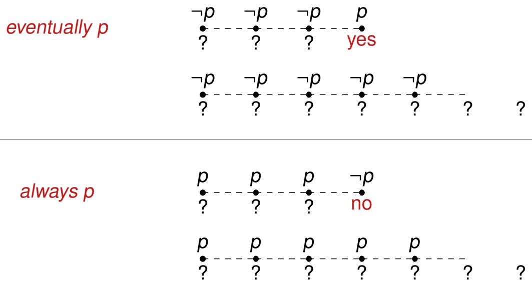

Runtime verification

monitoring a single partial execution and try to give a verdict

eventually p

$\neg p$	$\neg p$	$\neg p$	p			
?	?	?	yes			

$\neg p$	$\neg p$	$\neg p$	$\neg p$	$\neg p$		
?	?	?	?	?	?	?

---

always p

p	p	p	$\neg p$			
?	?	?	no			

p	p	p	p	p		
?	?	?	?	?	?	?

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p until q

p	p	p	q
?	?	?	yes

Runtime verification

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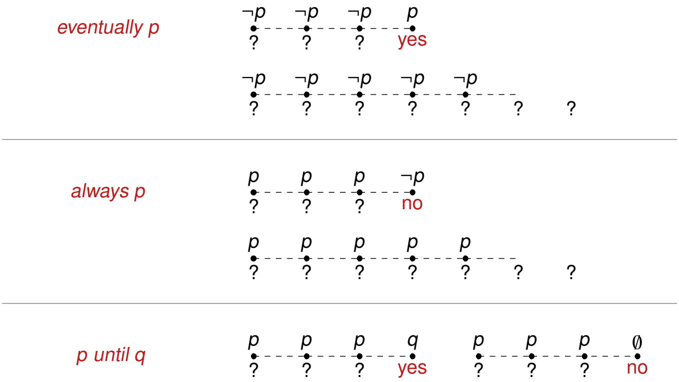

Runtime verification

monitoring a single partial execution and try to give a verdict

Monitorability

Definition (monitorability)

φ is monitorable $:=$ φ is suitable to be runtime verified

$:=$ either

there exists a **witness for φ-satisfaction** whenever φ is true

or

there exists a **witness for φ-violation** whenever φ is false

witness for φ-satisfaction: finite trace s.t. every system featuring it satisfies φ

witness for φ-violation: finite trace s.t. every system featuring it violates φ

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The branching time logic *HML*

$\varphi, \phi \in \mathit{HML} ::=$

tt	(truth)	$\mid \mathsf{ff}$	(falsehood)
$\mid \varphi \vee \phi$	(disjunction)	$\mid \varphi \wedge \phi$	(conjunction)
$\mid \langle\alpha\rangle\varphi$	(possibility)	$\mid [\alpha]\varphi$	(necessity)

The maximal monitorable subset [†]

$$\pi, \varpi \in \text{cHML} ::= \mathsf{tt} \quad \mid \mathsf{ff} \quad \mid \pi \vee \varpi \quad \mid \langle\alpha\rangle\pi$$

$$\theta, \vartheta \in \text{sHML} ::= \mathsf{tt} \quad \mid \mathsf{ff} \quad \mid \theta \wedge \vartheta \quad \mid [\alpha]\theta$$

[†] Francalanza, Aceto, Ingólfsdóttir, *On verifying Hennessy-Milner logic with recursion at runtime*. In **Runtime Verification**, 2015.

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$\pi, \varpi \in$ cHML::= tt $\mid$ ff $\mid \pi \vee \varpi$ $\mid \langle\alpha\rangle\pi$

$\theta, \vartheta \in$ sHML::= tt $\mid$ ff $\mid \theta \wedge \vartheta$ $\mid [\alpha]\theta$

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Monitorability: examples

The maximal monitorable subset

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$$\theta, \vartheta \in \text{sHML} ::= \text{tt} \quad | \; \text{ff} \quad | \; \theta \wedge \vartheta \quad | \; [\alpha] \theta$$

