A Tableau System for Right Propositional Neighborhood Logic over Finite Linear Orders: an Implementation

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Please notice: these slides have been mostly produced by Guido Sciavicco (University of Murcia)

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Outline

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1. Introduction on Interval Temporal Logic
2. Halpern-Shoham's modal logic HS
3. The tableau method for the logic A
4. Conclusions

Presentation

- this is an implementation and experimental work
    - no new theoretical results here
- not free of unexpected problems and difficulties
- this implementation, although particularly simple, is the only one of its kind

Time and logics

In AI usually time is formalized with languages (logics) that are:

- point-based: formulas interpreted over points
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  - the truth of a formula over an interval is independent from its truth over sub-intervals

Motivations

- properties intrinsically related to intervals (instead of points)
- points have no duration

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Several philosophical and logical paradoxes disappear:

- Zeno’s flying arrow paradox (“if at each instant the flying arrow stands still, how is movement possible?”)
- The dividing instant dilemma (“if the light is on and it is turned off, what is its state at the instant between the two events?”)

Reasoning parameters

Common parameters (independent from point/interval setting):

- linear or branching?
- discrete or dense?
- with or without beginning/end?

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Interval setting parameters:

- bounded/unbounded intervals?
- point intervals: yes/no?

Reasoning parameters

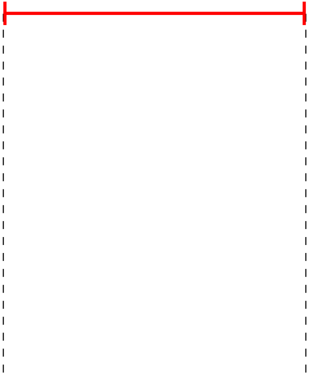
Common parameters (independent from point/interval setting):

- linear
- discrete
- with beginning/end

Interval setting parameters:

- bounded intervals
- point intervals: no

Binary interval relations on linear orders



J. F. Allen

Maintaining knowledge about temporal intervals.

Communications of the ACM, volume 26(11), pages 832-843, 1983.

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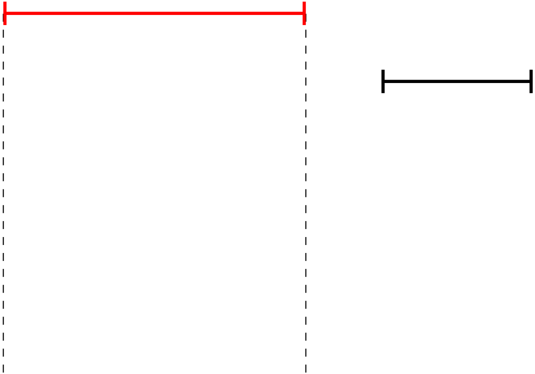

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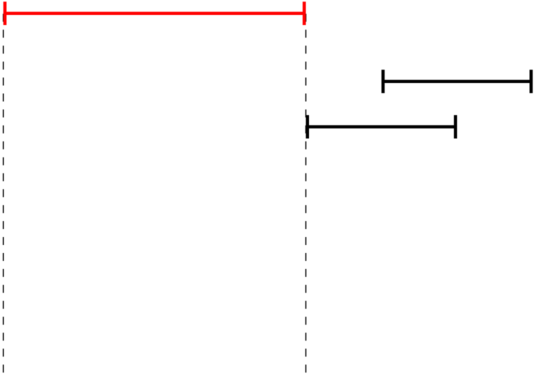

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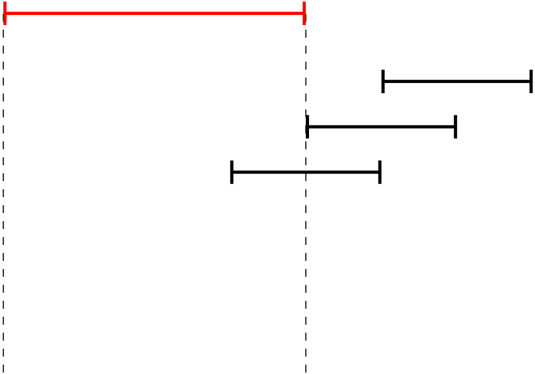

