Hybrid Metric Propositional Neighborhood Logics with Interval Length Binders

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Outline

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Time and logics

Studying time and its structure is of great importance in **computer science**:

- **Artificial Intelligence**.
  Planning, Natural Language Recognition, ...
- **Databases**.
  Temporal Databases.
- **Formal methods**.
  Specification and Verification of Systems and Protocols, Model Checking, ...

Points vs. intervals

Usually, time is formalized as a set of **time points** without duration.

But... this concept is extremely abstract:

time is actually viewed as a set of **intervals** (periods) with a duration.

Problem

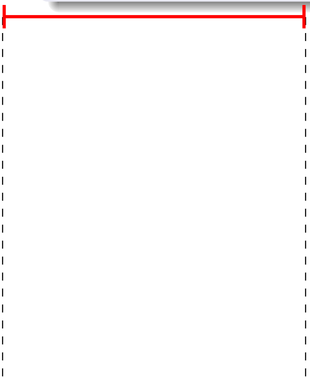
*It would be nice to have **temporal logics** that take time intervals as primary objects.*

Interval Temporal Logics

- The time period, instead of the time instant, is the primitive temporal entity
- Propositional letters are evaluated over pairs of points (instead of individual points)
- Relations between worlds are more complicate than the point-based case

Allen's relations

J. F. Allen
Maintaining knowledge about temporal intervals.
Communications of the ACM, 1983.



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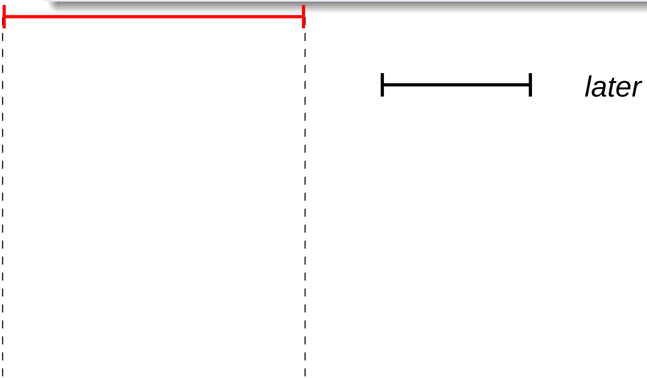

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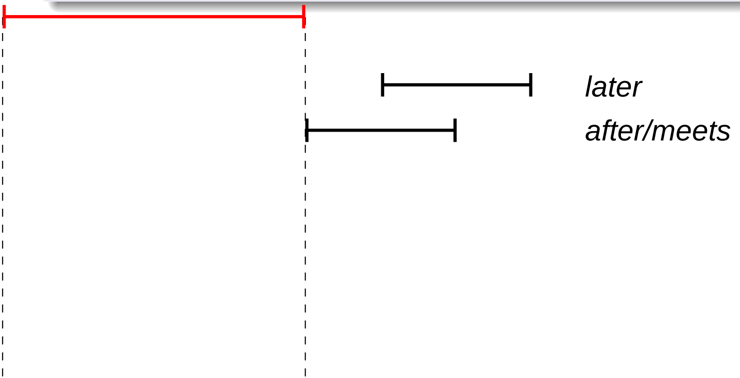

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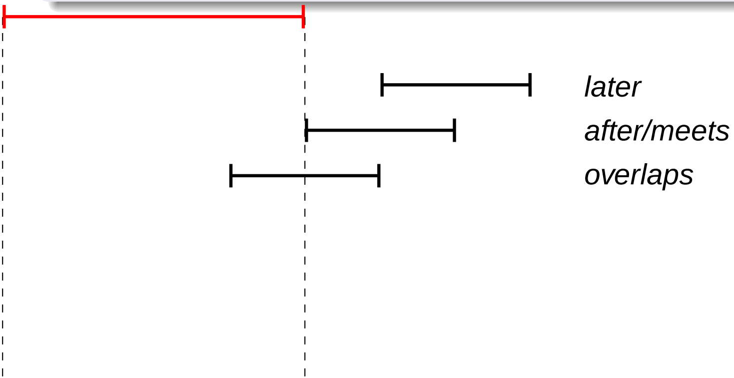