Examples

$\langle a \rangle \text{tt}$

positively monitorable

Monitorability: examples

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Examples

$\langle a \rangle \mathsf{tt}$ — positively monitorable

$[a]\mathsf{ff}$ — negatively monitorable

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$\langle a \rangle \mathsf{tt} \vee \langle b \rangle \mathsf{tt}$ — positively monitorable

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$\langle a \rangle [b] \mathsf{ff}$

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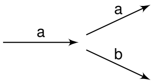
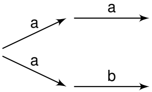
$\langle a \rangle [b]\mathsf{ff}$



$\langle a \rangle [b]\mathsf{ff}$ is false



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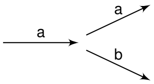
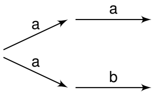
Examples

$\langle a \rangle \mathrm{tt}$	$[a]\mathrm{ff}$	$\langle a \rangle \mathrm{tt} \vee \langle b \rangle \mathrm{tt}$	$\cancel{\langle a \rangle [b] \mathrm{ff}}$
positively monitorable	negatively monitorable	positively monitorable	not monitorable



$\langle a \rangle [b]\mathrm{ff}$ is false



$\langle a \rangle [b]\mathrm{ff}$ is true

Limitations of monitoring

$\langle a \rangle(\langle b \rangle \mathrm{tt} \vee [c]\mathrm{ff})$: not monitorable, do model checking

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Limitations of monitoring

$\langle a \rangle (\langle b \rangle \mathrm{tt} \vee [c]\mathrm{ff})$: not monitorable, do model checking

$$\langle a \rangle (\langle b \rangle \mathrm{tt} \vee [c]\mathrm{ff}) \equiv \underbrace{\langle a \rangle \langle b \rangle \mathrm{tt}}_{\text{monitorable: do runtime verification}} \vee \underbrace{\langle a \rangle [c]\mathrm{ff}}_{\text{not monitorable: do model checking}}$$

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The problem: Maximal monitorable semantic sub-formula

Skip

Input: a formula φ of HML

$\psi ::= \mathsf{tt} \mid \mathsf{ff} \mid \psi \vee \psi \mid \psi \wedge \psi \mid \langle\alpha\rangle\psi \mid [\alpha]\psi$

Output: φ^{MON} (a maximal monitorable semantic sub-formula of φ)

The problem: Maximal monitorable semantic sub-formula

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Then $\varphi \equiv \varphi^{MON} \vee \varphi_{|\varphi^{MON}}$ where
- φ^{MON} is monitorable
- $\varphi_{|\varphi^{MON}}$ must be model checked

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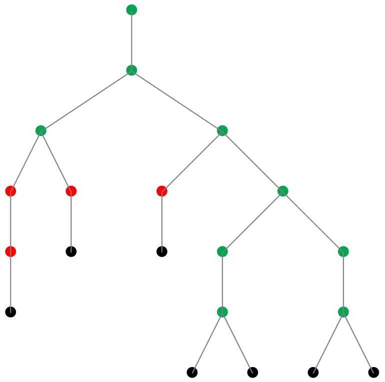
The decomposition procedure

1. Identify highest universal nodes
2. Remove subtrees rooted in highest universal nodes
3. Remove new leaves



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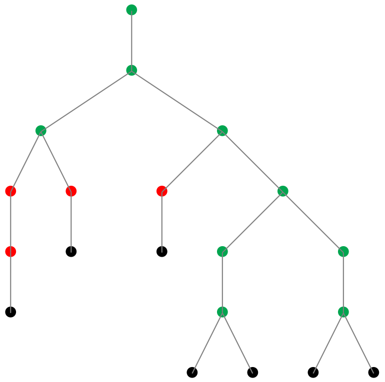

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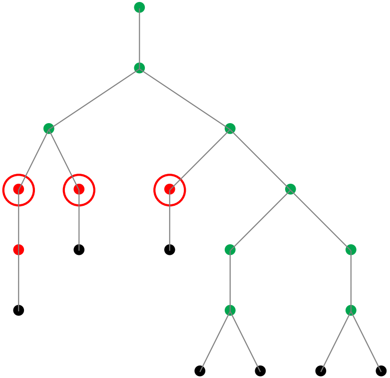