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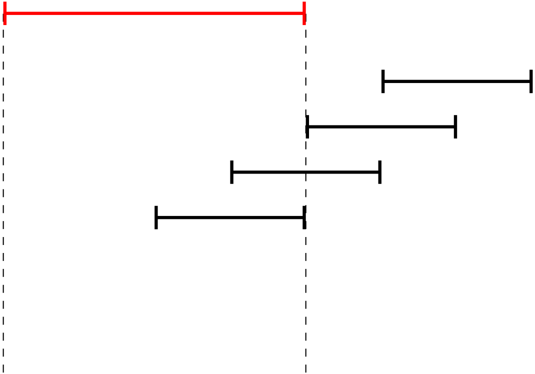

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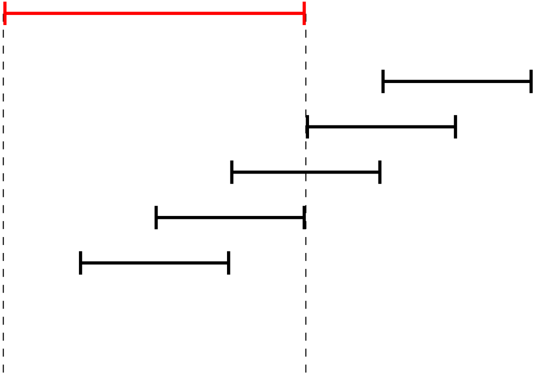

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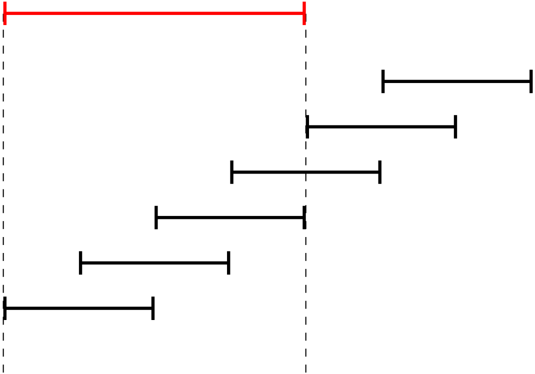

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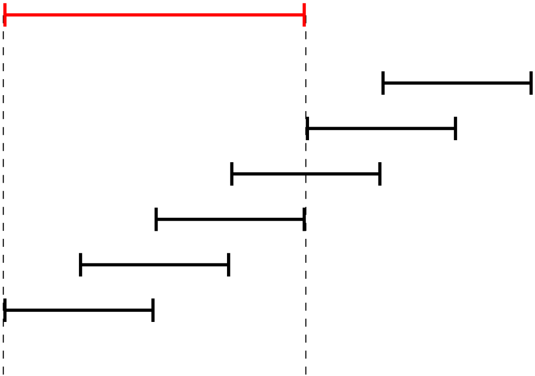

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6 relations + their inverses + equality = 13 Allen's relations.

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interval relations give rise to modal operators ➡ HS logic

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interval relations give rise to modal operators → HS logic

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Syntax:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \langle X \rangle \varphi$$

$$\langle X \rangle \in \{\langle A \rangle, \langle L \rangle, \langle B \rangle, \langle E \rangle, \langle D \rangle, \langle O \rangle, \langle \overline{A} \rangle, \langle \overline{L} \rangle, \langle \overline{B} \rangle, \langle \overline{E} \rangle, \langle \overline{D} \rangle, \langle \overline{O} \rangle\}$$

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Models:

$$\mathbf{M} = \langle \mathbb{I}(\mathbb{D}), V \rangle$$

$$V : \mathbb{I}(\mathbb{D}) \mapsto 2^{\mathcal{AP}}$$

$\mathcal{AP}$ atomic propositions (over intervals)

Formal semantics of HS

$\langle B \rangle$: **M**, $[d_0, d_1] \Vdash \langle B \rangle \phi$ iff there exists d_2 such that $d_0 \leq d_2 < d_1$ and **M**, $[d_0, d_2] \Vdash \phi$.

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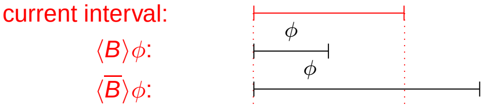

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$\langle E \rangle$: $\mathbf{M}, [d_0, d_1] \Vdash \langle E \rangle \phi$ iff there exists d_2 such that $d_0 < d_2 \leq d_1$ and $\mathbf{M}, [d_2, d_1] \Vdash \phi$.

