Model checking coalitional games in shortage resource scenarios

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Outline

1. Context
   - Multi-Agent Systems (MAS) — ***ATL***
   - MAS + resource constraints — ***RB-ATL / RAL***
2. Our proposal: *Priced* RB-ATL — ***PRB-ATL***
   - Model checking (lower bound)
   - Optimization problem
3. Conclusions

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- Several agents
- Intelligent (take decisions, moves)
- Independent
- Global state (union of single states)
- Next state univocally identified by moves

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Agents and coalitions

COALITION - modeling collective behaviors/strategies

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Logical Formalisms

Coalition Logic (CL) and Alternating-time Temporal Logic (ATL)

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Logical Formalisms

Coalition Logic (CL) and Alternating-time Temporal Logic (ATL)

Theorem (Goranko, TARK 2001)

CL *can be embedded into* ATL

ATL: syntax and semantics

Formulae of ATL are given by the grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\langle A\rangle\rangle \bigcirc \varphi \mid \langle\langle A\rangle\rangle\Box\varphi \mid \langle\langle A\rangle\rangle\varphi\mathcal{U}\varphi$$

Formulae of ATL predicate about abilities of coalitions of agents

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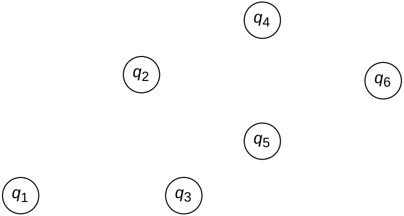
Formulae of ATL predicate about abilities of coalitions of agents

Formulae of ATL are evaluated wrt:

- a game structure (or game arena) G
- a location q of G

The arena of ATL

A game structure G is a state transition graph:



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- in vertices agents choose actions
- possible combinations $\rightarrow$ transitions (edges of the graph)

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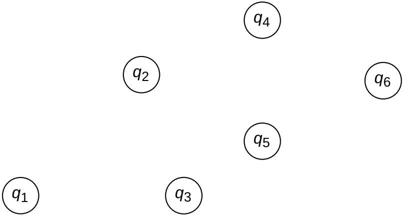

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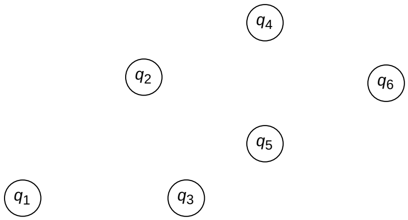

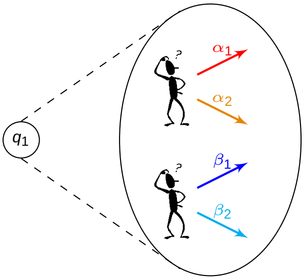

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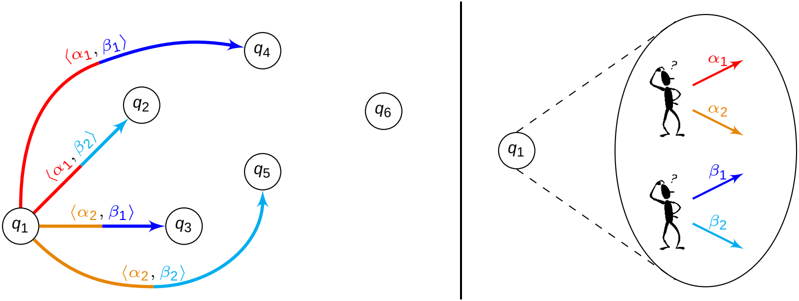

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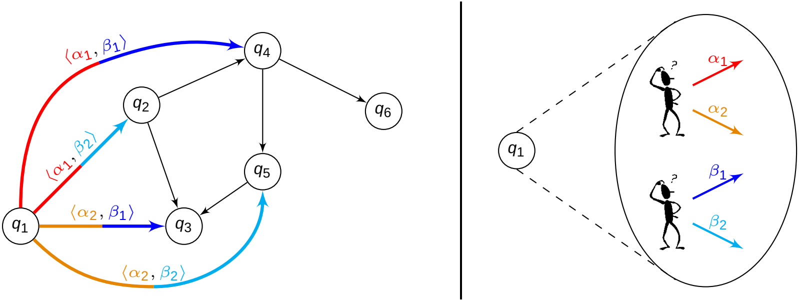

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Becoming friendly with ATL

Collective strategy for the proponent team to guarantee p holds

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regardless of actions performed by other agents (opponent)

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Addition of bounds on resources to ATL

