On a Logic for Coalitional Games with Priced-Resource Agents

D. Della Monica, M. Napoli, M. Parente

University of Salerno
ddellamonica@unisa.it

GAMES 2011
September 3, 2011 - Paris

Outline

Outline

1. Introduction
2. The logic *Priced* RB-ATL (PRB-ATL)
   - Model checking
   - Optimization problem
3. Conclusions

Agents and coalitions

- A Multi-Agent System (MAS) is a system with multiple agents/players
- Agents can join in coalitions/teams to collectively perform tasks/reach goals

Agents and coalitions

- A Multi-Agent System (MAS) is a system with multiple agents/players
- Agents can join in coalitions/teams to collectively perform tasks/reach goals

Agents and coalitions

- A Multi-Agent System (MAS) is a system with multiple agents/players
- Agents can join in coalitions/teams to collectively perform tasks/reach goals

Two sides of the same coin

Artificial Intelligence/Game theory

Agents and coalitions

- A Multi-Agent System (MAS) is a system with multiple agents/players
- Agents can join in coalitions/teams to collectively perform tasks/reach goals

Two sides of the same coin
Artificial Intelligence/Game theory

Logical Formalisms

Coalition Logic (CL) and Alternating-time Temporal Logic (ATL)

CL and ATL

CL [Pauly, Journal of Logic and Computation, 2002]

CL and ATL

CL [Pauly, Journal of Logic and Computation, 2002]

ATL [Alur, Henzinger, Kupferman, Journal of ACM, 2002]

CL and ATL

CL [Pauly, Journal of Logic and Computation, 2002]

ATL [Alur, Henzinger, Kupferman, Journal of ACM, 2002]

Theorem (Goranko, TARK 2001)

CL *can be embedded into* ATL

Addition of bounds on resources to ATL

World is small (resources are bounded)

World is small (resources are bounded)

Extensions of ATL **with bounds on resources:**

Addition of bounds on resources to ATL

World is small (resources are bounded)

Extensions of ATL **with bounds on resources:**

RB-ATL [Alechina, Logan, Nga, Rakib, AAMAS 2010]

Theorem: Model checking RB-ATL is decidable in $O(|\varphi|^{2 \cdot r+1} \times |G|)$
No lower bound

Addition of bounds on resources to ATL

World is small (resources are bounded)

Extensions of ATL **with bounds on resources:**

RB-ATL [Alechina, Logan, Nga, Rakib, AAMAS 2010]

Theorem: Model checking RB-ATL is decidable in $O(|\varphi|^{2 \cdot r+1} \times |G|)$
No lower bound

RAL [Bulling, Farwer, ECAI 2010]

If actions may produce resources,
then Model Checking becomes UNDECIDABLE

Team A

Endowment: $\eta : A \to \mathbb{N}^r$

$\langle\langle A^{\eta} \rangle\rangle \Diamond p$ whatever other agents do

A crucial property

Due to the nesting of the team operators in a formula, the agents can be provided with a new endowment of resources to perform subtasks

$\langle\langle A^{\eta}\rangle\rangle \bigcirc \langle\langle A^{\eta'}\rangle\rangle \Diamond p$ agents of team A, equipped with the endowment of resources η, can force the next state to be s.t. they can guarantee that p eventually holds equipped with the new endowment η'

Outline

Motivations

- In the current approaches there is not a notion of global availability of resources
  - new endowment for each subtask

Motivations

- In the current approaches there is not a notion of global availability of resources
  - new endowment for each subtask UNREALISTIC

Motivations

- In the current approaches there is not a notion of global availability of resources
  - new endowment for each subtask UNREALISTIC
- Very significant present-day issues related to procurement of resources:
  - resources are available on the market (or in nature) in limited amount
  - the cost for achieving them depends on such an availability (price of resources)

Our contributions

1. We introduce the global availability of resources on the market
   - acquisition of resources $\Rightarrow$ global availability is decreased
   - production of resources $\Rightarrow$ global availability is increased
2. We introduce the notion of price of resources
   - agents are equipped with an amount of money instead of an endowment of resources
   - they can use money for getting resources
   - price of resources can be any function of the several components into play (e.g., prices of resources depend on their global availability, the acting agent, and the physical location)

Our contributions

1. We introduce the global availability of resources on the market
   - acquisition of resources $\Rightarrow$ global availability is decreased
   - production of resources $\Rightarrow$ global availability is increased
2. We introduce the notion of price of resources
   - agents are equipped with an amount of money instead of an endowment of resources
   - they can use money for getting resources
   - price of resources can be any function of the several components into play (e.g., prices of resources depend on their global availability, the acting agent, and the physical location)

Our contributions

1. We introduce the global availability of resources on the market
   - acquisition of resources $\Rightarrow$ global availability is decreased
   - production of resources $\Rightarrow$ global availability is increased
2. We introduce the notion of price of resources
   - agents are equipped with an amount of money instead of an endowment of resources
   - they can use money for getting resources
   - price of resources can be any function of the several components into play (e.g., prices of resources depend on their global availability, the acting agent, and the physical location)

Our contributions

1. We introduce the global availability of resources on the market
   - acquisition of resources $\Rightarrow$ global availability is decreased
   - production of resources $\Rightarrow$ global availability is increased
2. We introduce the notion of price of resources
   - agents are equipped with an amount of money instead of an endowment of resources
   - they can use money for getting resources
   - price of resources can be any function of the several components into play (e.g., prices of resources depend on their global availability, the acting agent, and the physical location)

