Prompt Interval Temporal Logic

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Outline

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What is prompt

Intuition: to bound the delay with which a request is satisfied

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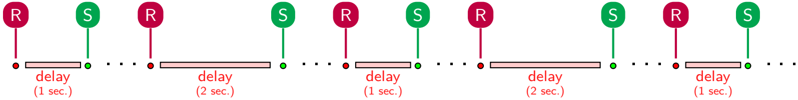

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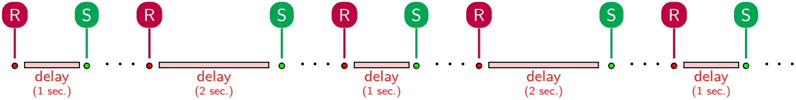

Delay sequence: 1, 2, 1, 2, 1, 2, ...

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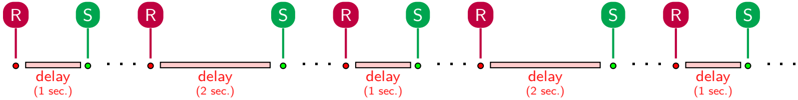

Delay sequence: 1, 2, 1, 2, 1, 2, ... Bound: 2

$[G](req \rightarrow Xsat \vee XXsat)$

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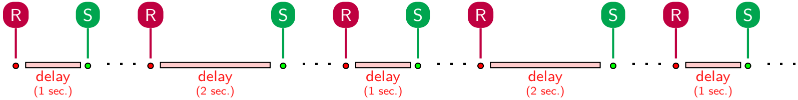

Delay sequence: 1, 2, 1, 2, 1, 2, . . . Bound: 2

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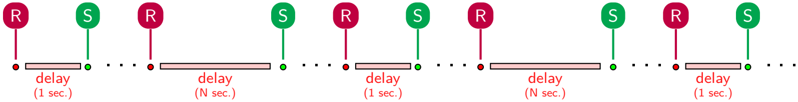
- ... but unknown (or arbitrarily large)



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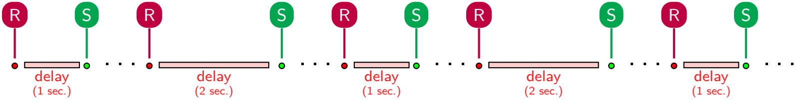

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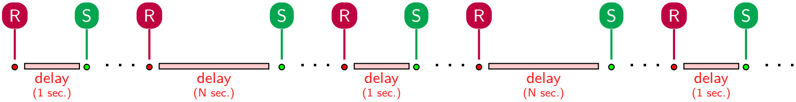

Delay sequence: 1, N, 1, N, 1, N, . . .

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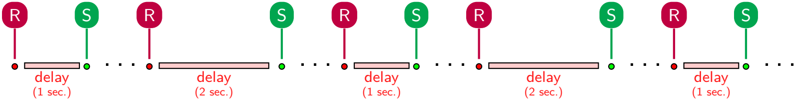

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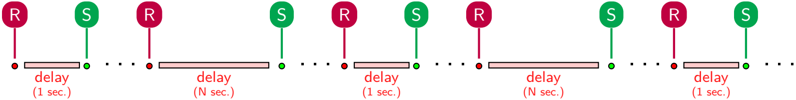
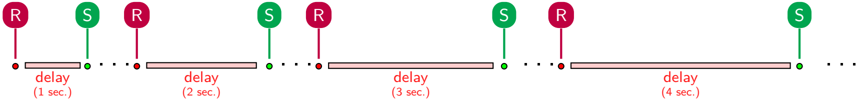


Delay sequence: 1, N, 1, N, 1, N, . . .   Bound: N (a constant)

What is not prompt



Delay sequence: 1, 2, 3, 4, 5, ...

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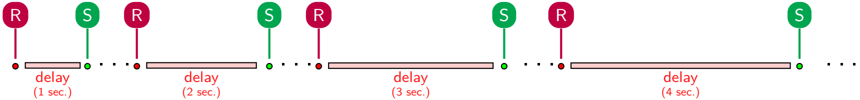
What is not prompt



Delay sequence: 1, 2, 3, 4, 5, . . .

