When are prime formulae characteristic?

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MFCS 2015
Milan, August 24–28, 2015

Outline

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Model checking vs. equivalence/preorder checking

Model checking	**Equivalence/preorder checking**
system property	system property

Model checking vs. equivalence/preorder checking

Model checking		**Equivalence/preorder checking**	
system	property	system	property
state-transition model ↓	↓ logical formalism		
p_S	φ		

Model checking vs. equivalence/preorder checking

Model checking

system

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state-transition model

logical formalism

$p_S \models \varphi$

Equivalence/preorder checking

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system — state-transition model — p_S

property — logical formalism — φ

$$p_S \models \varphi$$

Equivalence/preorder checking

system — state-transition model — p_S

property — state-transition model — p_{prop}

Model checking vs. equivalence/preorder checking

Model checking

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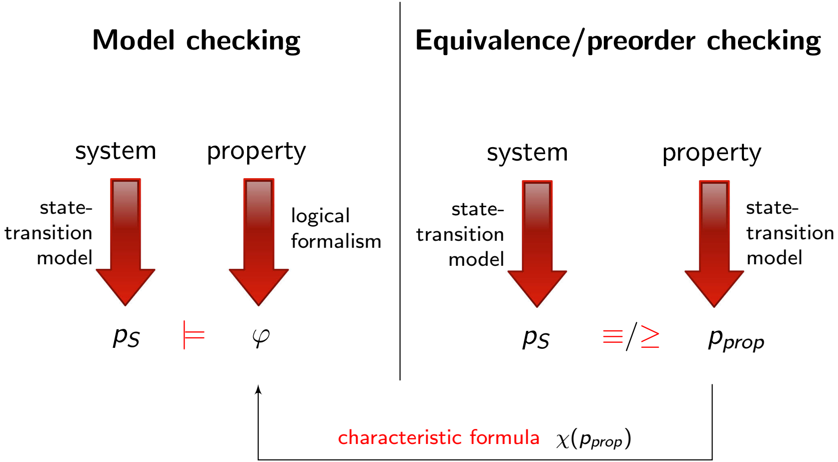
$p_S \equiv / \geq p_{prop}$

Model checking vs. equivalence/preorder checking



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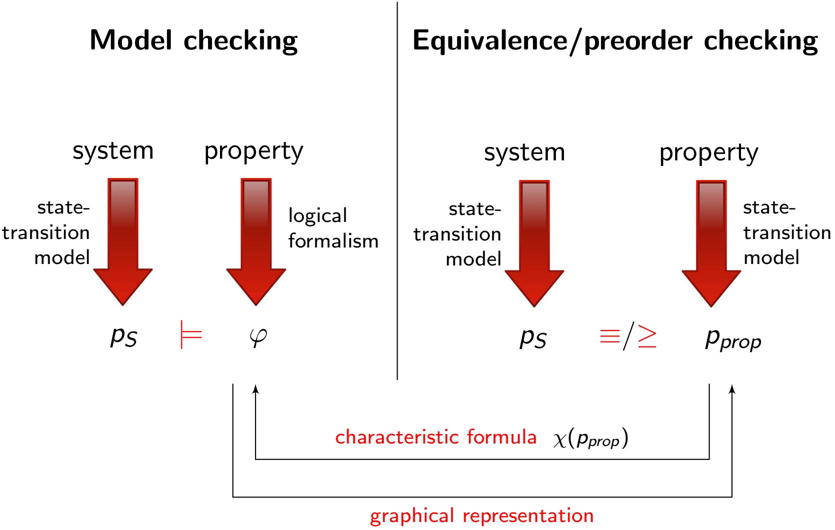

Characterizing the bridge between the two approaches

Question:

For what kind of logical specification is it possible to reduce model checking to preorder/equivalence checking?

Answer (attempt):

The ones expressed by consistent and prime formulae (?)

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Claim

A formula is characteristic iff it is consistent and prime

(characteristic $\Leftrightarrow$ consistent and prime)

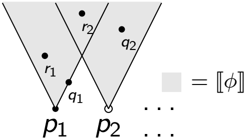
Not true in general.