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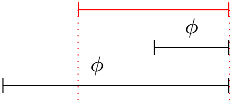

current interval:

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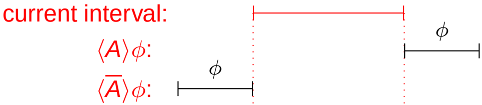

Formal semantics of HS - contd'

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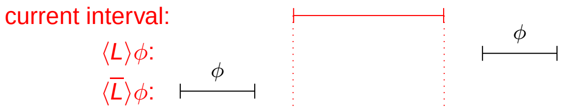

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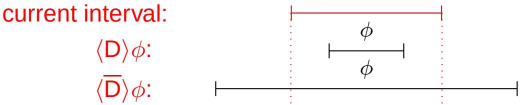

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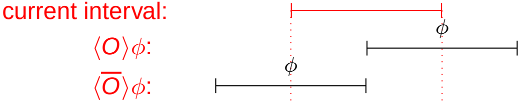

current interval:

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The zoo of fragments of HS

- $2^{12} = 4096$ fragments of HS
- several hundreds expressively different
- expressiveness and satisfiability issues wrt. class of interval structures (all, dense, discrete, finite, etc.)

Classifying HS fragments on finite linear orders

⏩ skip

Complexity class:

1: Non primitive recursive

2: EXPSPACE-complete

3: NEXPTIME-complete

4: NP-complete

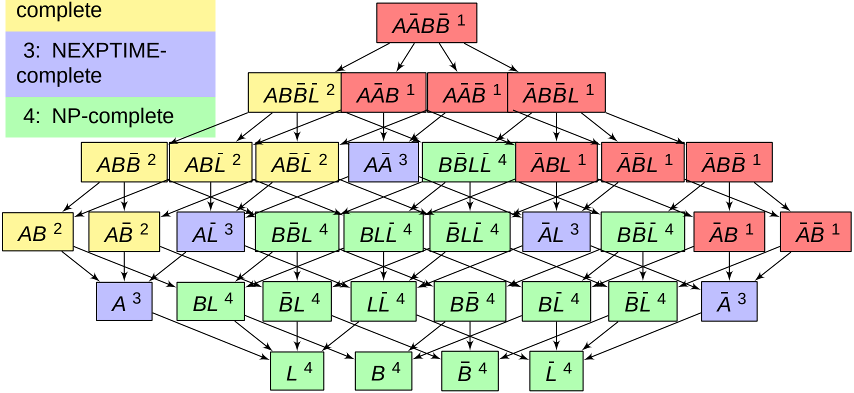

Figure: Hasse diagram of all and only decidable fragments of HS over finite linear orders.

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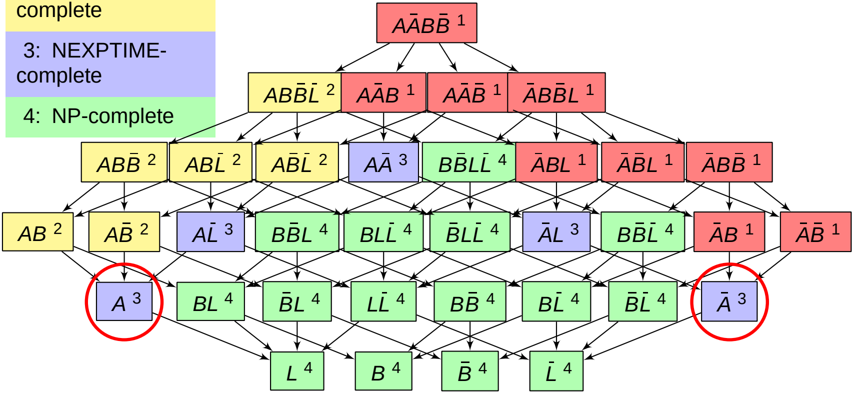

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F.Y.I., on this web address: *https://itl.dimi.uniud.it/content/logic-hs* you can find any information about HS-fragments and their fragments, updated to the latest advances, which are the results of over 10 years of research in this topic.

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In this work, we use the results we have that concern the fragment A in the finite case, and we put them at work to build a usable satisfiability checker.

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The logic A

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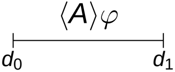
Semantics

$\mathbf{M}, [d_0, d_1] \Vdash \langle A \rangle \phi$



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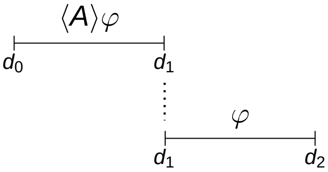

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- In this work we adapted the JUCS 2003 result (a classical tableaux) with the closing condition from the JAR 2007 result

Theoretical basis - contd’

The main theorem (small-model property) [JAR 2007]

Let φ be a A-formula. Then, φ is finitely satisfiable if and only if it is satisfiable on a model whose cardinality is strictly less than $2^m \cdot m + 1$, where m is the number of diamonds and boxes in φ.