Bound: ∞ (unbounded)

Prompt extensions of temporal logic

- PLTL
  [Alur-Etessami-La Torre-Peled, 2001]

- PROMPT-LTL
  [Kupferman-Piterman-Vardi, 2009]

Outline

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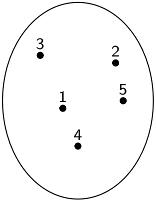
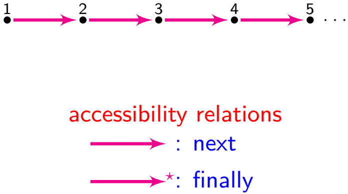
Temporal logics

- Temporal logics are (multi-)modal logics simplification

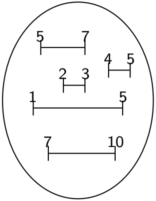


set of worlds
primitive temporal entity
time points/instants



A different approach: from points to intervals

- worlds are intervals (time period — pairs of points)



set of worlds
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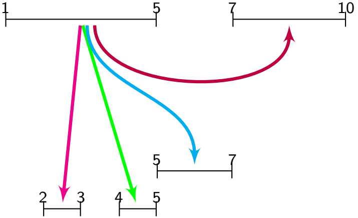
time intervals/periods



accessibility relations
all binary relations between pairs of intervals

The logic PNL

Syntax

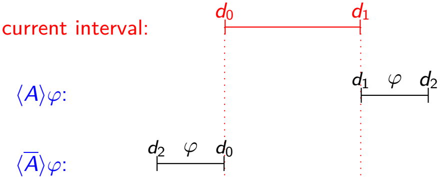
$$\varphi ::= \; p \;\mid\; \neg\varphi \;\mid\; \varphi \wedge \varphi \;\mid\; \langle A \rangle \varphi \;\mid\; \langle \overline{A} \rangle \varphi$$

Semantics

Models: $\mathbf{M} = \langle \mathbb{I}(\mathbb{D}), V \rangle$
(intervals over a linear order + atomic propositions eval.)

$\langle A \rangle$: $\mathbf{M}, [d_0, d_1] \Vdash \langle A \rangle \varphi$ iff $\exists d_2$ s.t. $d_1 < d_2$ and $\mathbf{M}, [d_1, d_2] \Vdash \varphi$

$\langle \overline{A} \rangle$: $\mathbf{M}, [d_0, d_1] \Vdash \langle \overline{A} \rangle \varphi$ iff $\exists d_2$ s.t. $d_2 < d_0$ and $\mathbf{M}, [d_2, d_0] \Vdash \varphi$



Outline

The logic PROMPT-PNL

Syntax

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle A \rangle \varphi \mid \langle \overline{A} \rangle \varphi \mid \langle A_x \rangle \varphi \mid \langle \overline{A}_x \rangle \varphi$$

Semantics

Models: $\mathbf{M} = \langle \mathbb{I}(\mathbb{D}), V, \delta, \sigma \rangle$
(intervals over a linear order + atomic propositions eval. + metric + bounding variables eval.)

$\langle A_x \rangle$: $\mathbf{M}, [d_0, d_1] \Vdash \langle A_x \rangle \varphi$ iff
$\exists d_2$ s.t. $d_1 < d_2$, $\mathrm{length}_\delta([\mathrm{d}_1, \mathrm{d}_2]) \leq \sigma(x)$, and $\mathbf{M}, [d_1, d_2] \Vdash \varphi$

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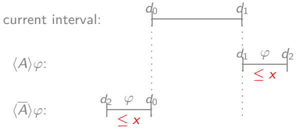

The satisfiability problem for PROMPT-PNL

Input: ▸ a PROMPT-PNL formula φ

Question: Are there

- a model $\mathbf{M} = \langle \mathbb{I}(\mathbb{D}), V, \delta, \sigma \rangle$ and
- an interval $[a, b] \in \mathbb{I}(\mathbb{D})$

that satisfy φ (i.e., $\mathbf{M}, [a, b] \Vdash \varphi$)

Outline

Undecidability of PROMPT−PNL

Theorem

The satisfiability problem for PROMPT−PNL is undecidable

Proof

By reduction from the *Finite Coloring Problem*
(aka. *Finite Tiling Problem*)

