Model Checking Coalitional Games with Priced-Resource Agents

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At a glance

- We presented the logic PRB-ATL [CILC 11 & GAMES 11]
- PRB-ATL is inspired to existing extensions of ATL
  - To deal with bounded resources scenarios in multi-agent systems
- We studied the model checking for PRB-ATL
  - It is in EXPTIME (upper bound) [LAMAS/M4M 11]
  - It is EXPTIME-hard (lower bound) [GAMES 12]
- We studied the optimal coalition problem [LAMAS/M4M 11]

Outline

1. Introduction to Multi-Agent Systems (MAS) - ***ATL***
   - Multi-Agent Systems and resource constraints - ***RB-ATL***
2. Our proposal: the logic *Priced* RB-ATL - ***PRB-ATL***
   - Model checking
   - Optimization problem
3. Conclusions

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Multi-Agent Systems (MAS)

- Several agents
- Intelligent (take decision)
- Independent
- Global state (union of single states)
- Move choices
- Next state

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Agents and coalitions

COALITION - modeling collective behaviors/strategies

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Logical Formalisms

Coalition Logic (CL) and Alternating-time Temporal Logic (ATL)

Agents and coalitions

COALITION - modeling collective behaviors/strategies

Logical Formalisms

Coalition Logic (CL) and Alternating-time Temporal Logic (ATL)

Theorem (Goranko, TARK 2001)

CL *can be embedded into* ATL

ATL: syntax and semantics

Formulae of ATL are given by the grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\langle A \rangle\rangle \bigcirc \varphi \mid \langle\langle A \rangle\rangle \varphi \mathcal{U} \varphi \mid \langle\langle A \rangle\rangle \Box \varphi$$

Formulae of ATL predicate about abilities of coalitions of agents

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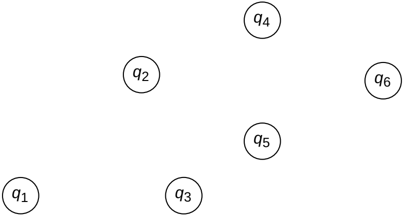
Formulae of ATL predicate about abilities of coalitions of agents

Formulae of ATL are evaluated wrt:

- a game structure (or game arena) G
- a location q of G

The arena of ATL

A game structure G is a state transition graph:



- locations labeled by atomic propositions
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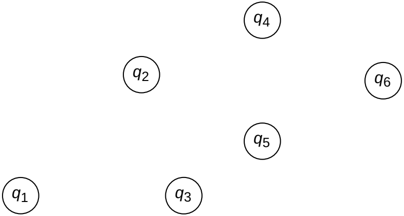

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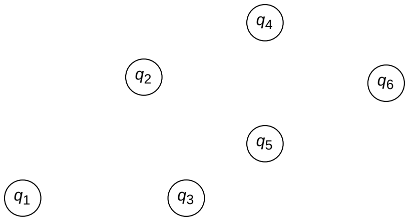

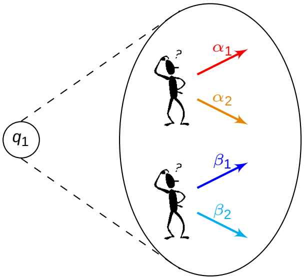

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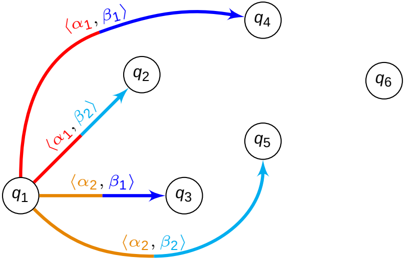

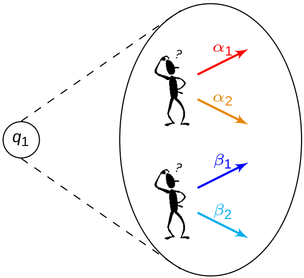

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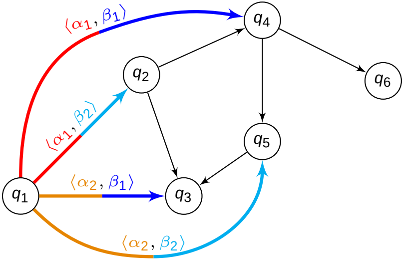

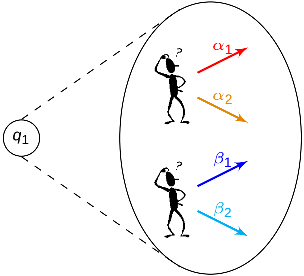

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Becoming friendly with ATL

Collective strategy to guarantee p holds

$\langle\langle A \rangle\rangle \bigcirc p$	next
$\langle\langle A \rangle\rangle \Box p$	always
$\langle\langle A \rangle\rangle p \mathcal{U} q$	until q

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regardless of actions performed by other agents (opponent)

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Addition of bounds on resources to ATL

o’ munn’ è piccirill!!!