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Theorem [JAR 2007]

A is NEXPTIME-hard, too

- A is NEXPTIME-complete

Satisfiability via tableaux

- satisfiability: given a A-formula φ, we want to establish if there exists a finite model **M** and an interval $[d_i, d_j]$ in it such that $\mathbf{M}, [d_i, d_j] \Vdash \varphi$

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- initial satisfiability
- tableaux rooted in φ
    - expansion rules
    - model production

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- a branch is *closed* if
  - contradiction, or
  - model too large (small-model theorem)

Branch Managing

- a *branch* is a list of nodes
  - enriched with an integer for the size of the associated model
- *nodes* are annotated formulas
  - (sub-)formula
  - interval
  - and a Boolean flag (node active or inactive)
- a node (its labeled formula $\varphi, [d_i, d_j]$) is *active* if
  - not a box-formula: rule has not been already applied to it
  - box-formula: a new point added since the last rule application
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- all branches are closed: *formula unsatisfiable*

Expansion rules

- classical operators are treated in the standard way
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$$(box)\ \frac{[A]\psi : [d_i, d_j]}{\psi : [d_j, d_{j+1}], \ldots, \psi : [d_j, d_N]},$$

$$(dia)\ \frac{\langle A\rangle\psi : [d_i, d_j]}{\psi : [d_j, d_{j+1}] \mid \ldots \mid \psi : [d_j, d_N] \mid \psi : [d_j, d_j'] \mid \ldots \mid \psi : [d_j, d_N']},$$

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$$(dia)\ \frac{\langle A\rangle\psi : [d_i, d_j]}{\psi : [d_j, d_{j+1}] \mid \ldots \mid \psi : [d_j, d_N] \mid \psi : [d_j, d'_j] \mid \ldots \mid \psi : [d_j, d'_N]},$$

- d_h ($j \leq h \leq N$) is a point in D
- d'_h ($j \leq h \leq N$) is a new point added to D ($d_h < d'_h d_{h+1}$)

Search Procedure

- initial tableau: 1 branch, 1 node, labeled with $\langle \varphi, [d_0, d_1] \rangle$

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  - if all branches are deleted, the formula is unsatisfiable
  - otherwise repeat the procedure

Priority Policies

To determine the priority of a branch in the queue, we implemented four different (and complete) policies:

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To determine the priority of a branch in the queue, we implemented four different (and complete) policies:

- FIFO: the standard first-in-first-out
- LDF: largest domain branches are expanded first
- SDF: smallest domain branches are expanded first
- GAN: branches with greatest number of active nodes are expanded first
- SDF: branches with smallest number of active nodes are expanded first

Formula representation

- input formula φ: conjunction of (sub-)formulas
  - input file: a line per each conjunct

- transformed into *negated normal form*

- stored into a *syntactic tree* (leaves are atomic propositions, and internal nodes are Boolean or modal operators).

Example

$$\boxed{\varphi = \langle A\rangle p \wedge [A](\neg p \vee \langle A\rangle p)}$$

$\varphi : [0, 1]$

Example

$$\boxed{\varphi = \langle A \rangle p \land [A](\neg p \lor \langle A \rangle p)}$$

→ $\varphi : [0,1]$

$\langle A \rangle p : [0,1]$

$[A](\neg p \lor \langle A \rangle p) : [0,1]$

Example

$$\boxed{\varphi = \langle A \rangle p \wedge [A](\neg p \vee \langle A \rangle p)}$$

$$\xcancel{\varphi : [0, 1]}$$

$$|$$

$$\rightarrow \langle A \rangle p : [0, 1]$$

$$|$$

$$[A](\neg p \vee \langle A \rangle p) : [0, 1]$$

$$|$$

$$p : [1, 2]$$

Example

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$\xcancel{\varphi : [0,1]}$

|

$\xcancel{\langle A \rangle p : [0,1]}$

|

→ $[A](\neg p \vee \langle A \rangle p) : [0,1]$

|

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|

$\neg p \vee \langle A \rangle p : [1,2]$

Example

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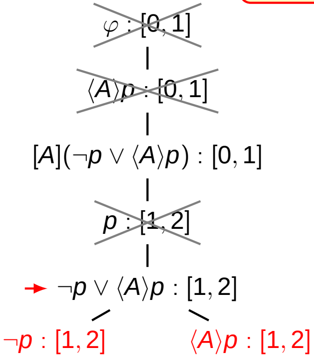