Finite Coloring Problem

Input:
- C: a set of colors
- H and V: two binary relations over colors
  (horizontal and vertical color compatibilities)
- c_i and c_f: two distinguished colors in C
  (initial and final color constraints)

Question: Are there
- naturals K and L, and
- a coloring function
  $\mathcal{C} : \{1, \ldots, K\} \times \{1, \ldots, L\} \to C$

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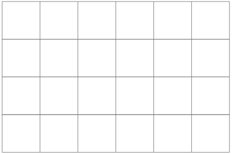

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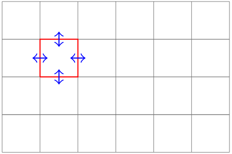

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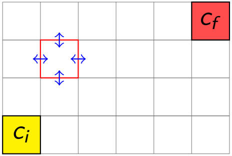

Overview of the proof

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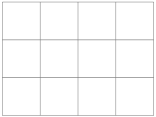
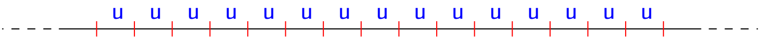
every u-interval “meets” a small u-interval

$$u \rightarrow \langle A_x \rangle u$$





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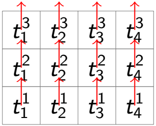

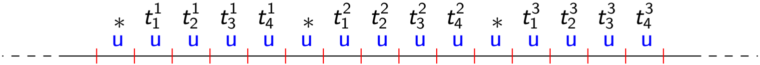

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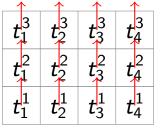

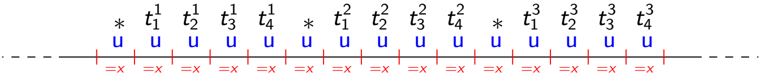

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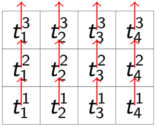
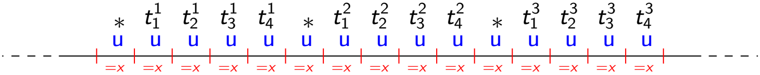
it is easy to give a length *upper bound*

$\langle A_x \rangle$u





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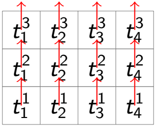
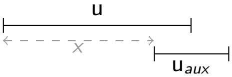
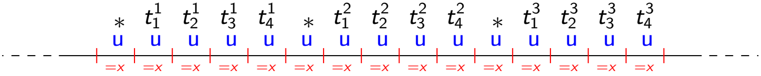


lower bound is trickier:

1 there is u_{aux}-interval starting at distance x from beginning of u-interval

$$\langle A\rangle u \to [A_x]\langle A\rangle u_{aux}$$





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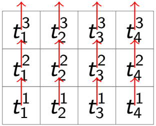
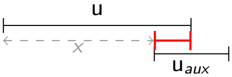
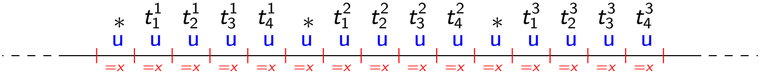


lower bound is trickier:

2 no small interval "meets" a u-interval while starting with a u_{aux}-interval

$$[G_x]\neg(\langle A\rangle u \wedge \langle\overline{A}\rangle\langle A\rangle u_{aux})$$





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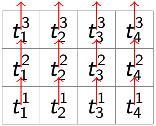

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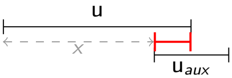

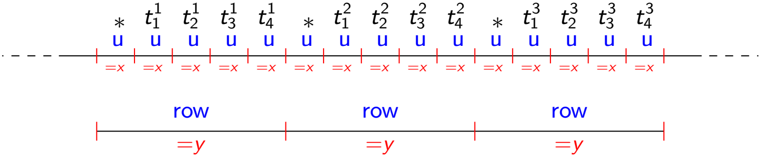

SAT is undecidable for PROMPT-PNL

Theorem.