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Quant’è piccolo il mondo!

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Resources are bounded

Extensions of ATL with bounds on resources

$\langle\langle A^{\eta}\rangle\rangle \Box p$

Endowment: $\eta : A \to \mathbb{N}^r$

The literature about Resource Bounded ATL (RB-ATL)

RB-ATL [Alechina, Logan, Nga, Rakib, AAMAS 2010]

Theorem: Model checking RB-ATL is decidable in $O(|\varphi|^{2 \cdot r+1} \times |G|)$
No lower bound

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RAL [Bulling, Farwer, ECAI 2010]

If actions may produce resources,
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Formulae of RB-ATL predicate about abilities of coalitions whose agents are equipped with a finite endowment of resources

Formulae of RB-ATL are evaluated wrt:

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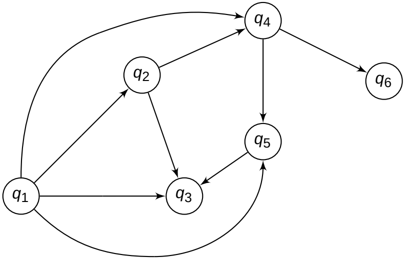

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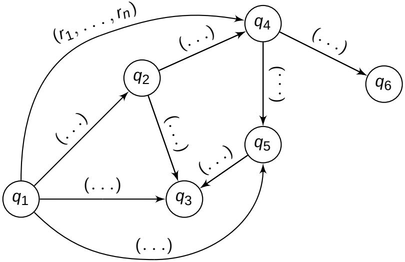

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Becoming friendly with RB-ATL

$$G, q \Vdash \langle\langle A^{\eta} \rangle\rangle \bigcirc \langle\langle A^{\eta'} \rangle\rangle \Box p$$

team A , equipped with endowment η , can force the next state to be s.t. the team A itself can guarantee that p always holds equipped with the new endowment η'

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An anomalous behavior

2 agents: **ag_1** and **ag_2**
1 resource type: **r_1**

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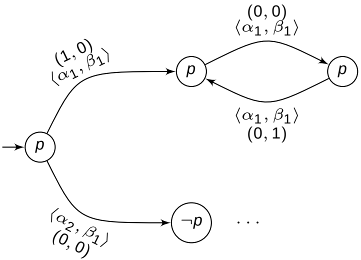

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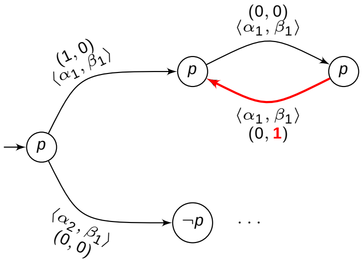

opponent consumes an infinite amount of resources

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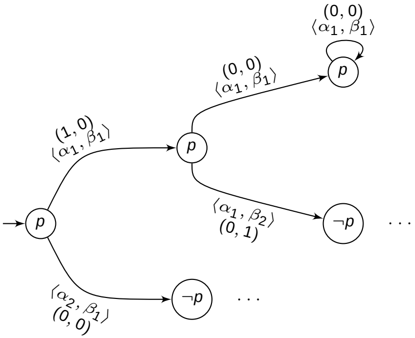

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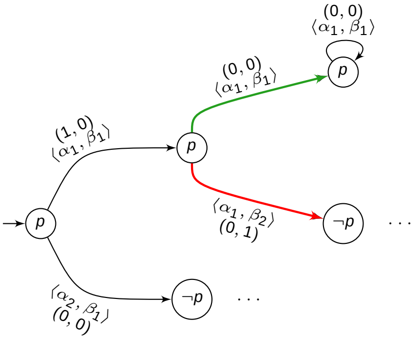

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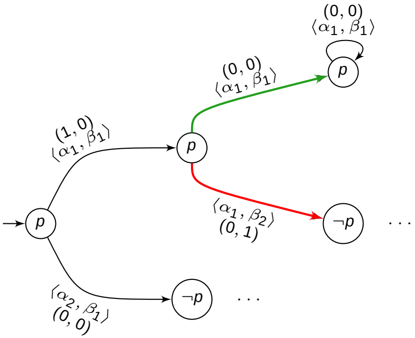

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opponent's moves should be constrained

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Weaknesses of previous approaches

- NO history (resources)
  - $G, q \Vdash \langle\langle A^{\eta} \rangle\rangle \bigcirc \langle\langle A^{\eta'} \rangle\rangle \Box p$ η and η' are independent
- opponent does NOT consume
  - opponent has no bounds on resources
  - consumption by opponent does not matter

What we want

- opponent's actions constrained
- consumption/production tracked
- a significant present-day issue $\Rightarrow$ procurement of resources
  - limited amount on the market (or in nature)
  - acquisition cost depending on current availability