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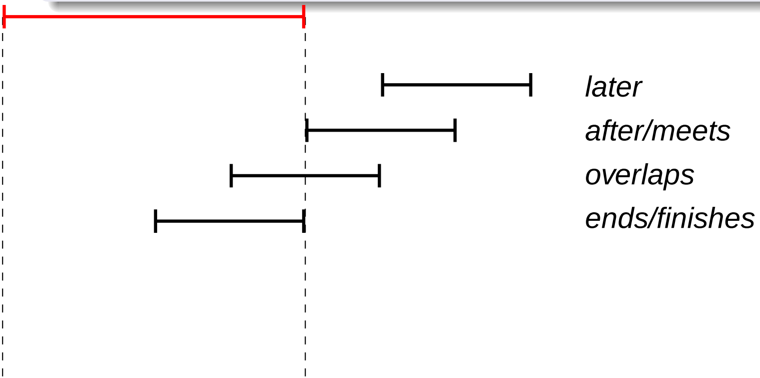

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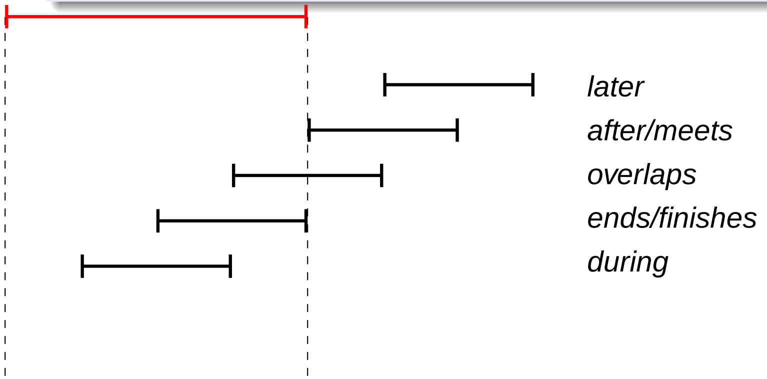

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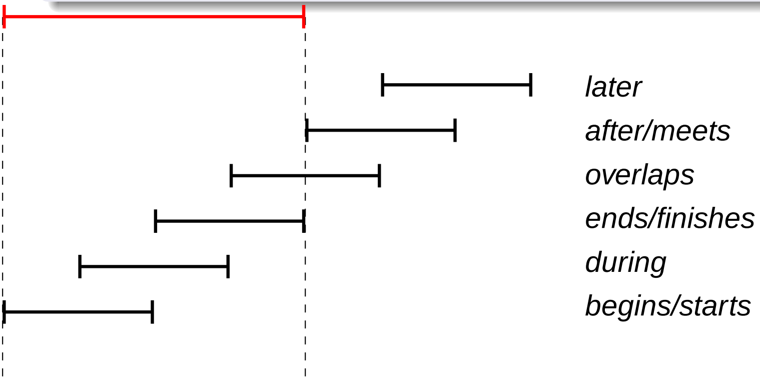

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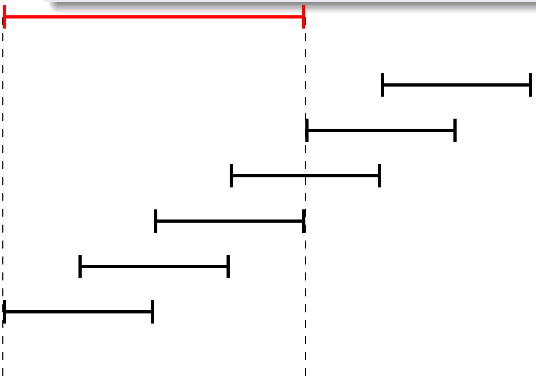

later	$\langle L\rangle$
after/meets	$\langle A\rangle$
overlaps	$\langle O\rangle$
ends/finishes	$\langle E\rangle$
during	$\langle D\rangle$
begins/starts	$\langle B\rangle$