Under which conditions? For which formalisms?

Logical characterization of preorders

Combination of the two views: logical characterization

A logic $\mathcal{L}$ characterizes $\leq$ if:
$p \leq q$ iff $\mathcal{L}(p) \subseteq \mathcal{L}(q)$



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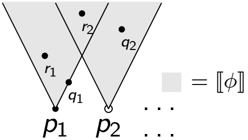

Characteristic formulae

A formula $\chi(p)$ is a characteristic formula for p with respect to $\leq$ iff:
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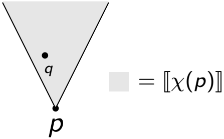

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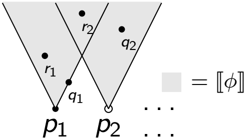

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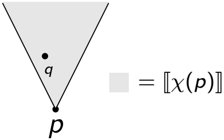

Consistent and prime formulae

- ϕ is consistent if $p \models \phi$ for some p
- ϕ is prime if for all ϕ_1, ϕ_2:
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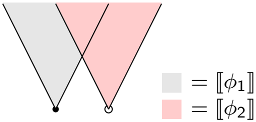

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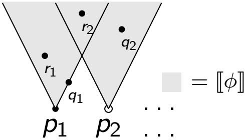

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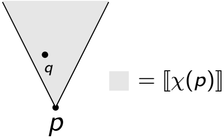

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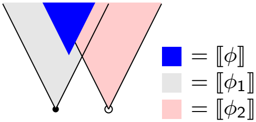

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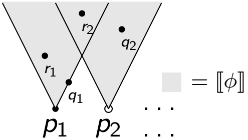

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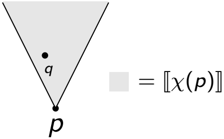

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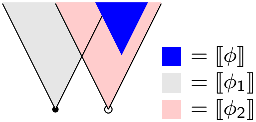

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Existing results

For preorders based on

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- covariant-contravariant simulation **[AFFIP12]**

[BL92] Boudol and Larsen, *Graphical versus logical specifications*, TCS 1992

[AFFIP12] Aceto, Fábregas, de Frutos-Escrig, Ingólfsdóttir, and Palomino, *Graphical representation of covariant-contravariant modal formulas*, EXPRESS 2011

Our quest

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- We use a purely logical approach

Isolating the logical view from the behavioural one

The logical formalisms

- $\mathcal{L}$ is a logic over a set of processes **Proc** (not necessarily LTS)
- interpretation function $[\![\_]\!] : \mathcal{L} \to \mathcal{P}(\mathbf{Proc})$

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The logically defined preorder $\sqsubseteq_{\mathcal{L}}$

- $p \sqsubseteq_{\mathcal{L}} q$ iff $\mathcal{L}(p) \subseteq \mathcal{L}(q)$
- $p \uparrow = \{q \mid p \sqsubseteq_{\mathcal{L}} q\}$

Our approach

- To derive some general properties of a logical preorder $\sqsubseteq_{\mathcal{L}}$ (for a generic logic $\mathcal{L}$) directly
- If for some behavioural preorder $\leq$ is characterized by $\mathcal{L}$ it automatically inherits some properties of $\sqsubseteq_{\mathcal{L}}$

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- ϕ is consistent if $[\![\phi]\!] \neq \emptyset$

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Theorem

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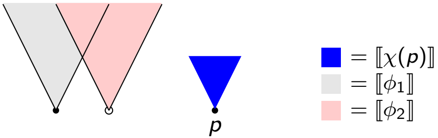

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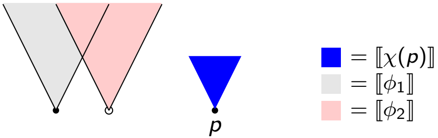

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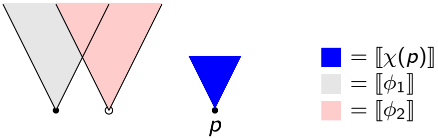