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$\xcancel{p : [1, 2]}$

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Example

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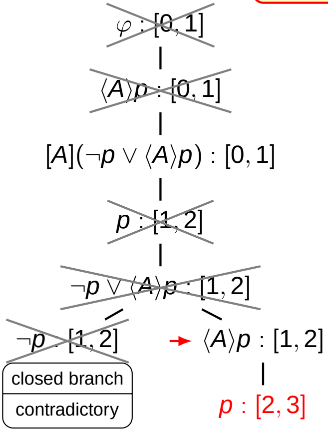

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~~$\neg p : [1, 2]$~~

closed branch

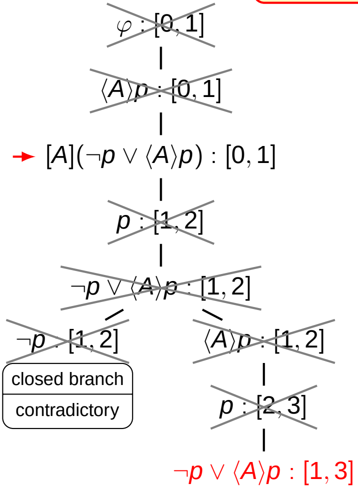
contradictory

→ $\langle A \rangle p : [1, 2]$

$p : [2, 3]$

Example

$\varphi = \langle A\rangle p \wedge [A](\neg p \vee \langle A\rangle p)$



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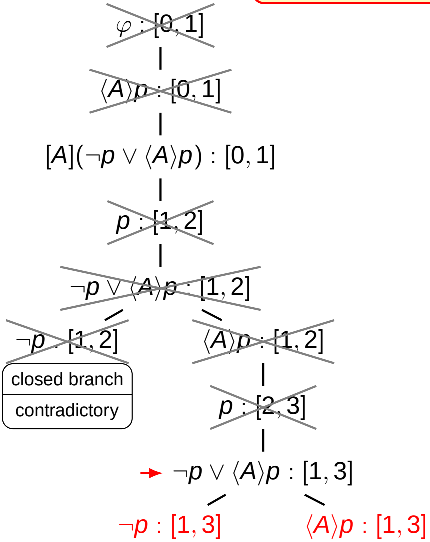

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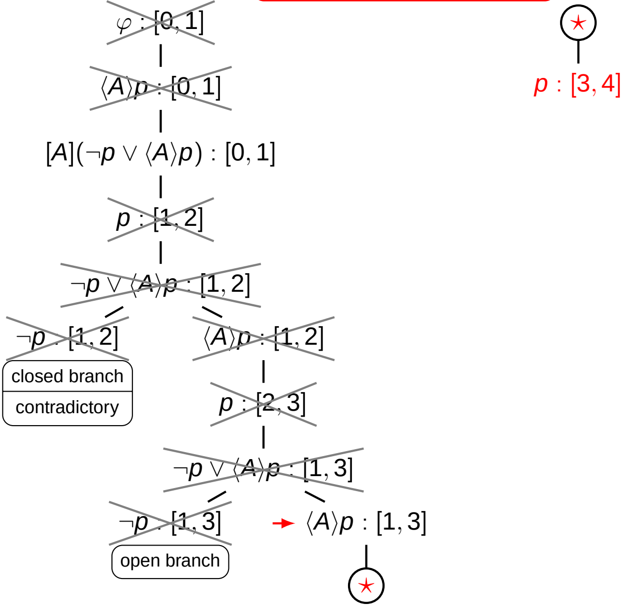