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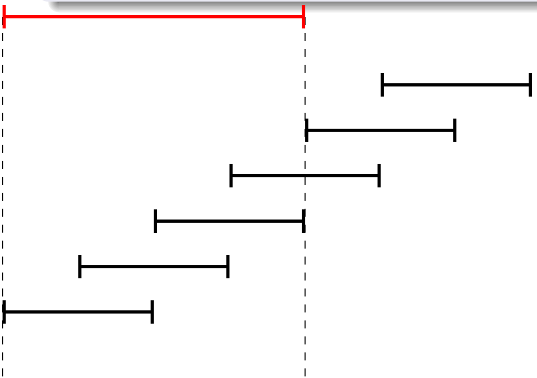

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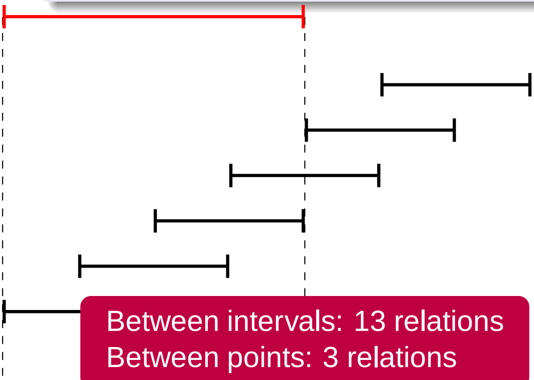

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Between intervals: 13 relations
Between points: 3 relations

First discouraging undecidability results

HS is undecidable

J. Halpern and Y. Shoham
A propositional modal interval logic.
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Undecidability of a small fragment of HS: BE

K. Lodaya
Sharpening the Undecidability of Interval Temporal Logic.
ASIAN 2000, volume 1961 of LNCS, pages 290-298. Springer, 2000.

Some decidable fragments

- **RPNL** (A)

D. Bresolin, A. Montanari, and G. Sciavicco
An optimal decision procedure for Right Propositional Neighborhood Logic.
Journal of Automated Reasoning, 2007.

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An optimal tableau-based decision algorithm for Propositional Neighborhood Logic.
STACS 2007, volume 4393 of LNCS, pages 549-560. Springer, 2007.

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PNL

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PNL

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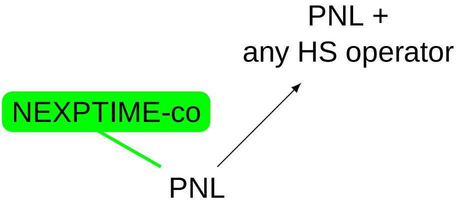

Extending PNL

Undecidable

PNL +
any HS operator

NEXPTIME-co

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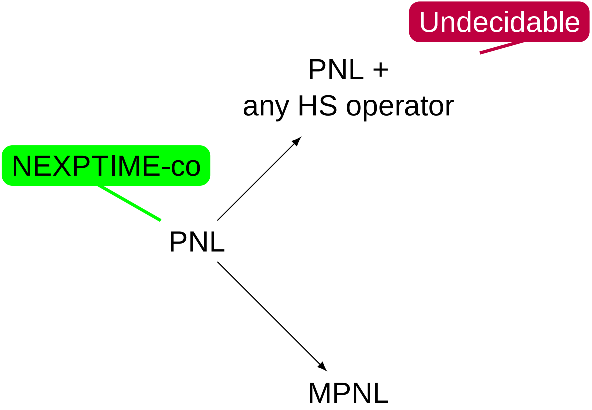

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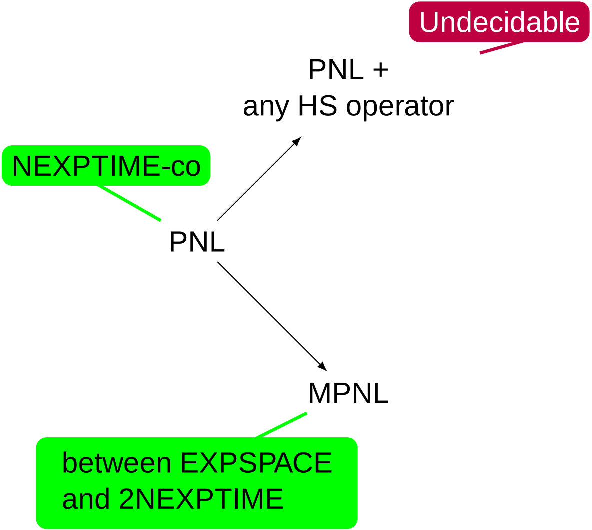