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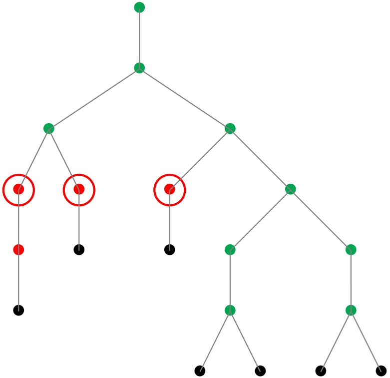

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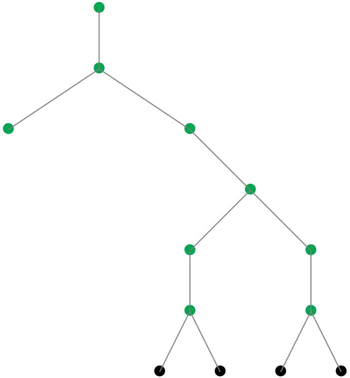

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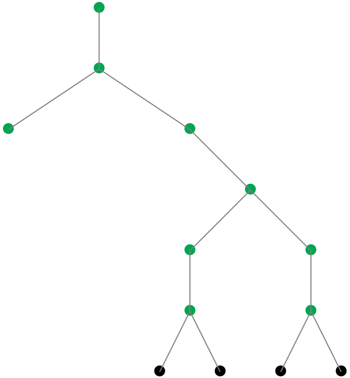

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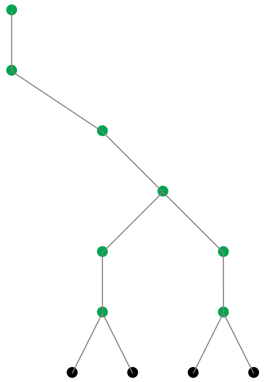
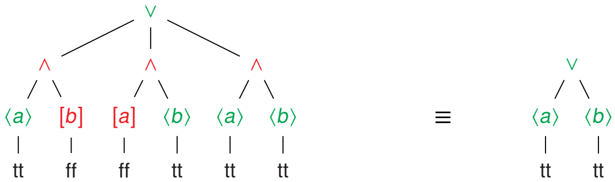


A counterexample

$$(\langle a\rangle tt \wedge [b]ff) \vee ([a]ff \wedge \langle b\rangle tt) \vee (\langle a\rangle tt \wedge \langle b\rangle tt) \quad \equiv \quad \langle a\rangle tt \vee \langle b\rangle tt$$



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$\vee$

$$\equiv \quad \begin{array}{ccc} & \vee & \\ / & & \backslash \\ \langle a\rangle & & \langle b\rangle \\ | & & | \\ \mathrm{tt} & & \mathrm{tt} \end{array}$$

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Modal transition systems might hold the answer

- Our approach was purely syntactic
- Formulas are semantics object but difficult to manipulate
- Syntactic trees of formulas can be manipulated but they are... well... too syntactic
- We needed a representation of formulas that
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Modal transition systems (MTS)

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- A more suitable graphical representation of formulas of *HML* is given by modal transition systems (MTS's) and modal refinement defined over them
  - every formula of *HML* is representable as a (finite set of) MTS
  - a translation back from MTS's into formulas also exists
  - modal refinement over MTS's is a preorder that carries the semantics information about formulas and their relationship

What are MTS's?

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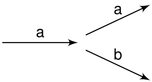
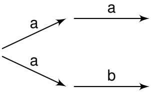
(as we consider *HML* rather than μ*HML*)





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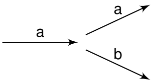
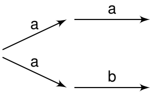
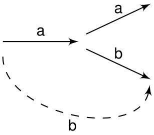
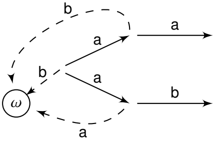
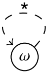




- An MTS is a triple $(P, \rightarrow, \dashrightarrow)$, where
  - P is a finite set of processes
  - $\rightarrow \subseteq P \times \Sigma \times P$
  - $\dashrightarrow \subseteq P \times \Sigma \times P$ and $\rightarrow \subseteq \dashrightarrow$







What is a modal refinement preorder relation?