Our contributions

1. We introduce the global availability of resources on the market
   - acquisition of resources $\Rightarrow$ global availability is decreased
   - production of resources $\Rightarrow$ global availability is increased
2. We introduce the notion of price of resources
   - agents are equipped with an amount of money instead of an endowment of resources
   - they can use money for getting resources
   - price of resources can be any function of the several components into play (e.g., prices of resources depend on their global availability, the acting agent, and the physical location)

Our contributions

1. We introduce the global availability of resources on the market
   - acquisition of resources $\Rightarrow$ global availability is decreased
   - production of resources $\Rightarrow$ global availability is increased
2. We introduce the notion of price of resources
   - agents are equipped with an amount of money instead of an endowment of resources
   - they can use money for getting resources
   - price of resources can be any function of the several components into play (e.g., prices of resources depend on their global availability, the acting agent, and the physical location)

Our contributions

1. We introduce the global availability of resources on the market
   - acquisition of resources $\Rightarrow$ global availability is decreased
   - production of resources $\Rightarrow$ global availability is increased
2. We introduce the notion of price of resources
   - agents are equipped with an amount of money instead of an endowment of resources
   - they can use money for getting resources
   - price of resources can be any function of the several components into play (e.g., prices of resources depend on their global availability, the acting agent, and the physical location)

Our contributions - cont'd

3. Model checking
   - PSPACE-hardness
   - Recover decidability even if actions produce resources
     - actions may produce a resource in a quantity that is not greater than the amount that has already been consumed so far
     - the global availability of the market will never be greater than the initial global availability
     - several significant real-world scenarios fit (e.g., memory usage, leasing a car)
4. Optimization problem
   - minimization of the amount of money needed to acquire the resources to perform a task

Our contributions - cont'd

3. Model checking
   - PSPACE-hardness
   - Recover decidability even if actions produce resources
     - actions may produce a resource in a quantity that is not greater than the amount that has already been consumed so far
     - the global availability of the market will never be greater than the initial global availability
     - several significant real-world scenarios fit (e.g., memory usage, leasing a car)
4. Optimization problem
   - minimization of the amount of money needed to acquire the resources to perform a task

Our contributions - cont'd

3. Model checking
   - PSPACE-hardness
   - Recover decidability even if actions produce resources
     - actions may produce a resource in a quantity that is not greater than the amount that has already been consumed so far
     - the global availability of the market will never be greater than the initial global availability
     - several significant real-world scenarios fit (e.g., memory usage, leasing a car)
4. Optimization problem
   - minimization of the amount of money needed to acquire the resources to perform a task

Our contributions - cont'd

3. Model checking
   - PSPACE-hardness
   - Recover decidability even if actions produce resources
     - actions may produce a resource in a quantity that is not greater than the amount that has already been consumed so far
     - the global availability of the market will never be greater than the initial global availability
     - several significant real-world scenarios fit (e.g., memory usage, leasing a car)
4. Optimization problem
   - minimization of the amount of money needed to acquire the resources to perform a task

Our contributions - cont'd

3. Model checking
   - PSPACE-hardness
   - Recover decidability even if actions produce resources
     - ★ actions may produce a resource in a quantity that is not greater than the amount that has already been consumed so far
     - ★ the global availability of the market will never be greater than the initial global availability
     - ★ several significant real-world scenarios fit (e.g., memory usage, leasing a car)
4. Optimization problem
   - minimization of the amount of money needed to acquire the resources to perform a task

Syntax and semantics

Formulae of PRB-ATL are given by the grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\langle A^{\vec{\$}} \rangle\rangle \bigcirc \varphi \mid \langle\langle A^{\vec{\$}} \rangle\rangle \varphi \mathcal{U} \varphi \mid \langle\langle A^{\vec{\$}} \rangle\rangle \Box \varphi$$

Formulae of PRB-ATL are given by the grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\langle A^{\vec{\$}} \rangle\rangle \bigcirc \varphi \mid \langle\langle A^{\vec{\$}} \rangle\rangle \varphi \mathcal{U} \varphi \mid \langle\langle A^{\vec{\$}} \rangle\rangle \Box \varphi$$

formulae of our logic state that a team of agents is able to perform a given task provided with a given amount of money

Syntax and semantics

Formulae of PRB-ATL are given by the grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle\langle A^{\vec{\$}} \rangle\rangle \bigcirc \varphi \mid \langle\langle A^{\vec{\$}} \rangle\rangle \varphi \mathcal{U} \varphi \mid \langle\langle A^{\vec{\$}} \rangle\rangle \Box \varphi$$

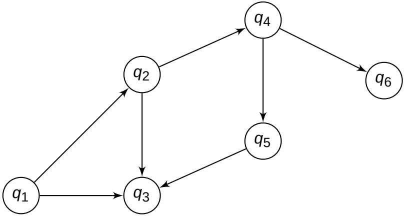
formulae of our logic state that a team of agents is able to perform a given task provided with a given amount of money

Formulae of PRB-ATL are evaluated wrt:

- a priced game structure G
- a location q of G
- an initial availability of resources $\vec{m}$