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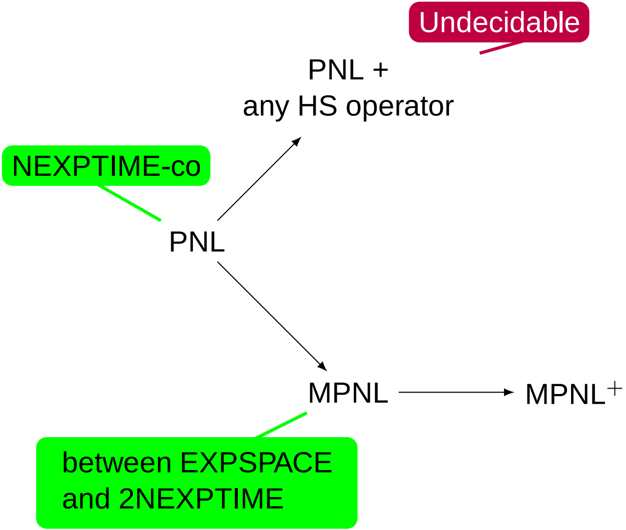

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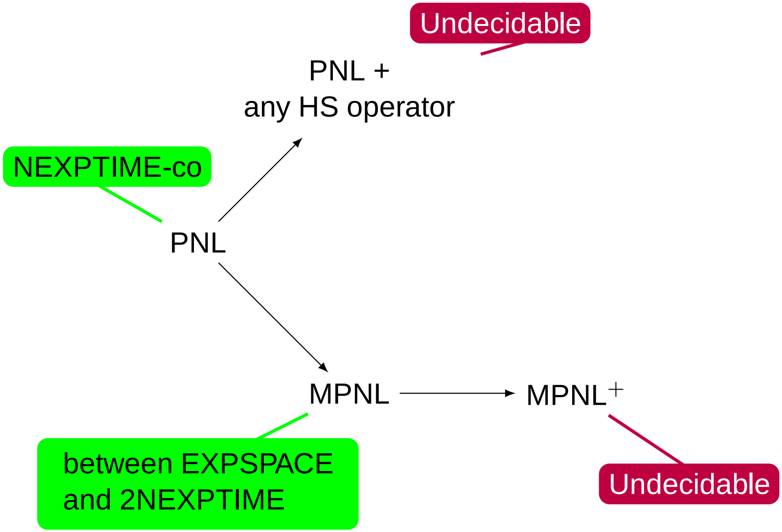

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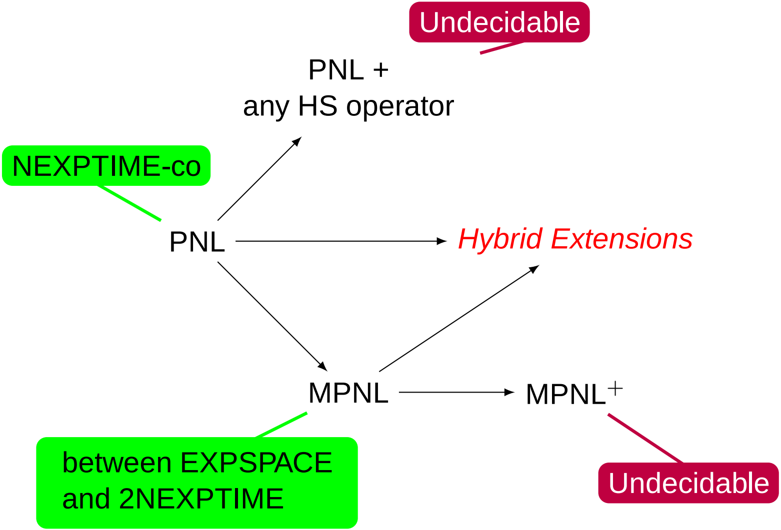