Extensions of ATL with bounds on resources

$$\langle\langle A^{\eta}\rangle\rangle \Box p$$

Endowment: $\eta : A \to \mathbb{N}^r$ (r = number of resources)

The literature about Resource Bounded ATL (RB-ATL)

RB-ATL [Alechina, Logan, Nga, Rakib, AAMAS 2010]

Theorem: Model checking RB-ATL is decidable in $O(|\varphi|^{2 \cdot r+1} \times |G|)$
No lower bound

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RAL [Bulling, Farwer, ECAI 2010]

Several logic variants, exploration of the (un)decidability border

E.g., if actions produce resources, then Model Checking becomes UNDECIDABLE

RB-ATL: syntax and semantics

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Formulae of RB-ATL predicate about abilities of coalitions whose agents are equipped with a finite endowment of resources

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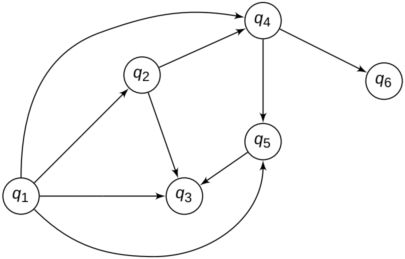

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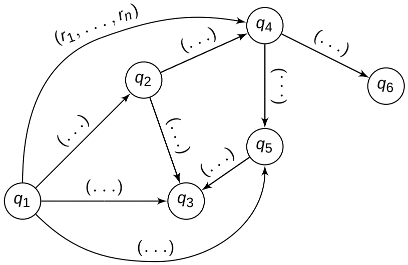

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$$G, q \Vdash \langle\langle A^{\eta} \rangle\rangle \bigcirc \langle\langle A^{\eta'} \rangle\rangle \Box p$$

team A, equipped with endowment η, can force the next state to be s.t. the team A itself can guarantee that p always holds equipped with the new endowment η'

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An anomalous behavior

$\eta(a) = 1$

$$G, q_0 \Vdash \langle\langle a^{\eta} \rangle\rangle \Box p$$

An anomalous behavior

2 agents: **a** and **b**
1 resource type: $\mathbf{r_1}$

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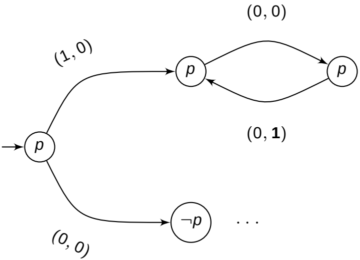

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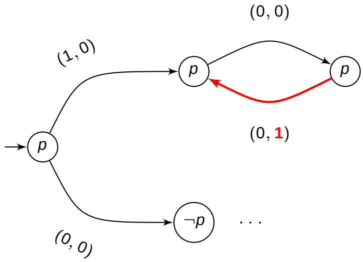

opponent consumes an infinite amount of resources

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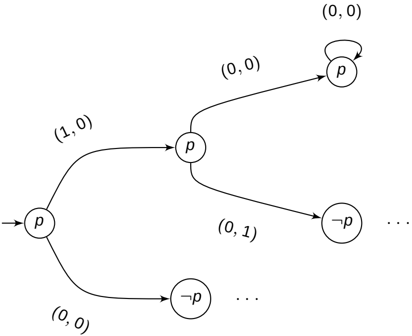

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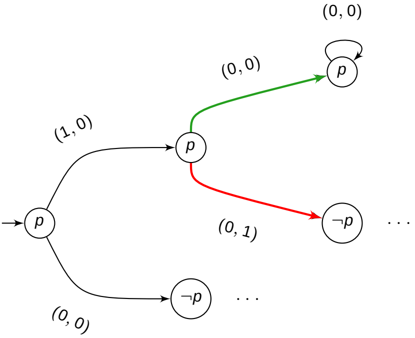

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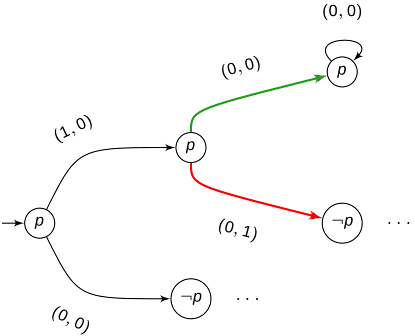

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opponent's moves should be constrained

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Weaknesses of previous formalism ***RB-ATL***

- NO history (resources)
  - $G, q \Vdash \langle\langle A^{\eta} \rangle\rangle \bigcirc \langle\langle A^{\eta'} \rangle\rangle \Box p$ η and η' are independent
- opponent does NOT consume
  - opponent has no bounds on resources
  - consumption by opponent does not matter