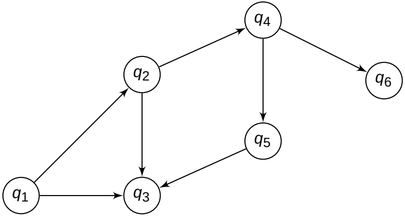
Priced game structure

A priced game structure G is a weighted graph:



Priced game structure

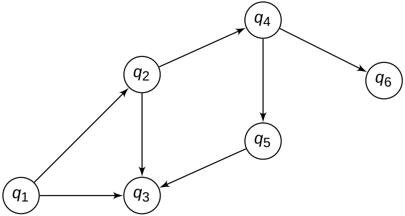
A priced game structure G is a weighted graph:



- locations are labeled by atomic propositions (represent the configurations of the system)
- in each location, each agent can choose among a non-empty set of actions to be performed
- any possible combination of actions gives rise to transitions (edges of the graph)
- actions consume and produce resources
- each resource has a price that is variable and depends on the current availability of that resource on the market, the location q of G and the acting agent
- a transition can be executed if the resources needed to perform the actions are available and the agents of a team have enough money to acquire them

Priced game structure

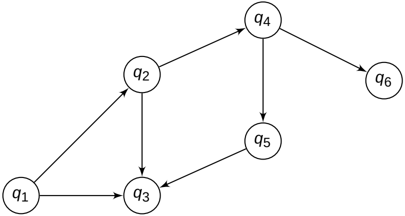
A priced game structure G is a weighted graph:



- locations are labeled by atomic propositions (represent the configurations of the system)
- in each location, each agent can choose among a non-empty set of actions to be performed
- any possible combination of actions gives rise to transitions (edges of the graph)
- actions consume and produce resources
- each resource has a price that is variable and depends on the current availability of that resource on the market, the location q of G and the acting agent
- a transition can be executed if the resources needed to perform the actions are available and the agents of a team have enough money to acquire them

Priced game structure

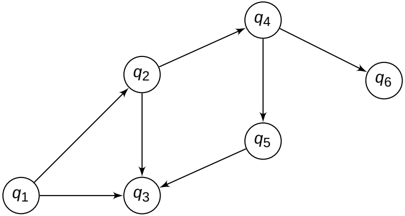
A priced game structure G is a weighted graph:



- locations are labeled by atomic propositions (represent the configurations of the system)
- in each location, each agent can choose among a non-empty set of actions to be performed
- any possible combination of actions gives rise to transitions (edges of the graph)
- actions consume and produce resources
- each resource has a price that is variable and depends on the current availability of that resource on the market, the location q of G and the acting agent
- a transition can be executed if the resources needed to perform the actions are available and the agents of a team have enough money to acquire them

Priced game structure

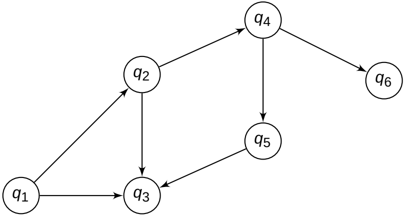
A priced game structure G is a weighted graph:



- locations are labeled by atomic propositions (represent the configurations of the system)
- in each location, each agent can choose among a non-empty set of actions to be performed
- any possible combination of actions gives rise to transitions (edges of the graph)
- actions consume and produce resources
- each resource has a price that is variable and depends on the current availability of that resource on the market, the location q of G and the acting agent
- a transition can be executed if the resources needed to perform the actions are available and the agents of a team have enough money to acquire them

Priced game structure

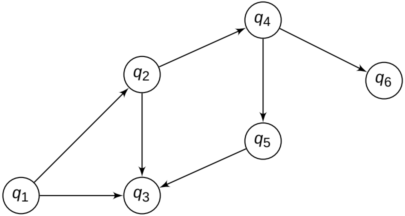
A priced game structure G is a weighted graph:



- locations are labeled by atomic propositions (represent the configurations of the system)
- in each location, each agent can choose among a non-empty set of actions to be performed
- any possible combination of actions gives rise to transitions (edges of the graph)
- actions consume and produce resources
- each resource has a price that is variable and depends on the current availability of that resource on the market, the location q of G and the acting agent
- a transition can be executed if the resources needed to perform the actions are available and the agents of a team have enough money to acquire them

Priced game structure

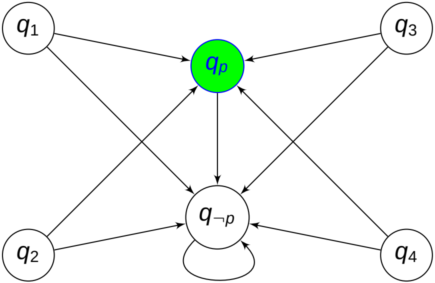
A priced game structure G is a weighted graph:



- locations are labeled by atomic propositions (represent the configurations of the system)
- in each location, each agent can choose among a non-empty set of actions to be performed
- any possible combination of actions gives rise to transitions (edges of the graph)
- actions consume and produce resources
- each resource has a price that is variable and depends on the current availability of that resource on the market, the location q of G and the acting agent
- a transition can be executed if the resources needed to perform the actions are available and the agents of a team have enough money to acquire them