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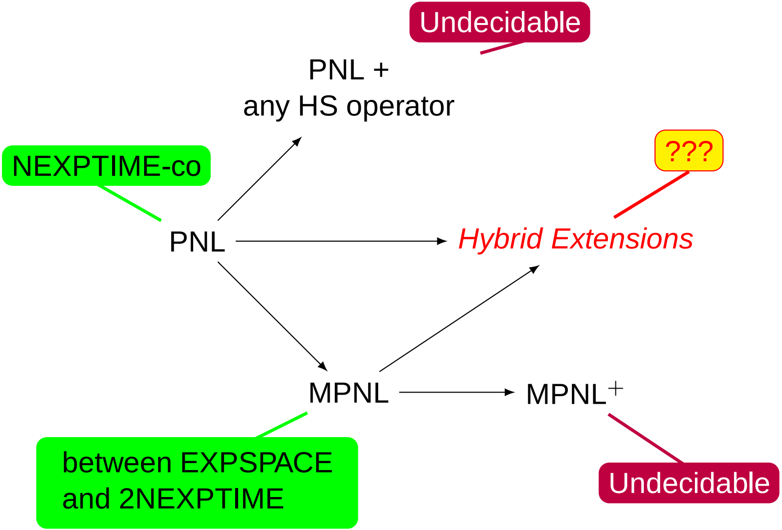

Possible hybrid extension of PNL and MPNL

Nominals are definable in PNL
(*Basic Hybrid PNL*)

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Binders over length of intervals
(*Weakly Hybrid MPNL*)

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PNL and MPNL: syntax and semantics

Syntax

- PNL: $\varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle A \rangle \varphi \mid \langle \overline{A} \rangle \varphi$

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- Operators *meets* ($\langle A\rangle$) and *met-by* ($\langle\overline{A}\rangle$):

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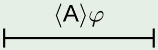

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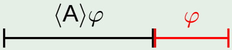

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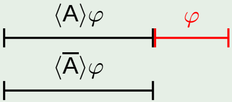

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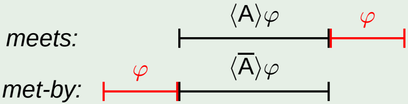

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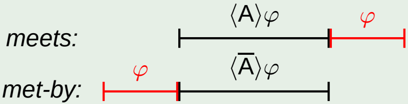

- Metric constraints over the length of the current interval:
  $\text{len}_{\sim k}$ holds over $[d_0, d_1]$ iff $d_1 - d_0 \sim k$

Weakly Hybrid MPNL (WHMPNL)

Metric constraints of MPNL use constants

$$\mathsf{len}_{=5}, \mathsf{len}_{>2}, \ldots$$

WHMPNL allows one to store the length of the current interval and to refer to it in sub-formulae

$$\downarrow_x (\ldots \mathsf{len}_{=\mathsf{x}}), \downarrow_x (\ldots \mathsf{len}_{\leq \mathsf{x}}), \ldots$$

WHMPNL fragments

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	set of hybrid constraints	constant constraints	# of length variables
WHMPNL$(<, \leq, =, \geq, >)$	$\{<, \leq, =, \geq, >\}$	YES	unbounded
WHPNL$(<, =)$	$\{<, =\}$	NO	unbounded
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The fragments $WHPNL(<)_1$ and $WHPNL(>)_1$

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$WHPNL(<)_1$ undecidability

$$\langle \mathrm{B} \rangle p := \downarrow_x \langle \overline{\mathrm{A}} \rangle \langle \mathrm{A} \rangle (\mathrm{len}_{<x} \wedge p)$$

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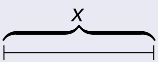

store length of current interval in x

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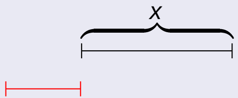

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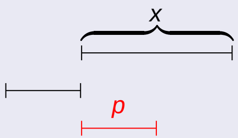

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$WHPNL(>)_1$ undecidability

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The fragments $WHPNL(\leq)_1$ and $WHPNL(\geq)_1$