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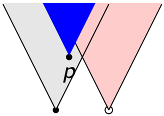

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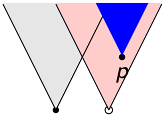

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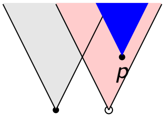

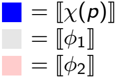

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Proc finite and $\wedge \in \mathcal{L}$: each process has a characteristic formula

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- Then $\chi(p)$ is characteristic for p

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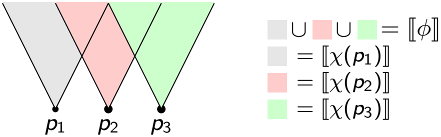

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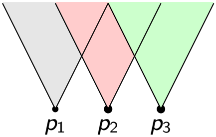

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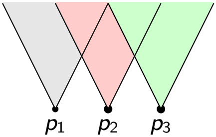

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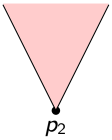

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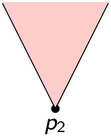

$= [\![\phi]\!]$

$= [\![\chi(p_2)]\!]$

$\square$

Outline

Characterization by primality: Generalization

Partial summary

- ALWAYS:

  characteristic $\Rightarrow$ prime (and consistent)

- **Proc** is finite and $\wedge \in \mathcal{L}$:

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No.

There exist formulae that are consistent and prime but not characteristic!

Characterization by primality: a counterexample

The claim characteristic $\Leftrightarrow$ consistent and prime
does not hold in general

Counterexample

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- $\mathbb{Q}$ is the set of processes
- $\mathbb{R}$ is the set of formulae

$\mathbb{R}$

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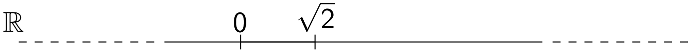

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- $\mathbb{Q}$ is the set of processes
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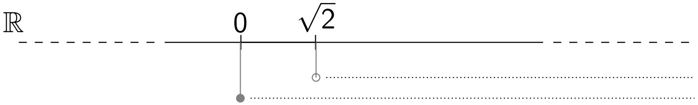

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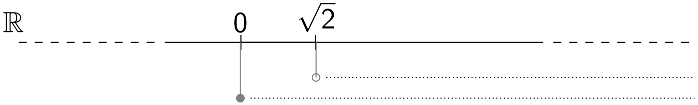

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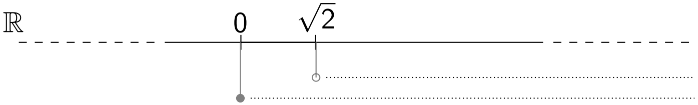

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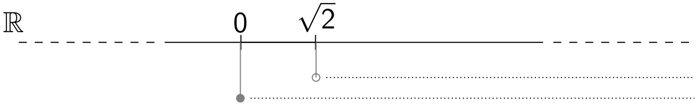
- On the other hand $\phi = \sqrt{2} \notin \mathbb{Q}$ cannot be characteristic for any process as $[\![\sqrt{2}]\!]$ does not have a least element



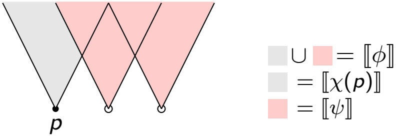
Outline

Decomposability

Definition

$\phi \in \mathcal{L}$ is decomposable if:

$[\![\phi]\!] = [\![\chi(p)]\!] \cup [\![\psi]\!]$ for some $\psi \in \mathcal{L}$ s.t. $p \notin [\![\psi]\!]$



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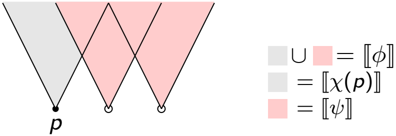

Theorem

- $\phi \in \mathcal{L}$ decomposable and prime $\Rightarrow$ characteristic
- $\mathcal{L}$ decomposable $\Rightarrow$ (characteristic $\Leftrightarrow$ consistent and prime)

Decomposability

Proof:

Assume ϕ decomposable, consistent and prime

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$[\![\chi(p)]\!] \cup [\![\psi]\!] \subseteq [\![\chi(p)]\!] \cup [\![\psi]\!]$

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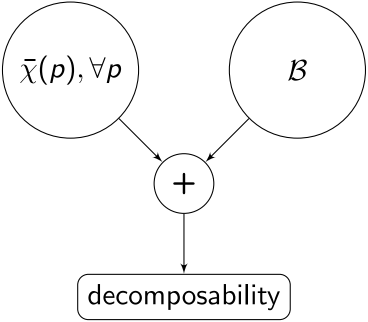
Therefore, $[\![\phi]\!] = [\![\chi(p)]\!] \cup [\![\psi]\!] = [\![\chi(p)]\!]$.

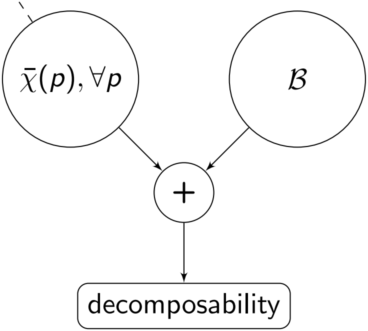
Outline

A path to decomposability



A path to decomposability

contra-characteristic formula $\bar{\chi}(p)$
captures somehow the complement of $\chi(p)$



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- $\{q \in P \mid q \not\sqsubseteq_{\mathcal{L}} p\} \subseteq [\![\bar{\chi}(p)]\!]$
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$$\begin{tikzcd}
\bar{\chi}(p), \forall p \arrow[dr] & & \mathcal{B} \arrow[dl] \\
& + \arrow[d] & \\
& \boxed{\text{decomposability}} &
\end{tikzcd}$$

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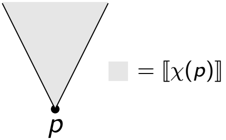

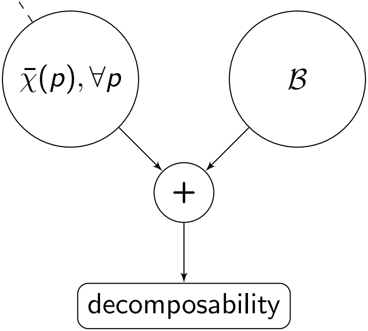

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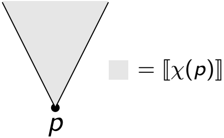
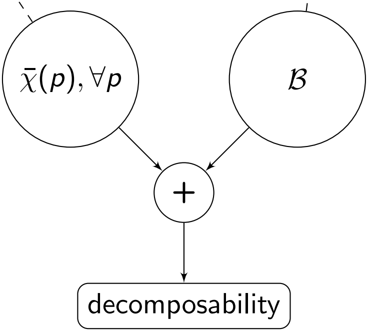
finite monotonic characterization
$\forall p, \mathcal{B}(p)$:
- is a finite set of truths implying all truths
- respects the preorder

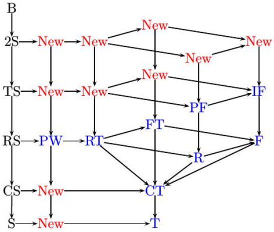




Outline

Application to semantics in van Glabbeek's spectrum

- Processes are finite acyclic graphs (finite trees)
- The semantics preorders are based on *simulation*, *complete simulation*, *ready simulation*, *trace simulation*, *2-nested simulation*, and *bisimulation*.



- All the preorders are characterized by subsets of the standard finite Hennessy-Milner logic.
- By checking the properties above we have proven that all these sub-logics are characterized by primality.

de Frutos-Escrig, Gregorio-Rodrguez, Palomino, and Romero-Hernández, *Unifying the Linear Time-Branching Time Spectrum of Process Semantics*, LMCS 2013

Outline

Conclusions

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Moreover

- We have gained some important insight into the properties of logically characterized preorders

Future work

- Use the result of this study to characterize more theories
- Find other properties that follow directly from a logical characterization of a preorder

The end

Thank you!