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M' is a refinement of M (denoted $M \sqsubseteq M'$) iff

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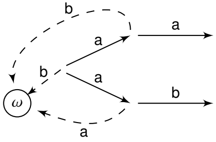
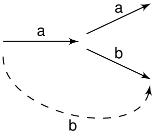


$\sqsubseteq$

$\not\sqsupseteq$



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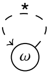

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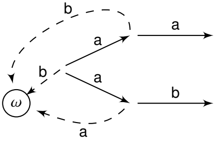

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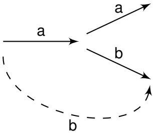

(we focus on acyclic MTS)

The problem: Maximal monitorable semantic sub-formula

$\psi ::= \mathrm{tt} \mid \mathrm{ff} \mid \psi \vee \psi \mid \psi \wedge \psi \mid \langle \alpha \rangle \psi \mid [\alpha] \psi$

Input: a formula φ of HML

Output: φ^{MON} (a maximal monitorable semantic sub-formula of φ)

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Then $\varphi \equiv \varphi^{MON} \vee \varphi_{|\varphi^{MON}}$ where
- φ^{MON} is monitorable
- $\varphi_{|\varphi^{MON}}$ must be model checked

The solution idea

- $\varphi \mapsto \mathrm{MTS}(\varphi)$ [†] *(transform φ into a set of MTS's)*
- $\mathrm{MTS}(\varphi) = \{M_1, \ldots, M_n\}$ is a finite set of MTS s.t.

  $$M \text{ satisfies } \varphi \quad \text{iff} \quad M_i \sqsubseteq M \text{ for some } i$$

  for all MTS M

  (wlog. we can assume $\mathrm{MTS}(\varphi) = \{M_\varphi\}$ to be a singleton)

[†] Boudol, Larsen, *Graphical Versus Logical Specifications*. **Theor. Comput. Sci.** 106(1): 3-20 (1992).

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- PROBLEM: the logical representation of an MTS M (characteristic formula of M, denoted $\chi(M)$) is not guaranteed to be in cHML
- consider almost-universal MTS's, i.e.,
  - every state has may transitions to ω (1-step-universal):
    characteristic formulas are in $cHML + \wedge$
  - every state (except ω) has exactly one outgoing must transition

  characteristic formulas are in *cHML*

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- Let *almost-un* be the set of almost-universal MTS's
- Let $\mathit{refinements}(M_\varphi)$ be the set of refinements of M_φ

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$$[\![\bigvee_{M \,\in\, \mathit{almost\text{-}un} \,\cap\, \mathit{refinements}(M_\varphi)} \chi(M)]\!] = [\![\bigvee_{M \,\in\, \mathit{almost\text{-}un} \,\cap\, \{M_\varphi\}} \chi(M)]\!] \subseteq [\![\varphi]\!]$$

The solution idea (cont'd)

Maximality follows from a continuity property

Lemma (claim)

Let M be an almost-universal MTS. If all of its ultimate refinements (i.e., $\dashrightarrow = \longrightarrow$) satisfy an HML formula ψ, then M satisfies ψ, too

Corollary (maximality)

$\bigvee_{M \,\in\, \textit{almost-un} \,\cap\, \mathrm{MTS}(\varphi)} \chi(M)$ *is the maximal monitorable semantic sub-formula of a given HML formula φ, i.e.,*

$$\bigvee_{M \,\in\, \textit{almost-un} \,\cap\, \mathrm{MTS}(\varphi)} \chi(M) = \varphi^{MON}$$

Outline

A quick introduction to runtime verification (monitoring)

Monitoring *HML*

Extending runtime verification applicability
- A failed attempt
- A promising road using process semantics

Conclusions

What is missing?

- To extend the approach to full μ*HML*
  - MTS's with cycles
- Even in the context of *HML*, extend monitoring abilities
  - a monitor knows when a process terminates (complete-simulation)
  - a monitor knows which are the next (1-step) possible states (ready-simulation)
- Complexity analysis
  - comparison with a (doubly exponential) recent approach (not published yet)