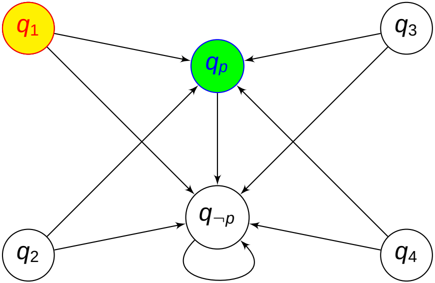
Outline

PSPACE-membership



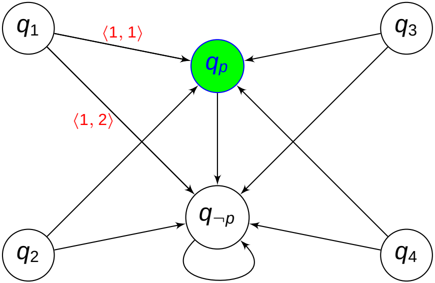
PSPACE-membership

$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$

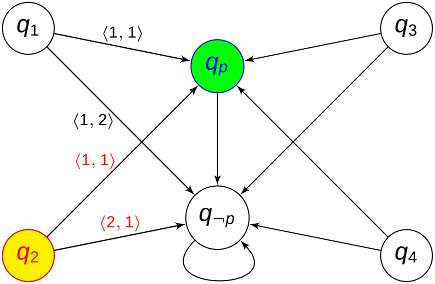


PSPACE-membership

$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$



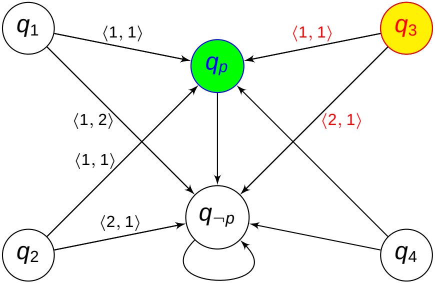
PSPACE-membership



$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$

$d(q_2, ag_1) = 2$
$d(q_2, ag_2) = 1$
$qty(q_2, ag_1, act_1) = \langle 1\rangle$
$\rho(q_2, ag_1) = \langle 1\rangle$

PSPACE-membership

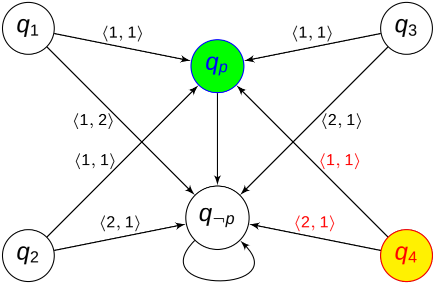


$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$

$d(q_2, ag_1) = 2$
$d(q_2, ag_2) = 1$
$qty(q_2, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_2, ag_1) = \langle 1 \rangle$

$d(q_3, ag_1) = 2$
$d(q_3, ag_2) = 1$
$qty(q_3, ag_1, act_1) = \langle 2 \rangle$
$\rho(q_3, ag_1) = \langle \frac{1}{2} \rangle$

PSPACE-membership



$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$

$d(q_2, ag_1) = 2$
$d(q_2, ag_2) = 1$
$qty(q_2, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_2, ag_1) = \langle 1 \rangle$

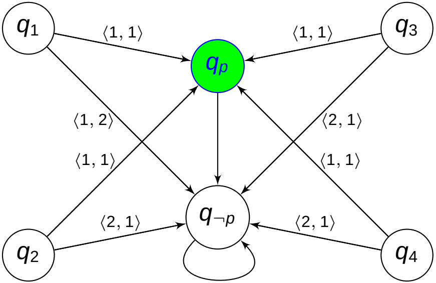
$d(q_3, ag_1) = 2$
$d(q_3, ag_2) = 1$
$qty(q_3, ag_1, act_1) = \langle 2 \rangle$
$\rho(q_3, ag_1) = \langle \frac{1}{2} \rangle$

$d(q_4, ag_1) = 2$
$d(q_4, ag_2) = 1$
$qty(q_4, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_4, ag_1) = \langle 2 \rangle$

PSPACE-membership

$G, q_4, \langle 2 \rangle \models \varphi$

$\varphi = \langle\langle 1^1 \rangle\rangle \bigcirc p$



$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$

$d(q_2, ag_1) = 2$
$d(q_2, ag_2) = 1$
$qty(q_2, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_2, ag_1) = \langle 1 \rangle$

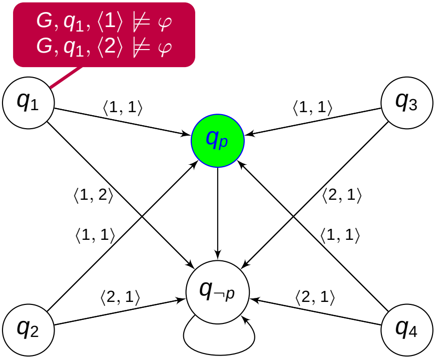
$d(q_3, ag_1) = 2$
$d(q_3, ag_2) = 1$
$qty(q_3, ag_1, act_1) = \langle 2 \rangle$
$\rho(q_3, ag_1) = \langle \frac{1}{2} \rangle$

$d(q_4, ag_1) = 2$
$d(q_4, ag_2) = 1$
$qty(q_4, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_4, ag_1) = \langle 2 \rangle$

