

# Parity-energy ATL for qualitative and quantitative reasoning in MAS (extended abstract)<sup>†‡</sup>

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In this paper, we introduce a new logic suitable to reason about strategic abilities of multi-agent systems where (teams of) agents are subject to qualitative (parity) and quantitative (energy) constraints and where goals are represented, as usual, by means of temporal properties. We formally define such a logic, named parity-energy-ATL (pe-ATL, for short), and we study its model checking problem, which we prove to be decidable with different complexity upper bounds, depending on different choices for the energy range.

In recent years, game theory has been proved to be very useful in open-system verification, where the game evolution emerges from the coordination of different parts viewed as autonomous and proactive agents [5, 11]. This has encouraged the development of several frameworks aimed at reasoning about strategies and their interaction [1, 8, 10, 9, 13]. An important contribution in this field has been the development of *Alternating-Time Temporal Logic* (ATL, for short) by Alur, Henzinger, and Kupferman [1], whose model checking problem is proved to be solvable in polynomial time [1]. In the context of games equipped with quantitative objectives, an important contribution is given by *energy parity games* [4], zero-sum 2-player games played on a weighted game arena where parity are associated to states and energy values are associated to transitions. Player 0's goal is to satisfy the parity (the least parity occurring infinitely often along the run is even) and the energy (the energy level stays positive along the run) condition. The problem of deciding these games is known to lie in  $\text{NP} \cap \text{coNP}$ , as it is for parity games.

In this paper we combine energy parity games and ATL specifications in a new logical formalism, named *parity-energy-ATL* (pe-ATL), and we solve the related model checking question. Roughly speaking, pe-ATL allows one to check the satisfaction of a parity condition while keeping the energy level within a given range along system evolutions determined by coalitions along the ATL formula. We show that the addressed model checking question lies in NPTIME or NEXPTIME, depending on the type of energy range given in input. The conceived framework can be successfully used in several contexts (e.g., smart-city applications [3, 14] and systems for task and resource allocation [12, 7]).

We assume the reader is familiar with ATL and related notion (e.g., concurrent game structures, strategies, outcomes); see [6] for more details.

Intuitively, a concurrent game structure (CGS) is a labeled transition system, whose transitions correspond to join actions of all agents. A strategy for an agent is a function mapping a finite path on a CGS to an action choice.

The logic pe-ATL we propose and study share the same syntax with ATL, but it is interpreted

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over CGS extended with support for parity and energy conditions.

**Definition 1** (energy and parity condition). *Let  $G$  be a CGS. A parity condition is a function  $p: Q \rightarrow \mathbb{N}$  assigning natural numbers to states in  $G$ . An energy condition is a triple  $e = \langle w, \mathcal{E}^{init}, [a, b] \rangle$ , where  $w$  is a weight assignment of rational weights to transitions of  $G$ ,  $\mathcal{E}^{init} \in [a, b]$  is the initial energy level of  $e$ , and  $[a, b]$  is its energy bound, with  $a \in \mathbb{Q} \cup \{-\infty\}$ ,  $b \in \mathbb{Q} \cup \{+\infty\}$ , and  $a \leq b$ .*

A *parity-energy CGS* (*pe-CGS*) is a CGS extended with a parity and a energy condition.

We extend the notion of strategy to comply with parity and energy conditions as expected, that is, we constrain strategies for agents to produce outcomes along which the energy level stays within the energy range specified by the energy condition and such that the smallest parity occurring infinitely often is even.

**Definition 2** (model checking problem). *The model checking problem for pe-ATL consists in verifying, given a pe-CGS  $\mathcal{G}$  and a pe-ATL formula  $\varphi$ , whether  $\mathcal{G}$  satisfies  $\varphi$ .*

The complexity of the model checking problem depends on the type of energy range specified by the energy condition.

Solving the model checking problem for input energy ranges  $[a, b]$  that are either bounded or unbounded to both left and right (i.e., either  $a \neq -\infty$  and  $b \neq +\infty$  or  $a = -\infty$  and  $b = +\infty$ ) is (technically) easier, as one can reduce the search for a strategy to memoryless or uniform ones: in memoryless strategies the choice of the next action only depends on the current state (regardless to the previous states in the run), in uniform strategies the choice of the next action depends on the current state and the energy level produced by the run up to that state.

