Towards automaton-based characterisations of FO over trees

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First-Order Logic (FO) over trees is a well-studied subject from many points of view. It enjoys different algebraic characterisations, even though not effective, like the one based on forest algebras provided by Bojańczyk, Straubing and Walukiewicz [2], as well as equivalent presentations in terms of branching-time temporal logics. However, automata-theoretic characterisations are missing, in particular for FO over infinite trees. Automaton-based characterisations instead exist for Monadic Second Order Logic (MSO) [7] and some of its fragments, like Monadic Chain Logic and Monadic Path Logic [1].

In this talk, I will first present a generalization of a result by Hafer and Thomas [4], establishing the equivalence between FO and a variant of CTL*, denoted CTL_f*, with path quantification restricted to finite paths. Their work focuses on infinite binary trees, while we consider infinite arbitrary (but finitely) branching trees. Specifically, we relate FO to $cCTL_f$*, i.e. the extension of CTL_f* with operators for counting distinct successors. Our result complements another characterization by Schlingoff [6], who proved that a variant of CTL enriched with past operators is equivalent to FO.

Using these two temporal logics as a starting point, I will present the developments in the quest for automaton-based characterizations of FO over infinite trees, in the same spirit as recent works on automata for CTL [3], Monadic Chain Logic, and Monadic Path Logic [1]. All these characterisations use variants of alternating automata with logically-defined transition functions and structural restrictions (e.g., the *hesitant* partition on the state set, originally introduced in [5]). Finally, I will briefly discuss the impact on expressiveness of one-way vs. two-way head movements, also in comparison with the simpler setting of infinite words.

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[1] M. Benerecetti, L. Bozzelli, F. Mogavero, A. Peron, Automata-Theoretic characterisations of branching-time temporal logics, 51st International Colloquium on Automata, Languages and Programming, (2024).

[2] M. Bojańczyk, H. Straubing, and I. Walukiewicz, Wreath products of forest algebras, with applications to tree logics, Logical Methods in Computer Science, 8 (2012).

[3] U. Boker and Y. Shaulian, Automaton-based criteria for membership in CTL, Proceedings of the 33rd Annual ACM/IEEE Symposium on Logic in Computer Science (2018).

[4] T. Hafer and W. Thomas, Computation tree logic CTL* and path quantifiers in the monadic theory of the binary tree, International colloquium on automata, languages, and programming, (1987).

[5] O. Kupferman, M.Y. Vardi, and P. Wolper. An Automata Theoretic Approach to Branching-Time Model Checking. JACM, 47(2), (2000).

[6] B. H. Schlingoff, Expressive completeness of temporal logic of trees, Journal of Applied Non-Classical Logics, 2(2), (1992).

[7] J. W. Thatcher and J. B. Wright, Generalized finite automata theory with an application to a decision problem of Second-Order Logic, Mathematical Systems Theory, 2.1 (1968).