On the expressiveness of the interval logic of Allen's relations over finite and discrete linear orders

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Joint work with L. Aceto, A. Ingólfsdóttir, A. Montanari, G.Sciavicco

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Outline

Interval Temporal Logics

Halpern-Shoham's modal logic HS

Expressiveness of HS fragments over discrete/finite linear orders

Conclusions

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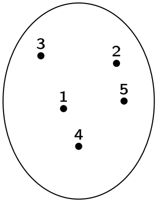
Temporal logics: origins and application fields

- Temporal logics play a major role in computer science
  - automated system verification
- Temporal logics are (multi-)modal logics



set of worlds
primitive temporal entity
time points/instants

accessibility relations

$\longrightarrow$: next

$\longrightarrow^{\star}$: finally

Temporal logics: origins and application fields

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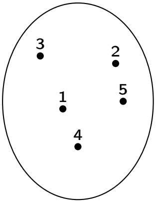
simplification



set of worlds
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$\longrightarrow$: next

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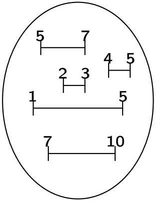
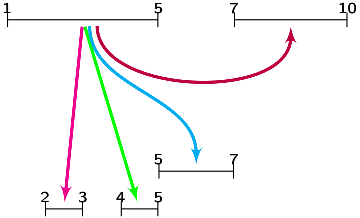
A different approach: from points to intervals

- worlds are intervals (time period — pairs of points)

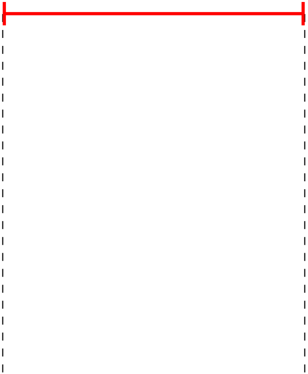


set of worlds
primitive temporal entity
time intervals/periods



accessibility relations
all binary relations between pairs of intervals

Binary interval relations on linear orders



J. F. Allen
Maintaining knowledge about temporal intervals
Communications of the ACM, volume 26(11), pages 832-843, 1983

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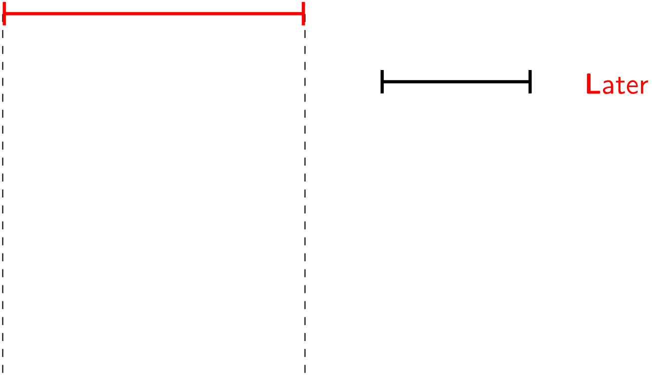

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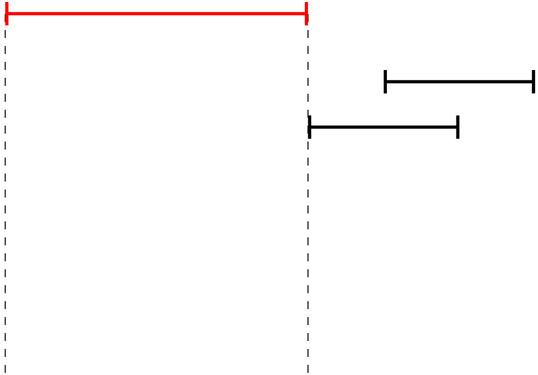

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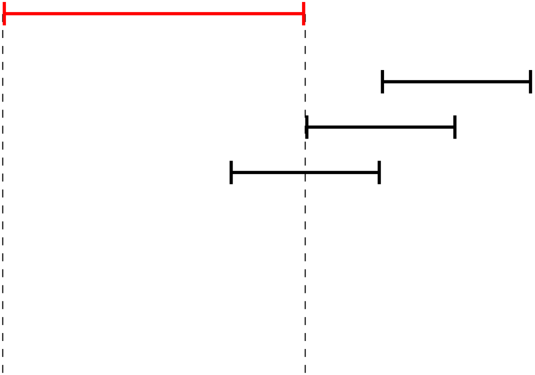

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Overlaps (to right)