PSPACE-membership

$G, q_4, \langle 2 \rangle \models \varphi$

$\varphi = \langle\langle 1^1 \rangle\rangle \bigcirc p$



$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$

$d(q_2, ag_1) = 2$
$d(q_2, ag_2) = 1$
$qty(q_2, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_2, ag_1) = \langle 1 \rangle$

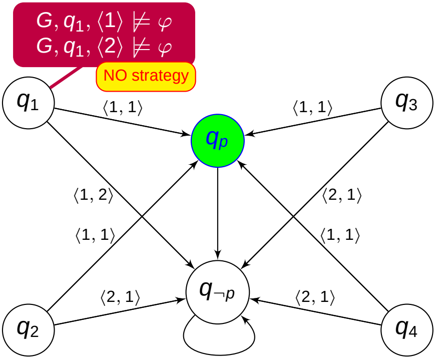
$d(q_3, ag_1) = 2$
$d(q_3, ag_2) = 1$
$qty(q_3, ag_1, act_1) = \langle 2 \rangle$
$\rho(q_3, ag_1) = \langle \frac{1}{2} \rangle$

$d(q_4, ag_1) = 2$
$d(q_4, ag_2) = 1$
$qty(q_4, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_4, ag_1) = \langle 2 \rangle$

PSPACE-membership

$G, q_4, \langle 2 \rangle \models \varphi$

$\varphi = \langle\langle 1^1 \rangle\rangle \bigcirc p$



$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$

$d(q_2, ag_1) = 2$
$d(q_2, ag_2) = 1$
$qty(q_2, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_2, ag_1) = \langle 1 \rangle$

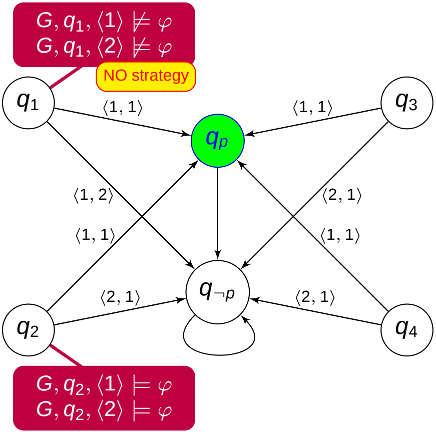
$d(q_3, ag_1) = 2$
$d(q_3, ag_2) = 1$
$qty(q_3, ag_1, act_1) = \langle 2 \rangle$
$\rho(q_3, ag_1) = \langle \frac{1}{2} \rangle$

$d(q_4, ag_1) = 2$
$d(q_4, ag_2) = 1$
$qty(q_4, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_4, ag_1) = \langle 2 \rangle$

PSPACE-membership

$G, q_4, \langle 2 \rangle \models \varphi$

$\varphi = \langle\langle 1^1 \rangle\rangle \bigcirc p$



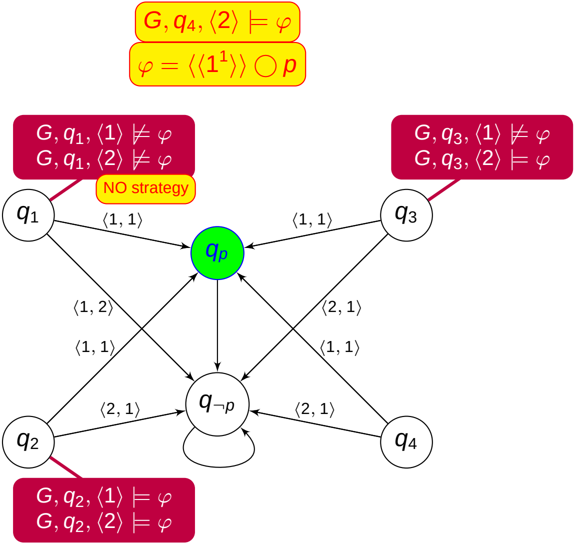
$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$

$d(q_2, ag_1) = 2$
$d(q_2, ag_2) = 1$
$qty(q_2, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_2, ag_1) = \langle 1 \rangle$

$d(q_3, ag_1) = 2$
$d(q_3, ag_2) = 1$
$qty(q_3, ag_1, act_1) = \langle 2 \rangle$
$\rho(q_3, ag_1) = \langle \frac{1}{2} \rangle$

$d(q_4, ag_1) = 2$
$d(q_4, ag_2) = 1$
$qty(q_4, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_4, ag_1) = \langle 2 \rangle$

PSPACE-membership



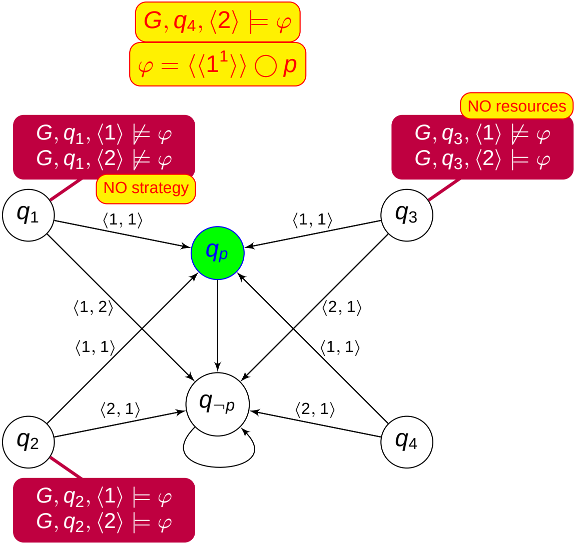
$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$