What we want

- opponent's actions constrained
- consumption/production tracked
- a significant present-day issue $\Rightarrow$ procurement of resources
  - limited amount on the market (or in nature)
  - acquisition cost depending on current availability

How we get it

Key notion $\Rightarrow$ global availability of resources on the market

- a semantic component (part of the arena)
- evolves depending on agents' actions (also opponent)
- affects the choice of the actions (also opponent)

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Auxiliary notion $\Rightarrow$ price of resources

- agents equipped with money instead of resources
- money for getting resources
- price of resources function of several components (take into account the history of the system)

Money vs. resources - our proposal

Money

- inside the formula
- assigned to agents
- private: any agent has his own amount of money
- unknown
- availability checked for proponent's agents only

Resources

- part of the model
- represent the market (nature)
- public: agents draw on resources from a shared pool
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Money is a ***meta-resource***

- buy resources
  - money like resources in previous approaches
- unit of measurement

Resource production and decidability

Alechina, Logan, Nga, Rakib

Actions can **only consume** resources

Bulling, Farwer

If actions may produce resources,
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Actions may produce resources...

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Actions may produce resources...
...but ***not so much***!!!

- model checking decidable
- several models fit
  (e.g. memory usage, leasing a car, releasing resources previously acquired)

Formulae of PRB-ATL are given by the grammar:

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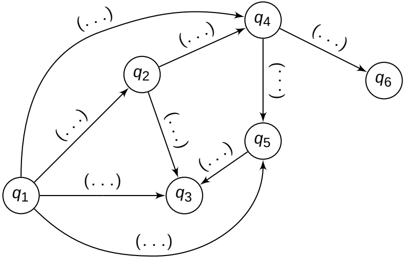
Priced game structure

A priced game structure G is a weighted state transition graph:



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- transition guards: also opponent

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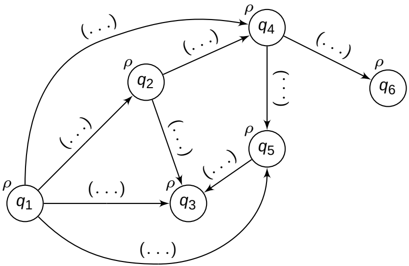

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Theorem

The model checking problem for PRB-ATL *is EXPTIME-complete*

- *membership (upper bound)* [LAMAS 2011]
- *hardness (lower bound)*

- Same asymptotic complexity as RB-ATL

Model checking complexity

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Reduction from the acceptance problem for Linearly-Bounded Alternating Turing Machine

Parametrized reduction

Model checking is exponential time in

- n (number of agents)
- r (number of resources)
- size of M (maximum component in endowment)

1st reduction: parametric in the size of M (n and r are constant)

2nd reduction: parametric in r (n and M are constant)

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Parametric PRB-ATL formulae

- PRB-ATL: $\varphi = \langle\langle A_1^{\$_1}\rangle\rangle\Diamond(\langle\langle A_2^{\$_2}\rangle\rangle \bigcirc p \vee \langle\langle A_3^{\$_3}\rangle\rangle q\mathcal{U}p)$

Definition (Cost of a PRB-ATL formula)

$$f\_cost(\varphi) = \$_1(A_1) + \$_2(A_2) + \$_3(A_3)$$

- parametric PRB-ATL: $\varphi_{\vec{X}} = \langle\langle X_1^{\$_1}\rangle\rangle\Diamond(\langle\langle X_2^{\$_2}\rangle\rangle \bigcirc p \vee \langle\langle A_3^{\$_3}\rangle\rangle q\mathcal{U}p)$

The *Optimal Coalition* problem

Definition (Optimal Coalition problem)

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Input:

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A logic for modeling multi-agent systems with bounds on resources

ATL: abilities of coalitions of agents

RB-ATL: abilities of coalitions whose agents are equipped with a finite endowment of resources

PRB-ATL: abilities of coalitions whose agents are equipped with an amount of money

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- money - price of resources

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Theorem: Model checking PRB-ATL is EXPTIME-complete
Reachability for PRB-ATL is EXPTIME-complete

Future work

- 3rd reduction: parametric in the n (r and M are constant)
- Exact complexity when action cannot produce resources
  - Reachability is NP-hard
  - Model checking is PSPACE-hard
- Resource-bounded extensions of other classical formalisms
  - μ-calculus [Della Monica, Lenzi - ICAART 2012]
  - ATL* ???
  - ...

The end

Thanks for the attention