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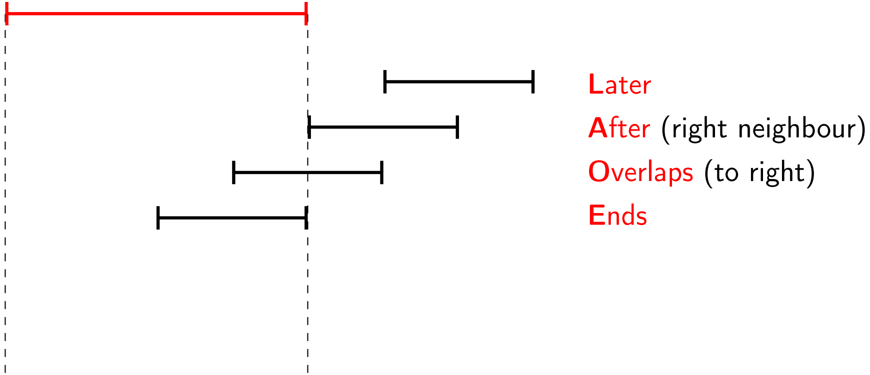

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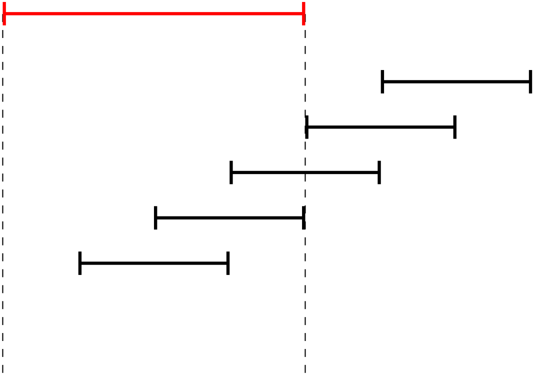

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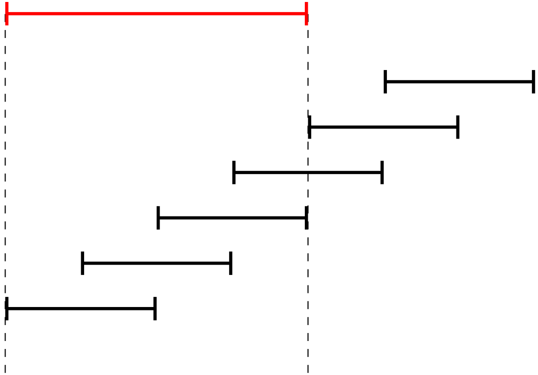

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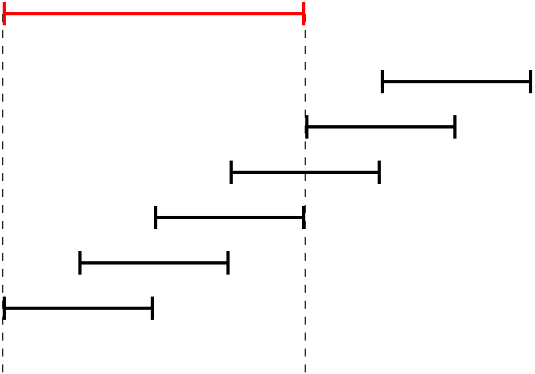

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6 relations + their inverses = 12 Allen's relations

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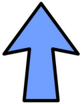
Halpern-Shoham's modal logic of interval relations

interval relations give rise to modal operators



HS logic

Halpern-Shoham's modal logic of interval relations

interval relations give rise to modal operators

HS logic

HS is undecidable over all significant classes of linear orders

J. Halpern and Y. Shoham
A propositional modal logic of time intervals
Journal of the ACM, volume 38(4), pages 935-962, 1991

Halpern-Shoham's modal logic of interval relations

interval relations give rise to modal operators → HS logic

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Syntax:
$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle X \rangle \varphi$$
$$\langle X \rangle \in \{\langle \mathrm{A} \rangle, \langle \mathrm{L} \rangle, \langle \mathrm{B} \rangle, \langle \mathrm{E} \rangle, \langle \mathrm{D} \rangle, \langle \mathrm{O} \rangle, \langle \overline{\mathrm{A}} \rangle, \langle \overline{\mathrm{L}} \rangle, \langle \overline{\mathrm{B}} \rangle, \langle \overline{\mathrm{E}} \rangle, \langle \overline{\mathrm{D}} \rangle, \langle \overline{\mathrm{O}} \rangle\}$$

Halpern-Shoham's modal logic of interval relations

interval relations give rise to modal operators ⇑ HS logic

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Models:

$$\mathbf{M} = \langle \mathbb{I}(\mathbb{D}), V \rangle$$

$$V : \mathbb{I}(\mathbb{D}) \mapsto 2^{\mathcal{AP}}$$

$\mathcal{AP}$ atomic propositions (over intervals)

Formal semantics of HS

$\langle B \rangle$: $\mathbf{M}, [d_0, d_1] \Vdash \langle B \rangle \phi$ iff there exists d_2 such that $d_0 \leq d_2 < d_1$ and $\mathbf{M}, [d_0, d_2] \Vdash \phi$.