$d(q_2, ag_1) = 2$
$d(q_2, ag_2) = 1$
$qty(q_2, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_2, ag_1) = \langle 1 \rangle$

$d(q_3, ag_1) = 2$
$d(q_3, ag_2) = 1$
$qty(q_3, ag_1, act_1) = \langle 2 \rangle$
$\rho(q_3, ag_1) = \langle \frac{1}{2} \rangle$

$d(q_4, ag_1) = 2$
$d(q_4, ag_2) = 1$
$qty(q_4, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_4, ag_1) = \langle 2 \rangle$

PSPACE-membership



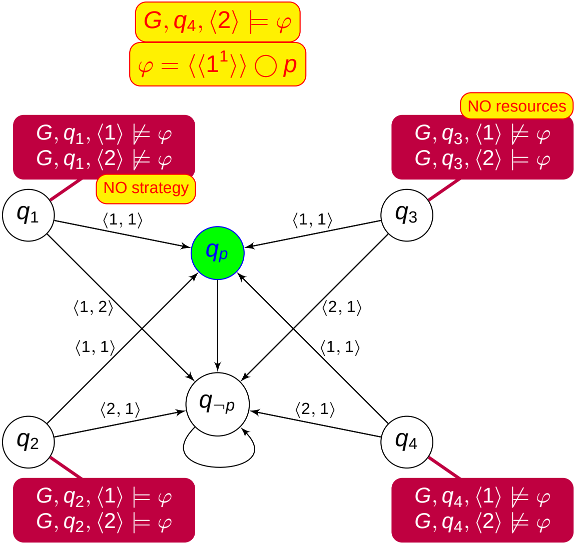
$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$

$d(q_2, ag_1) = 2$
$d(q_2, ag_2) = 1$
$qty(q_2, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_2, ag_1) = \langle 1 \rangle$

$d(q_3, ag_1) = 2$
$d(q_3, ag_2) = 1$
$qty(q_3, ag_1, act_1) = \langle 2 \rangle$
$\rho(q_3, ag_1) = \langle \frac{1}{2} \rangle$

$d(q_4, ag_1) = 2$
$d(q_4, ag_2) = 1$
$qty(q_4, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_4, ag_1) = \langle 2 \rangle$

PSPACE-membership



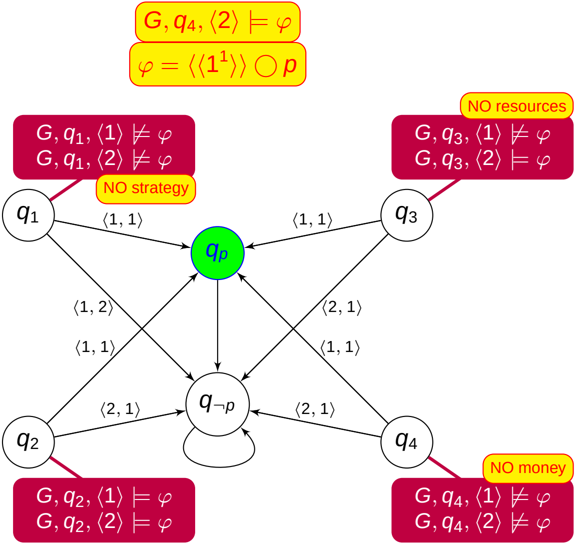
$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$

$d(q_2, ag_1) = 2$
$d(q_2, ag_2) = 1$
$qty(q_2, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_2, ag_1) = \langle 1 \rangle$

$d(q_3, ag_1) = 2$
$d(q_3, ag_2) = 1$
$qty(q_3, ag_1, act_1) = \langle 2 \rangle$
$\rho(q_3, ag_1) = \langle \frac{1}{2} \rangle$

$d(q_4, ag_1) = 2$
$d(q_4, ag_2) = 1$
$qty(q_4, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_4, ag_1) = \langle 2 \rangle$

PSPACE-membership



$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$

$d(q_2, ag_1) = 2$
$d(q_2, ag_2) = 1$
$qty(q_2, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_2, ag_1) = \langle 1 \rangle$

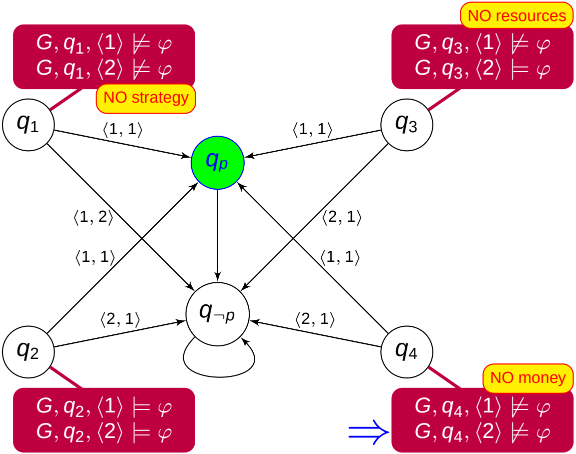
$d(q_3, ag_1) = 2$
$d(q_3, ag_2) = 1$
$qty(q_3, ag_1, act_1) = \langle 2 \rangle$
$\rho(q_3, ag_1) = \langle \frac{1}{2} \rangle$