$WHPNL(\leq)_1$ and $WHPNL(\geq)_1$ undecidability

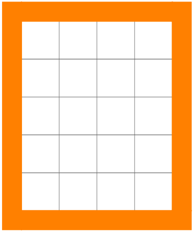
Immediately from:

- $\mathrm{len}_{<x} \Leftrightarrow \neg \mathrm{len}_{\geq x}$
- $\mathrm{len}_{>x} \Leftrightarrow \neg \mathrm{len}_{\leq x}$

The fragment $WHPNL(=)_1$

Reduction from the Finite Tiling Problem

This is the problem of establishing whether, for a given finite set of tile types $\mathcal{T} = \{t_1, \ldots, t_k\}$, there exists a finite rectangle $\mathcal{R}$ having the border colored with a fixed color ■ such that $\mathcal{T}$ can tile $\mathcal{R}$ respecting the color constraints.



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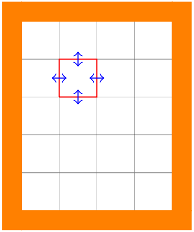
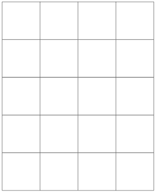


Proof overview

1. Encoding the rectangle
2. Encoding the neighbourhood relations



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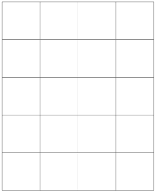

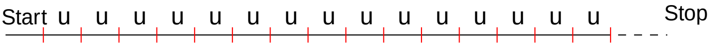

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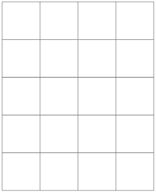

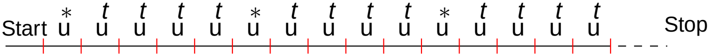

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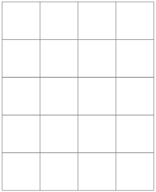
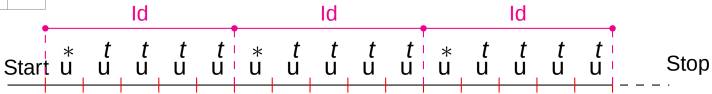
2. Encoding the neighbourhood relations





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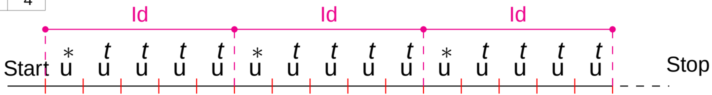
t_1^5	t_2^5	t_3^5	t_4^5
t_1^4	t_2^4	t_3^4	t_4^4
t_1^3	t_2^3	t_3^3	t_4^3
t_1^2	t_2^2	t_3^2	t_4^2
t_1^1	t_2^1	t_3^1	t_4^1



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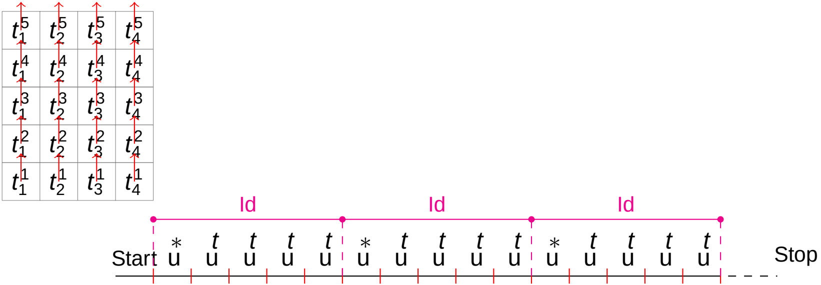

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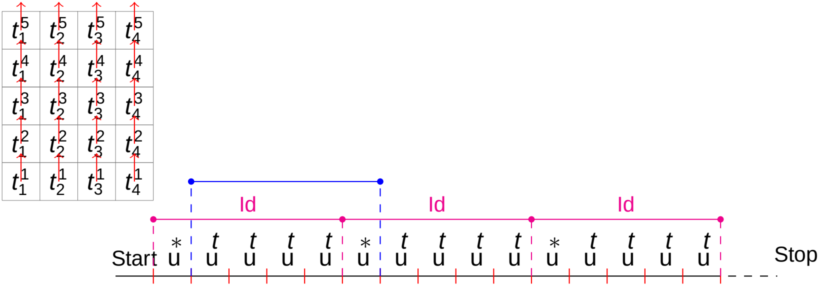