$\langle \overline{B} \rangle$: $\mathbf{M}, [d_0, d_1] \Vdash \langle \overline{B} \rangle \phi$ iff there exists d_2 such that $d_1 < d_2$ and $\mathbf{M}, [d_0, d_2] \Vdash \phi$.

current interval:

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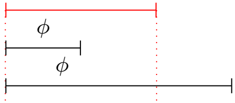

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$\langle E \rangle$: $\mathbf{M}, [d_0, d_1] \Vdash \langle E \rangle \phi$ iff there exists d_2 such that $d_0 < d_2 \leq d_1$ and $\mathbf{M}, [d_2, d_1] \Vdash \phi$.

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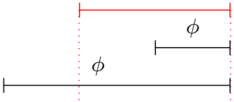

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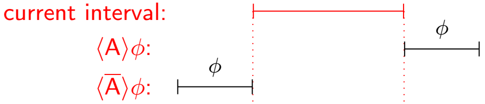

Formal semantics of HS - contd'

$\langle L \rangle$: $\mathbf{M}, [d_0, d_1] \Vdash \langle L \rangle \phi$ iff there exists d_2, d_3 such that $d_1 < d_2 < d_3$ and $\mathbf{M}, [d_2, d_3] \Vdash \phi$.

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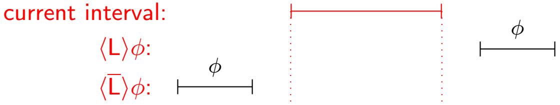

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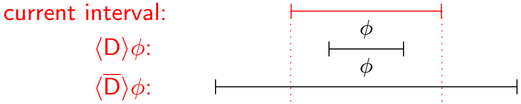

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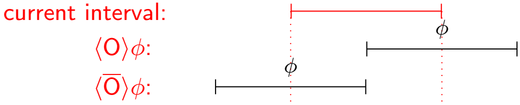
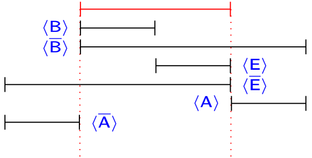


Definabilities among modalities

All modalities are definable in terms of $\langle B\rangle$, $\langle \overline{B}\rangle$, $\langle E\rangle$, $\langle \overline{E}\rangle$, $\langle A\rangle$, $\langle \overline{A}\rangle$

$HS \equiv B\overline{B}E\overline{E}A\overline{A}$



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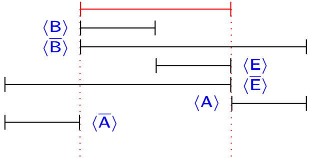

Defining the other interval modalities:

- Later: $\langle L \rangle \varphi \equiv \langle A \rangle \langle A \rangle \varphi$
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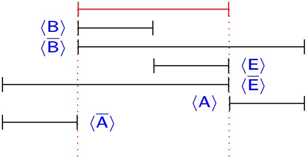

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- During (strict sub-interval): $\langle D\rangle\varphi \equiv \langle B\rangle\langle E\rangle\varphi$ $(\equiv \langle E\rangle\langle B\rangle\varphi)$
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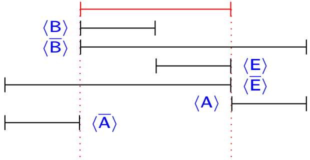

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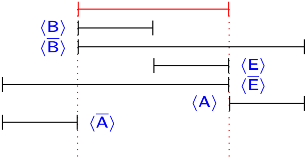

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In general, it is possible defining HS modalities in terms of others

The zoo of fragments of HS

- $2^{12} = 4096$ fragments of HS (syntactic)
- only ~ 1000 expressively different fragments
- expressiveness classification wrt. several classes of interval structures
  - all, dense, discrete, finite, ???