$d(q_4, ag_1) = 2$
$d(q_4, ag_2) = 1$
$qty(q_4, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_4, ag_1) = \langle 2 \rangle$

PSPACE-membership

$G, q_4, \langle 2 \rangle \models \varphi$ $\Rightarrow$ NO

$\varphi = \langle\langle 1^1 \rangle\rangle \bigcirc p$



$d(q_1, ag_1) = 1$
$d(q_1, ag_2) = 2$

$d(q_2, ag_1) = 2$
$d(q_2, ag_2) = 1$
$qty(q_2, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_2, ag_1) = \langle 1 \rangle$

$d(q_3, ag_1) = 2$
$d(q_3, ag_2) = 1$
$qty(q_3, ag_1, act_1) = \langle 2 \rangle$
$\rho(q_3, ag_1) = \langle \frac{1}{2} \rangle$

$d(q_4, ag_1) = 2$
$d(q_4, ag_2) = 1$
$qty(q_4, ag_1, act_1) = \langle 1 \rangle$
$\rho(q_4, ag_1) = \langle 2 \rangle$

PSPACE-hardness

Theorem

The model checking problem for PRB-ATL *is PSPACE-hard*

Reduction from the *TQBF* problem

Fully Quantified Boolean Formulae

Fully Quantified Boolean Formula a Boolean formula in which all the Boolean variables occur inside the scope of an existential or universal quantifier

Prenex Normal Form all the quantifiers appear at the beginning of the formula and each quantifier's scope is everything following it

- Any formula may be put into prenex normal form
- we assume that the Boolean quantifier-free part is in conjunctive normal form

Fully Quantified Boolean Formulae

Fully Quantified Boolean Formula a Boolean formula in which all the Boolean variables occur inside the scope of an existential or universal quantifier

Prenex Normal Form all the quantifiers appear at the beginning of the formula and each quantifier's scope is everything following it

- Any formula may be put into prenex normal form
- we assume that the Boolean quantifier-free part is in conjunctive normal form

Fully Quantified Boolean Formulae

Fully Quantified Boolean Formula a Boolean formula in which all the Boolean variables occur inside the scope of an existential or universal quantifier

Prenex Normal Form all the quantifiers appear at the beginning of the formula and each quantifier's scope is everything following it

- Any formula may be put into prenex normal form
- we assume that the Boolean quantifier-free part is in conjunctive normal form

Fully Quantified Boolean Formulae

Fully Quantified Boolean Formula a Boolean formula in which all the Boolean variables occur inside the scope of an existential or universal quantifier

Prenex Normal Form all the quantifiers appear at the beginning of the formula and each quantifier's scope is everything following it

- Any formula may be put into prenex normal form
- we assume that the Boolean quantifier-free part is in conjunctive normal form

Fully Quantified Boolean Formulae

Fully Quantified Boolean Formula a Boolean formula in which all the Boolean variables occur inside the scope of an existential or universal quantifier

Prenex Normal Form all the quantifiers appear at the beginning of the formula and each quantifier's scope is everything following it

- Any formula may be put into prenex normal form
- we assume that the Boolean quantifier-free part is in conjunctive normal form

Fully Quantified Boolean Formulae in prenex conjunctive normal form

- $\forall x \exists y [(x \vee y) \wedge (\neg x \vee \neg y)]$
- $\exists x_1 \forall x_2 \exists x_3 [(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)]$

Reduction - the idea

- Given a fully quantified Boolean formula Φ
- We provide
  - a priced game structure G
  - a location q in G
  - an initial availability of resources $\vec{m}$
  - a PRB-ATL formula φ

  such that

$$G, q, \vec{m} \models \varphi \text{ iff } \Phi \text{ is true}$$

Reduction - an example

- Fully quantified Boolean formula:
  $\Phi = \exists x_1 \forall x_2 \exists x_3 [(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)]$
- Initial availability of resources (6 resources - 2 for each Boolean variable):
  $\vec{m} = \langle 1, 1, 1, 1, 1, 1 \rangle$ (only 1 item available for each resource)
- Priced game structure G_Φ corresponding to Φ (numer of agents: 1):

- PRB-ATL formula φ_Φ corresponding to Φ:
  $\langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle \emptyset^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle \emptyset^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc p,$

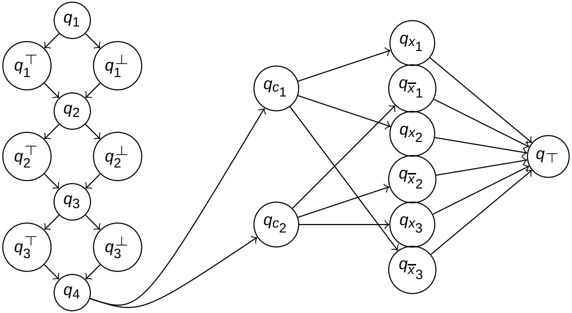
Reduction - an example

- Fully quantified Boolean formula:
  $\Phi = \exists x_1 \forall x_2 \exists x_3 [(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)]$
- Initial availability of resources (6 resources - 2 for each Boolean variable):
  $\vec{m} = \langle 1,1,1,1,1,1 \rangle$ (only 1 item available for each resource)
- Priced game structure G_Φ corresponding to Φ (numer of agents: 1):