The satisfiability problem for the future fragment of PROMPT-PNL is undecidable

Outline

The culprit for undecidability

- using bound x both in existential and universal modalities
- this gives the ability of expressing lower and upper bound for the length of intervals
- thus we can define special chains of intervals
- ... and we can use such special chains as a ruler to suitably encode vertical color compatibility relation

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Recipe for decidability

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   - split X into two sets $X_{\Diamond}$ (existential modalities) and $X_{\Box}$ (universal modalities)

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5. Solve infinite satisfiability

Infinite satisfiability

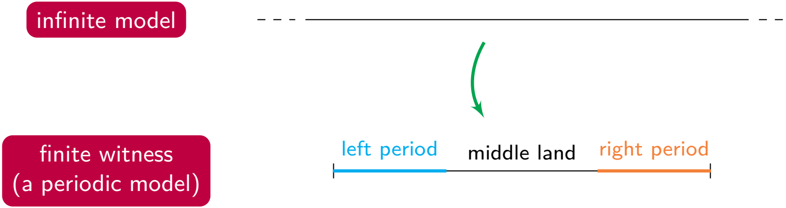
Proof via small model theorem

infinite model

finite witness
(a periodic model)

Infinite satisfiability

Proof via small model theorem



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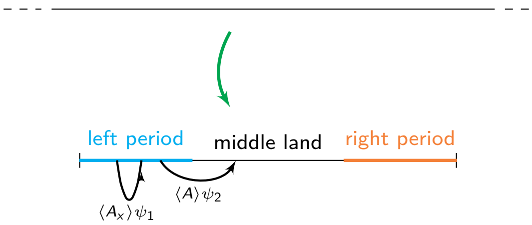

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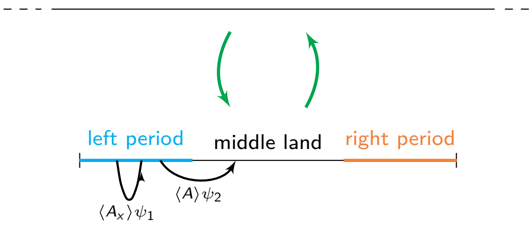

Infinite satisfiability

Proof via small model theorem

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finite witness
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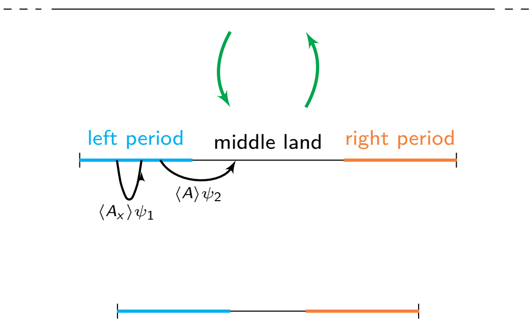
minimal periodic
model



"useless" points are removed
without loss of information

Infinite satisfiability

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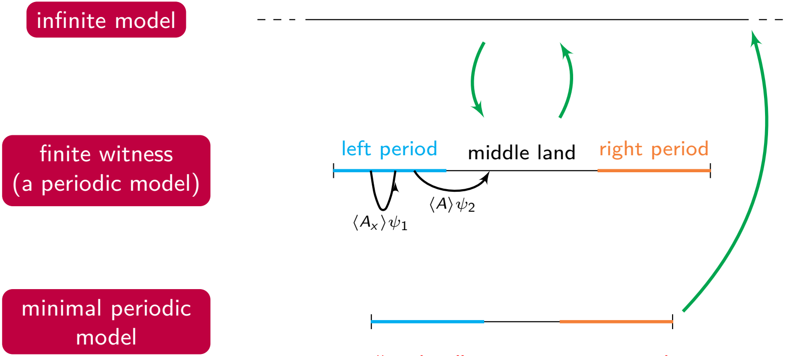

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SAT is decidable for $\mathtt{PROMPT}^d_{-}\mathtt{PNL}$

Theorem.

The satisfiability problem for $\mathtt{PROMPT}^d_{-}\mathtt{PNL}$ is decidable (NEXPTIME-complete)

Outline

Conclusions and future work

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two prompt extensions of Interval Temporal Logic PNL

- full logic PROMPT-PNL is undecidable
- its syntactic restriction $\text{PROMPT}\overset{d}{-}\text{PNL}$ is decidable
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Future work

- which is the minimum number of variables to make PROMPT−PNL undecidable
  - the *unrestricted* two variable fragment might be expressive and decidable
- parametric extensions of PNL
  - e.g., allowing both upper and lower bound
- comparison between PROMPT−PNL and *metric PNL*

Thank you!