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Classification over all linear orders/dense linear orders

L. Aceto, D. Della Monica, V. Goranko, A. Ingólfsdóttir, A. Montanari, and G. Sciavicco

A Complete Classification of the Expressiveness of Interval Logics of Allen's Relations: The General and the dense cases

ACTA Informatica, 2014 (to appear)

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In this paper:

- finite
- discrete

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The expressiveness classification programme

Expressiveness classification programme: classify the fragments of HS with respect to their expressiveness, relative to classes of finite/discrete interval models.

Comparing expressive power of HS fragments

L_1, L_2 HS-fragments

L_1 L_2

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$$L_1 \ \{\prec, \equiv, \succ, \not\approx\} \ L_2$$

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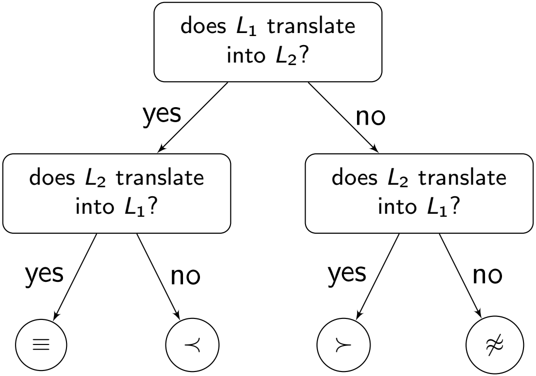

Truth-preserving translation

There exists a truth-preserving translation of L_1 into L_2
iff
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Each modality $\langle X \rangle$ of L_1 is definable in L_2
(i.e., $\exists$ a L_2-formula φ s.t. $\langle X \rangle p \equiv \varphi$)

Example: $\langle \mathrm{L} \rangle p \equiv \langle \mathrm{A} \rangle \langle \mathrm{A} \rangle p$

Truth-preserving translation

There exists a truth-preserving translation of L_1 into L_2
iff
L_2 is at least as expressive as L_1
$(L_1 \preceq L_2)$

Each modality $\langle X \rangle$ of L_1 is definable in L_2
(i.e., $\exists$ a L_2-formula φ s.t. $\langle X \rangle p \equiv \varphi$)

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2^{12} fragments... $\frac{2^{12} \cdot (2^{12} - 1)}{2}$ comparisons

Our approach

Solution:
To find a complete set of definabilities among modalities

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Notation:

$\mathsf{X_1 X_2 \ldots X_n}$
$=$
HS-fragment with modalities
$\langle X_1 \rangle, \langle X_2 \rangle, \ldots, \langle X_n \rangle$

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$$\overbrace{\mathsf{X_1X_2\ldots X_n}}^{\mathcal{X}} \qquad \overbrace{\mathsf{Y_1Y_2\ldots Y_m}}^{\mathcal{Y}}$$

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$$\overbrace{\mathsf{X_1X_2\ldots X_n}}^{\mathcal{X}} \qquad \overset{\{\prec, \equiv, \succ, \not\approx\}}{??} \qquad \overbrace{\mathsf{Y_1Y_2\ldots Y_m}}^{\mathcal{Y}}$$

Our approach

Solution:
To find a complete set of definabilities among modalities

Notation:

$$\mathsf{X_1 X_2 \ldots X_n} = \text{HS-fragment with modalities } \langle X_1\rangle, \langle X_2\rangle, \ldots, \langle X_n\rangle$$

$$\overbrace{\mathsf{X_1 X_2 \ldots X_n}}^{\mathcal{X}} \quad \overset{\{\prec, \equiv, \succ, \not\approx\}}{??} \quad \overbrace{\mathsf{Y_1 Y_2 \ldots Y_m}}^{\mathcal{Y}}$$

$\langle X_1\rangle \sqsubseteq \mathsf{Y_1 \ldots Y_m}$??

$\ldots$??

$\langle X_n\rangle \sqsubseteq \mathsf{Y_1 \ldots Y_m}$??

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To find a complete set of definabilities among modalities

Notation:

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$$\overbrace{\mathsf{X_1X_2 \dots X_n}}^{\mathcal{X}} \qquad \overset{\{\prec, \equiv, \succ, \not\approx\}}{??} \qquad \overbrace{\mathsf{Y_1Y_2 \dots Y_m}}^{\mathcal{Y}}$$

$$\begin{array}{cc}
\langle X_1 \rangle \sqsubseteq \mathsf{Y_1 \dots Y_m} & ?? \\
 & \wedge \\
\dots & ?? \\
 & \wedge \\
\langle X_n \rangle \sqsubseteq \mathsf{Y_1 \dots Y_m} & ?? \\
\hline
\mathcal{X} \preceq \mathcal{Y} & ??
\end{array}$$