- PRB-ATL formula φ_Φ corresponding to Φ:
  $\langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle \emptyset^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle \emptyset^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc p,$

Reduction - an example

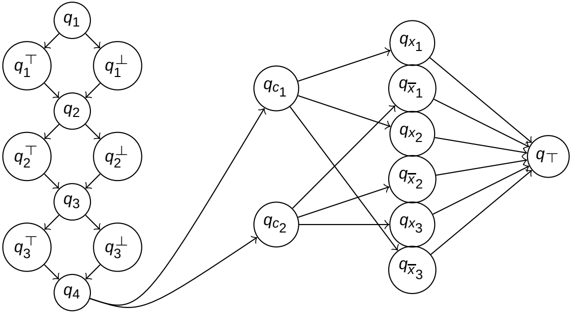
- Fully quantified Boolean formula:
  $\Phi = \exists x_1 \forall x_2 \exists x_3 [(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)]$
- Initial availability of resources (6 resources - 2 for each Boolean variable):
  $\vec{m} = \langle 1, 1, 1, 1, 1, 1 \rangle$ (only 1 item available for each resource)
- Priced game structure G_Φ corresponding to Φ (numer of agents: 1):



- PRB-ATL formula φ_Φ corresponding to Φ:
  $\langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle \emptyset^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle \emptyset^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc p,$

Reduction - an example

- Fully quantified Boolean formula:
  $\Phi = \exists x_1 \forall x_2 \exists x_3 [(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3)]$
- Initial availability of resources (6 resources - 2 for each Boolean variable):
  $\vec{m} = \langle 1, 1, 1, 1, 1, 1 \rangle$ (only 1 item available for each resource)
- Priced game structure G_Φ corresponding to Φ (numer of agents: 1):



- PRB-ATL formula φ_Φ corresponding to Φ:
  $\langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle \emptyset^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle \emptyset^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc \langle\langle 1^{\vec{0}} \rangle\rangle \bigcirc p,$

Reduction - the main lemma

Lemma

$G_\Phi, q_1, \vec{m} \models \varphi_\Phi$ *if and only if* Φ *is true*

Outline

Parametric PRB-ATL formulae

- PRB-ATL: $\varphi = \langle\langle A_1^{\vec{\$}_1} \rangle\rangle \Diamond (\langle\langle A_2^{\vec{\$}_2} \rangle\rangle \bigcirc p \vee \langle\langle A_3^{\vec{\$}_3} \rangle\rangle q \mathcal{U} p)$

Definition (Cost of a PRB-ATL formula)

$$f\_cost(\varphi) = A_1 \cdot \vec{\$}_1 + A_2 \cdot \vec{\$}_2 + A_3 \cdot \vec{\$}_3$$

- parametric PRB-ATL: $\varphi_{\vec{X}} = \langle\langle X_1^{\vec{\$}_1} \rangle\rangle \Diamond (\langle\langle X_2^{\vec{\$}_2} \rangle\rangle \bigcirc p \vee \langle\langle A_3^{\vec{\$}_3} \rangle\rangle q \mathcal{U} p)$

The *Optimal Coalition* problem

Definition (Optimal Coalition problem)

To determine optimal coalitions that satisfy a PRB-ATL formula

The *Optimal Coalition* problem

Definition (Optimal Coalition problem)

To determine optimal coalitions that satisfy a PRB-ATL formula

Input:

- a parametric PRB-ATL formula $\varphi_{\vec{X}}$, with $\vec{X} = \langle X_1, \ldots, X_k \rangle$
- a priced game structure G
- a location q in G
- an initial availability of resources $\vec{m}$

The *Optimal Coalition* problem

Definition (Optimal Coalition problem)

To determine optimal coalitions that satisfy a PRB-ATL formula

Input:

- a parametric PRB-ATL formula $\varphi_{\vec{X}}$, with $\vec{X} = \langle X_1, \ldots, X_k \rangle$
- a priced game structure G
- a location q in G
- an initial availability of resources $\vec{m}$

A brute force algorithm only requires polynomial space

The *Optimal Coalition* problem

Definition (Optimal Coalition problem)

To determine optimal coalitions that satisfy a PRB-ATL formula

Input:

- a parametric PRB-ATL formula $\varphi_{\vec{X}}$, with $\vec{X} = \langle X_1, \ldots, X_k \rangle$
- a priced game structure G
- a location q in G
- an initial availability of resources $\vec{m}$

A brute force algorithm only requires polynomial space

Theorem

The Optimal Coalition problem is PSPACE-complete

Outline

Conclusions and future work

Conclusions:

- PRB-ATL: a formalism to model scenarios with bounded, priced resources
  - Model checking PRB-ATL is PSPACE-complete
  - Determine the optimal coalitions formation is PSPACE-complete

Conclusions and future work

Conclusions:

- PRB-ATL: a formalism to model scenarios with bounded, priced resources
  - Model checking PRB-ATL is PSPACE-complete
  - Determine the optimal coalitions formation is PSPACE-complete

Future work:

- To study variants of the logic (e.g., agents can be viewed as resources)
- Resource-bounded extensions of other classical formalisms (e.g., μ-calculus)