How we get it

Key notion $\Rightarrow$ global availability of resources on the market

- a semantic component (part of the arena)
- evolves depending on agents' actions (also opponent)
- affects the choice of the actions (also opponent)

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Auxiliary notion ⇒ price of resources

- agents equipped with money instead of resources
- money for getting resources
- price of resources function of several components
  (take into account the history of the system)

Money vs. resources - our proposal

Money

- inside the formula
- assigned to agents
- private: any agent has his own amount of money
- unknown
- availability checked for proponent's agents only

Resources

- part of the model
- represent the market (nature)
- public: agents draw on resources from a shared pool
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Money is a *meta-resource*

- buy resources
  - money like resources in previous approaches
- unit of measurement

Resource production and decidability

Alechina, Logan, Nga, Rakib

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Bulling, Farwer

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Actions may produce resources...
...but ***not so much***!!!

- model checking decidable
- several models fit (memory usage, leasing a car, releasing resources previously acquired)

Syntax and semantics

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Priced game structure

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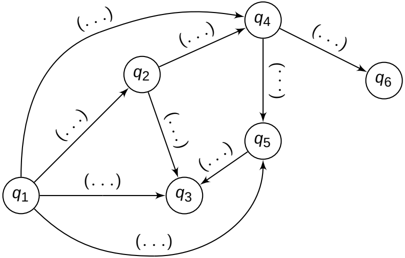

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- resources have a variable prices
- transition guards: also opponent

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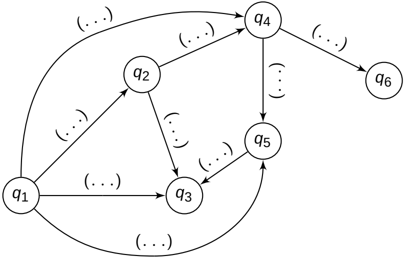

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Model checking complexity

Theorem

The model checking problem for PRB-ATL *is EXPTIME-complete*

- *membership (upper bound):* [LAMAS 2011]
- *hardness (lower bound):* [GAMES 2012]

- Same asymptotic complexity as RB-ATL
- Exponential blow-up compared to ATL

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Reduction from the acceptance problem for Linearly-Bounded Alternating Turing Machine

Reduction

- Given a LB-ATM $\mathcal{A}$ and an input s_0
- We provide
  - a priced game structure $G_{\mathcal{A},s_0}$
  - an initial location q_0 in $G_{\mathcal{A},s_0}$
  - an initial availability of resources $\vec{m}_0$
  - a PRB-ATL formula $\varphi_{\mathcal{A},s_0}$

  such that

$$G_{\mathcal{A},s_0}, q_0, \vec{m}_0 \models \varphi_{\mathcal{A},s_0} \text{ iff } \mathcal{A} \text{ accepts on input } s_0$$

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Parametric PRB-ATL formulae

- PRB-ATL: $\varphi = \langle\langle A_1^{\$_1} \rangle\rangle \Diamond (\langle\langle A_2^{\$_2} \rangle\rangle \bigcirc p \vee \langle\langle A_3^{\$_3} \rangle\rangle q \mathcal{U} p)$

Definition (Cost of a PRB-ATL formula)

$$f\_cost(\varphi) = \$_1(A_1) + \$_2(A_2) + \$_3(A_3)$$

- parametric PRB-ATL: $\varphi_{\vec{X}} = \langle\langle X_1^{\$_1} \rangle\rangle \Diamond (\langle\langle X_2^{\$_2} \rangle\rangle \bigcirc p \vee \langle\langle A_3^{\$_3} \rangle\rangle q \mathcal{U} p)$

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Definition (Optimal Coalition problem)

To determine minimal-cost coalitions that satisfy a PRB-ATL formula

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Input:

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Input:

- a parametric PRB-ATL formula
- a priced game structure
- a location
- an initial availability of resources

Theorem

The Optimal Coalition problem is EXPTIME-complete

Outline

1. Introduction to Multi-Agent Systems (MAS) - ***ATL***
   - Multi-Agent Systems and resource constraints - ***RB-ATL***
2. Our proposal: the logic *Priced* RB-ATL - ***PRB-ATL***
   - Model checking
   - Optimization problem
3. Conclusions

Conclusions

A logic for modeling multi-agent systems with bounds on resources

ATL: abilities of coalitions of agents

RB-ATL: abilities of coalitions whose agents are equipped with a finite endowment of resources

PRB-ATL: abilities of coalitions whose agents are equipped with an amount of money

- global availability of resources
- money - price of resources

Future work

- Resource-bounded extensions of other classical formalisms
  - e.g., μ-calculus [Della Monica, Lenzi - ICAART 2012]
- Hierarchical, MAS, and resources???
- An attempt towards tractability: the *reachability problem*

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