Our approach

Solution:
To find a complete set of definabilities among modalities

Notation:

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$$\begin{array}{ccc}
\langle X_1\rangle \sqsubseteq \mathsf{Y_1\ldots Y_m} & ?? & \texttt{true} \\
 & \wedge & \wedge \\
\ldots & ?? & \texttt{true} \\
 & \wedge & \wedge \\
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$\langle X_1\rangle \sqsubseteq \mathsf{Y_1\ldots Y_m}$	??	true	...
	$\wedge$	$\wedge$	$\wedge$
...	??	true	false
	$\wedge$	$\wedge$	$\wedge$
$\langle X_n\rangle \sqsubseteq \mathsf{Y_1\ldots Y_m}$	??	true	...
$\mathcal{X} \preceq \mathcal{Y}$	??	true	false

Our approach - cont'd

$\mathcal{Y} \preceq \mathcal{X}$?

$\mathcal{X} \preceq \mathcal{Y}$?

Our approach - cont'd

		$\mathcal{Y} \preceq \mathcal{X}$? yes	$\mathcal{Y} \preceq \mathcal{X}$? no
$\mathcal{X} \preceq \mathcal{Y}$?	yes	$\mathcal{X} \equiv \mathcal{Y}$	$\mathcal{X} \prec \mathcal{Y}$
	no	$\mathcal{X} \succ \mathcal{Y}$	$\mathcal{X} \not\equiv \mathcal{Y}$

Complete sets of definabilities among modalities

$\langle L \rangle \sqsubseteq A$ $\quad \langle L \rangle p \equiv \langle A \rangle \langle A \rangle p$

$\langle D \rangle \sqsubseteq BE$ $\quad \langle D \rangle p \equiv \langle B \rangle \langle E \rangle p$

$\langle O \rangle \sqsubseteq \overline{B}E$ $\quad \langle O \rangle p \equiv \langle E \rangle \langle \overline{B} \rangle p$

J. Halpern and Y. Shoham
A propositional modal logic of time intervals
Journal of the ACM, 1991

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D. Della Monica, V. Goranko, A. Montanari, and G. Sciavicco

Expressiveness of the Interval Logics of Allen's Relations on the Class of all Linear Orders: Complete Classification

IJCAI 2011

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\end{array}
\right\} \text{class of all linear orders}
$$

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IJCAI 2011

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class of all linear orders

$\langle A \rangle \sqsubseteq \overline{B}E$ $\quad \langle A \rangle p \equiv \varphi(p) \vee \langle E \rangle \varphi(p)^{\dagger}$

$^{\dagger}\varphi(p) := [E]\bot \wedge \langle \overline{B} \rangle ([E][E]\bot \wedge \langle E \rangle (p \vee \langle \overline{B} \rangle p))$

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classes of discrete/finite linear orders (except for $\langle O \rangle$)

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class of all linear orders

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classes of discrete/finite linear orders (except for $\langle O \rangle$)

$\langle O \rangle \sqsubseteq ???$ $\quad \langle O \rangle p \equiv ???$

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class of all linear orders

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classes of discrete/finite linear orders (except for $\langle \mathrm{O} \rangle$)

$\langle \mathrm{O} \rangle \sqsubseteq$??? $\quad \langle \mathrm{O} \rangle p \equiv$???

under investigation

$^{\dagger}\varphi(p) := [\mathrm{E}]\bot \wedge \langle \overline{\mathrm{B}} \rangle ([\mathrm{E}][\mathrm{E}]\bot \wedge \langle \mathrm{E} \rangle (p \vee \langle \overline{\mathrm{B}} \rangle p))$

Proving non-existence

Existence is easy... **...non-existence is hard**

Proving non-existence

Existence is easy...

a new
land

...non-existence is hard

Proving non-existence

Existence is easy... **...non-existence is hard**

a new land

a bearded Icelander

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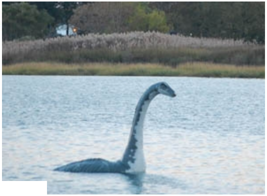


Nessy

elfs

Proving non-existence

Existence is easy...