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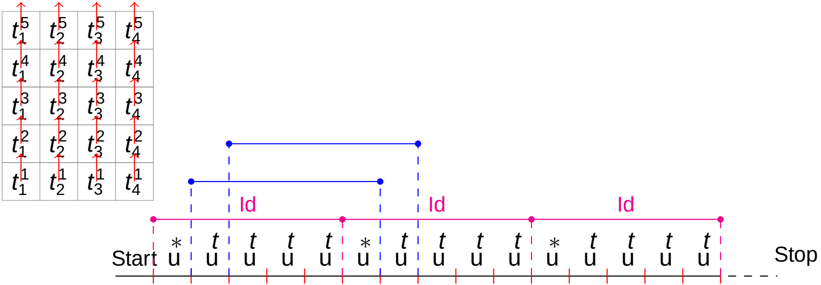

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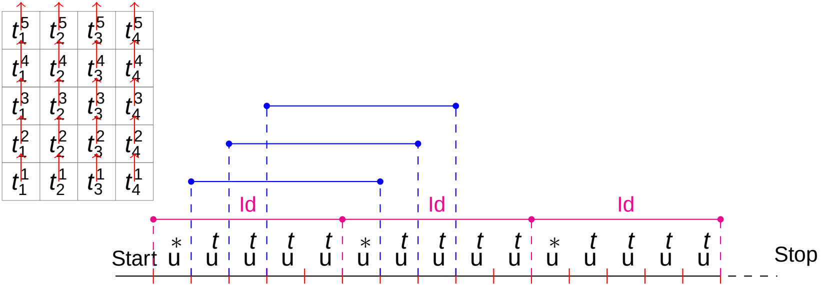

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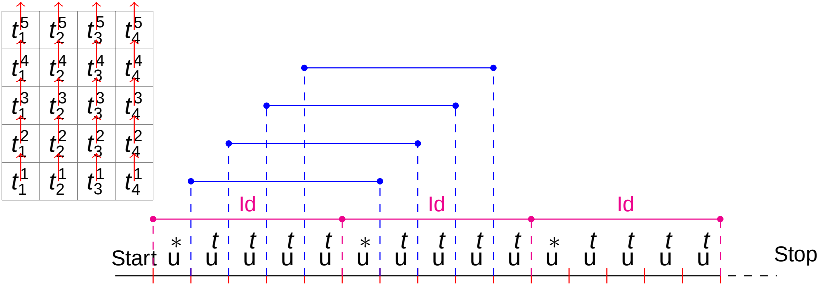

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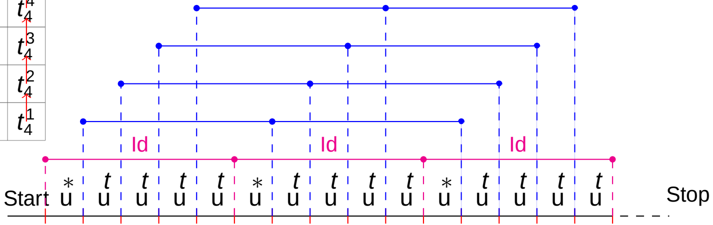
t_1^1	t_2^1	t_3^1	t_4^1



Regaining decidability

Binder can store into variables a bounded number of values (up to k)

Two semantic restrictions to the binder (when length is greater then k):

1. in restricted semantic, the binder stores into the variable a non-deterministic value greater than k (hybrid constraints occur in positive form)
2. in truncated semantic, the binder stores into the variable the length $k+1$

Both logics can be translated into MPNL (size at most exponential)

Complexity is in 3NEXPTIME

Outline

Conclusions

- Analyzed a number of hybrid extension of (M)PNL
- Also very weak extensions lead to undecidability
- Proposed decidable extension
  - no actual gain in expressivity wrt MPNL

Open question

Hybrid machinery increasing expressive power without losing decidability

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