The problem is technically much more involved when considering *mixed* energy ranges  $[a, b]$  (i.e., either  $a \in \mathbb{Z}$  and  $b = +\infty$  or  $a = -\infty$  and  $b \in \mathbb{Z}$ ). In these cases, it is not possible to focus on memoryless or uniform strategies, and we use a model theoretic argument instead.

We define appropriate finite structures, named *witnesses*, which are shown to be expressively complete for strategies (which are infinite objects), meaning that every such witness corresponds to a particular strategy, and, vice versa, every strategy can be compactly encoded into a witness which keeps enough information about the strategy itself. As a consequence, the search for a strategy amounts to looking for a suitable witness. Finally, we establish a bound for the size of a witness corresponding to a strategy; thus, the search space to search for witnesses is finite, and a decision procedure follows.

Thus, our main technical contribution is summarized in the following theorem.

**Theorem 1.** *The model checking problem for pe-ATL is:*

- *in NEXPTIME if  $a, b \in \mathbb{Z}$  (bounded instances),*
- *in NPTIME if  $[a, b] = [-\infty, +\infty]$  (unbounded instances),*
- *in NPTIME if either  $a \in \mathbb{Z}$  and  $b = +\infty$  or  $a = -\infty$  and  $b \in \mathbb{Z}$  (mixed instances).*

The proposed setting follows a recent and promising trend devoted to the study of systems enabling qualitative and quantitative reasoning in MAS. Before the last decade, these two aspects have been mostly kept separate, despite their interplay in many natural application scenarios (e.g., allocation systems subject to energy constraints). Our proposal aims at developing a logical system able to deal with these two aspects jointly.

As future work, we aim at establishing tight complexity bounds for the problems considered here, as well as considering different choices for modeling energy condition: at least another option is worth being considered, according to which energy level evolves while trying to satisfy the formula along the entire game (in our setting the energy level is reset whenever a new search for strategy by a possibly different team begins—see [2] for a comparison on the two approaches in the setting of ATL without parity condition).

## References

- [1] R. Alur, T.A. Henzinger, and O. Kupferman. Alternating-Time Temporal Logic. *JACM*, 49(5):672–713, 2002.
- [2] N. Bulling and B. Farwer. On the (un-)decidability of model checking resource-bounded agents. In *Proc. of the 19th European Conference on Artificial Intelligence (ECAI)*, volume 215 of *Frontiers in Artificial Intelligence and Applications*, pages 567–572. IOS Press, 2010.
- [3] C.F. Calvillo, A. Sánchez-Mirallas, and J. Villar. Energy management and planning in smart cities. *Renewable and Sustainable Energy Reviews*, 55:273–287, 2016.
- [4] K. Chatterjee and L. Doyen. Energy parity games. *Theor. Comput. Sci.*, 458:49–60, 2012.
- [5] E.M. Clarke, O. Grumberg, and D.A. Peled. *Model Checking*. MIT Press, 2002.
- [6] Dario Della Monica and Aniello Murano. Parity-energy ATL for qualitative and quantitative reasoning in MAS. In *Proc. of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*, 2018.
- [7] N. Fijalkow and M. Zimmermann. Parity and streett games with costs. *Logical Methods in Computer Science*, 10(2), 2014.
- [8] S. Ghosh and R. Ramanujam. Strategies in Games: A Logic-Automata Study. In *ESSLLI’11*, LNCS 7388, pages 110–159. Springer, 2011.
- [9] W. Jamroga and W. Penczek. Specification and Verification of Multi-Agent Systems. In *ESSLLI’11*, LNCS 7388, pages 210–263. Springer, 2011.
- [10] W. Jamroga and W. van der Hoek. Agents that Know How to Play. *FI*, 63(2-3):185–219, 2004.
- [11] O. Kupferman, M.Y. Vardi, and P. Wolper. Module Checking. *IC*, 164(2):322–344, 2001.
- [12] F. Mogavero, A. Murano, and L. Sorrentino. On promptness in parity games. *Fundam. Inform.*, 139(3):277–305, 2015.
- [13] F. Mogavero, A. Murano, and M.Y. Vardi. Relentful Strategic Reasoning in Alternating-Time Temporal Logic. In *LPAR’10*, LNAI 6355, pages 371–387. Springer, 2010.
- [14] L. Nenzi, L. Bortolussi, V. Ciancia, M. Loreti, and M. Massink. Qualitative and quantitative monitoring of spatio-temporal properties. In *Runtime Verification*, pages 21–37. Springer, 2015.