a ne
la

Provide a witness

a beard
Icela

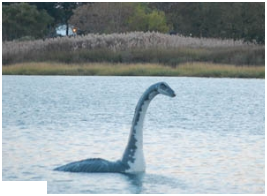
 

an Italian
in Reykjavik

...non-existence is hard

aliens

Nessy



elfs

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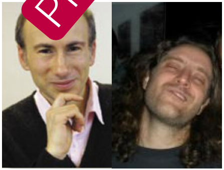

an Italian
in Reykjavik

...non-existence is hard

aliens

Ne

Bisimulations

elfs

Bisimulation between interval structures

$Z \subseteq M_1 \times M_2$ is a bisimulations wrt the fragment $\mathsf{X}_1\mathsf{X}_2\ldots\mathsf{X}_n$ iff

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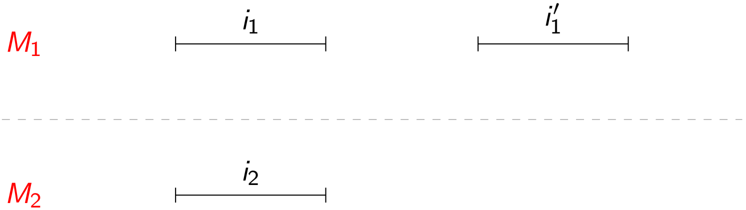
2. bisimulation relation "preserved" by modal operators, i.e., for every modal operator $\langle X \rangle$:



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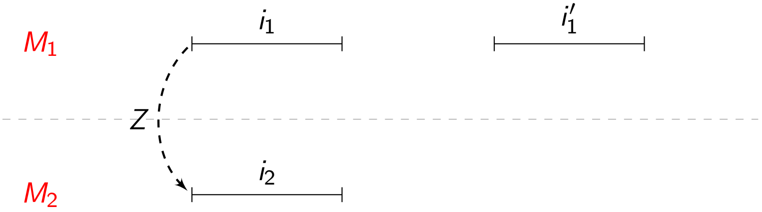
$$(i_1, i_2) \in Z$$



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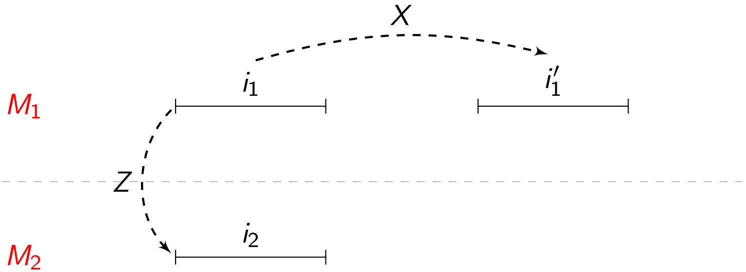
$$(i_1, i_2) \in Z$$
$$(i_1, i'_1) \in X$$



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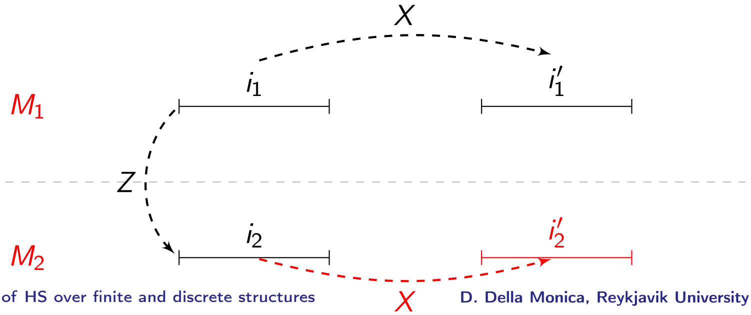

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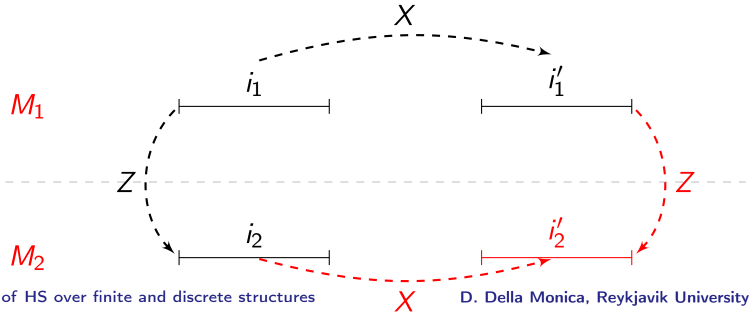

Bisimulation between interval structures - cont'd

Theorem A bisimulation for $\mathcal{L}$ preserves the truth of $\mathcal{L}$-formulae

$[a, b]$ and $[c, d]$ are bisimilar
φ is a $\mathcal{L}$-formula

φ is true in $[a, b]$ iff φ is true in $[c, d]$

How to use bisimulations to disprove definability

Suppose that we want to prove:

$\langle X \rangle$ is not definable in terms of $\mathcal{L}$

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We must provide:

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By contradiction

If $\langle X \rangle$ is definable in terms of $\mathcal{L}$, then $\langle X \rangle p$ is

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Truth of $\langle X \rangle p$ preserved by Z,

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   b. $M_1, i_1 \Vdash \langle X \rangle p$ and $M_2, i_2 \Vdash \neg\langle X \rangle p$

By contradiction
If $\langle X \rangle$ is definable in terms of $\mathcal{L}$, then $\langle X \rangle p$ is
Truth of $\langle X \rangle p$ preserved by Z,
but $\langle X \rangle p$ is true in i_1 (in M_1) and false in i_2 (in M_2)

How to use bisimulations to disprove definability

Suppose that we want to prove:

$\langle X \rangle$ is not definable in terms of $\mathcal{L}$

We must provide:

1. two models M_1 and M_2
2. a bisimulation $Z \subseteq M_1 \times M_2$ wrt fragment $\mathcal{L}$
3. two interval $i_1 \in M_1$ and $i_2 \in M_2$ such that
   a. i_1 and i_2 are Z-related
   b. $M_1, i_1 \Vdash \langle X \rangle p$ and $M_2, i_2 \Vdash \neg \langle X \rangle p$

By contradiction

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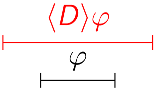
but $\langle X \rangle p$ is true in i_1 (in M_1) and false in i_2 (in M_2)
$\Rightarrow$ contradiction

An example: the operator $\langle D \rangle$

skip

Semantics:

$$M, [a, b] \Vdash \langle D \rangle \varphi \stackrel{def}{\Leftrightarrow} \exists c, d \text{ such that } a < c < d < b \text{ and } M, [c, d] \Vdash \varphi$$



An example: the operator $\langle D \rangle$

skip

Semantics:

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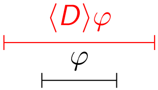

Operator $\langle D \rangle$ is definable in terms of BE $\quad \langle D \rangle \varphi \equiv \langle B \rangle \langle E \rangle \varphi$

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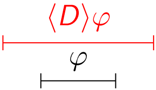


Operator $\langle D \rangle$ is definable in terms of BE $\qquad \langle D \rangle \varphi \equiv \langle B \rangle \langle E \rangle \varphi$

To prove that $\langle D \rangle$ is not definable in terms of any other fragment, we must prove that:

1) $\langle D \rangle$ is not definable in terms of ALBO$\overline{\text{ALBEDO}}$

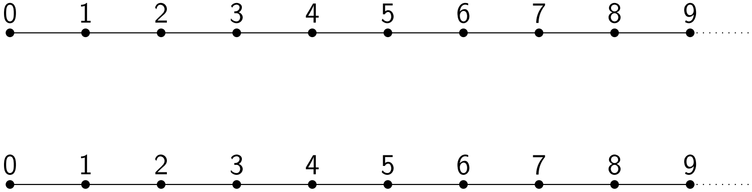
2) $\langle D \rangle$ is not definable in terms of ALEO$\overline{\text{ALBEDO}}$

$\langle D \rangle$ is not definable in terms of A

A bisimulation wrt fragment A but not D

Bisimulation wrt A ($\mathcal{AP} = \{p\}$):

- models: $M_1 = \langle \mathbb{I}(\mathbb{N}), V_1 \rangle$, $M_2 = \langle \mathbb{I}(\mathbb{N}), V_2 \rangle$



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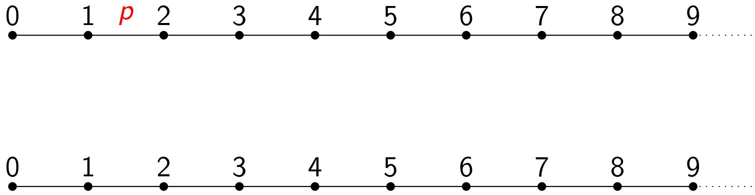

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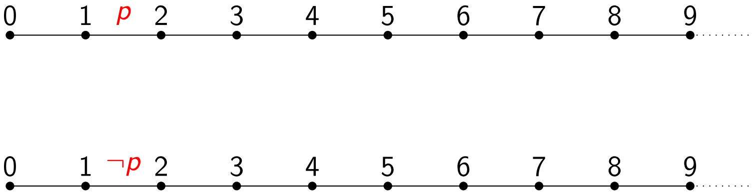

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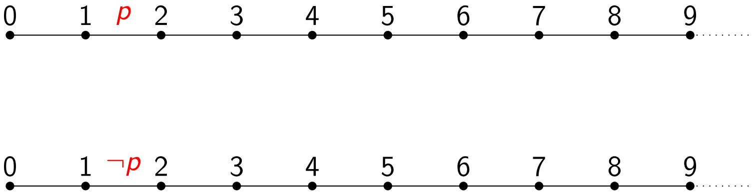
- bisimulation relation Z: $([x, y], [w, z]) \in Z$ iff



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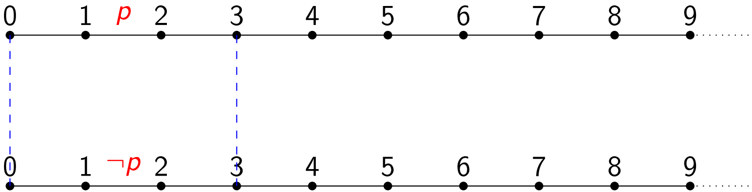

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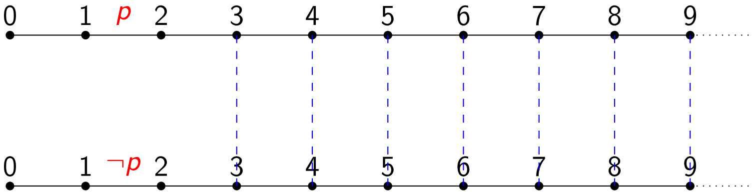

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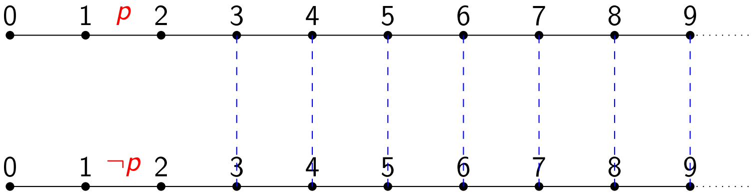

$\boxed{M_1, [0,3] \Vdash \langle D \rangle p \text{ and } M_2, [0,3] \Vdash \neg \langle D \rangle p}$

$\langle D \rangle$ is not definable in terms of A

A bisimulation wrt fragment A but not D

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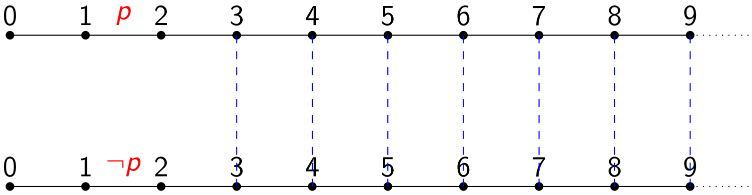

$\boxed{M_1, [0,3] \Vdash \langle D \rangle p \text{ and } M_2, [0,3] \Vdash \neg \langle D \rangle p}$ $\Rightarrow$ the thesis

Outline

Interval Temporal Logics

Halpern-Shoham's modal logic HS

Expressiveness of HS fragments over discrete/finite linear orders

Conclusions

Expressiveness classification: results and TODOs

DONE:

- class of all linear orders (1347 fragments) [IJCAI 11]
- classes of dense linear orders (966 fragments) [TIME 13]

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- $\langle O \rangle$ over finite/discrete linear orders — $\langle \overline{O} \rangle$ for free

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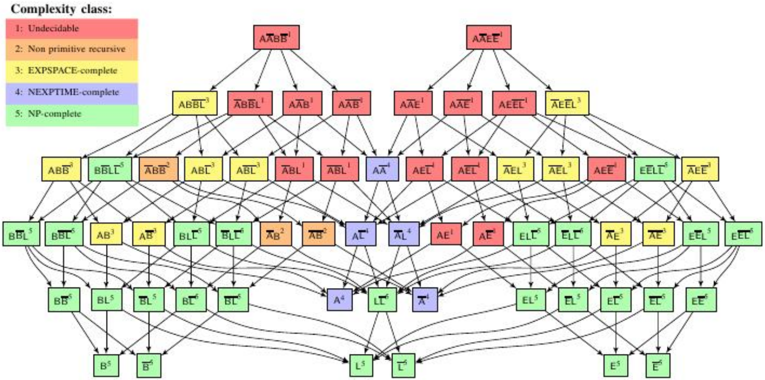
Bisimulation as a technique to disprove existence of definabilities

Expressiveness classification over natural numbers



The end

Thank you