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open branch

→ $\langle A \rangle p : [1,3]$

⋆

⋆

$p : [3,4]$

Example

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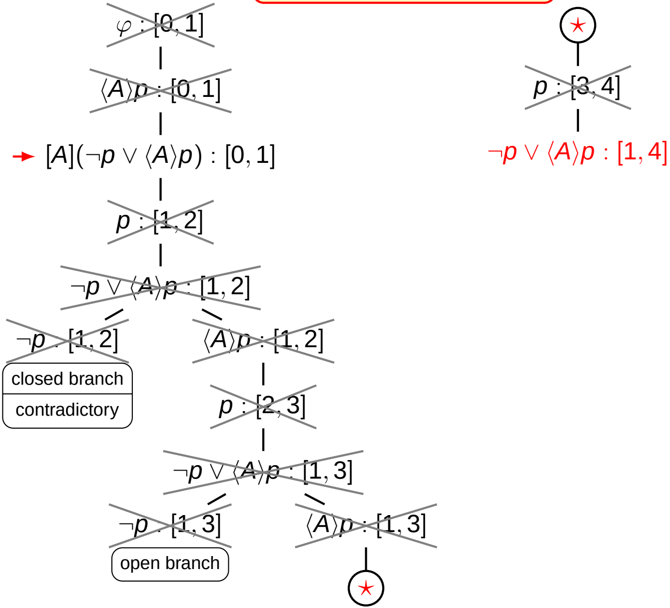

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closed branch

contradictory

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~~$\neg p \vee \langle A \rangle p : [1,3]$~~

~~$\neg p : [1,3]$~~

open branch

~~$\langle A \rangle p : [1,3]$~~

$\star$

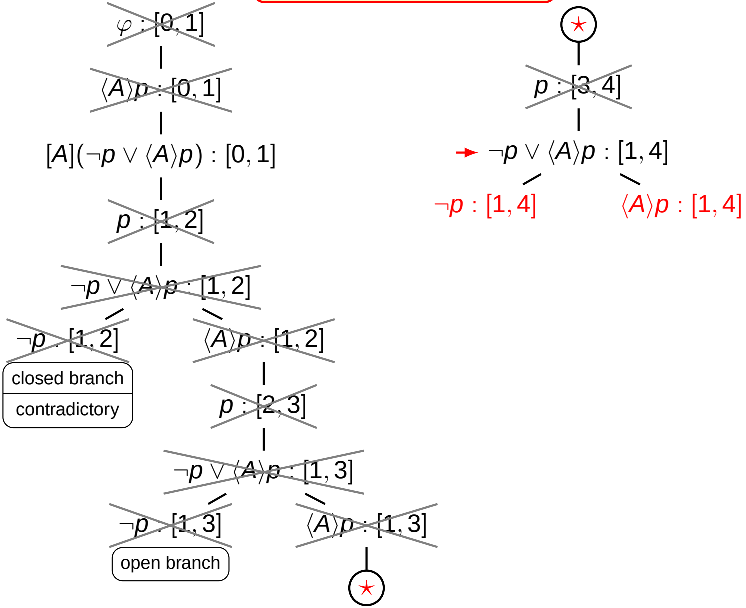
$\star$

~~$p : [3,4]$~~

$\neg p \vee \langle A \rangle p : [1,4]$

Example

$\varphi = \langle A\rangle p \wedge [A](\neg p \vee \langle A\rangle p)$



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closed branch

contradictory

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open branch

⋆

⋆

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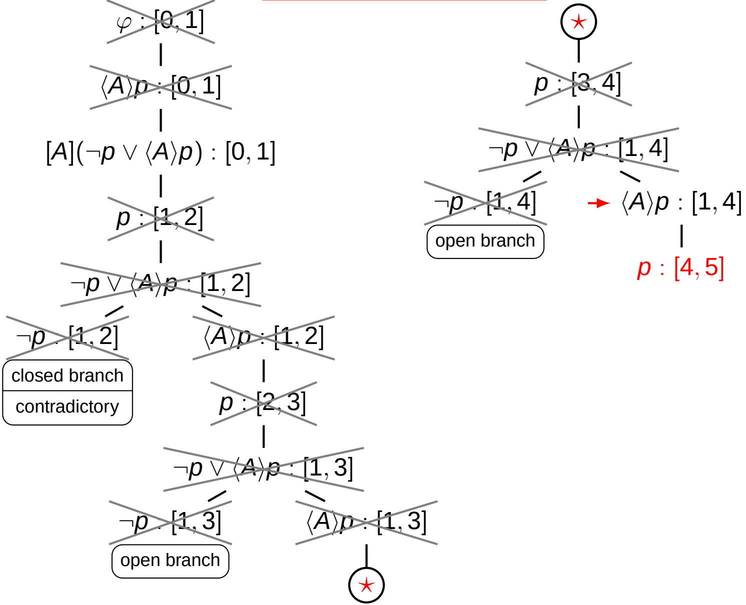

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closed branch

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open branch

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$\star$

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open branch

→ $\langle A\rangle p : [1,4]$

$p : [4,5]$

Example

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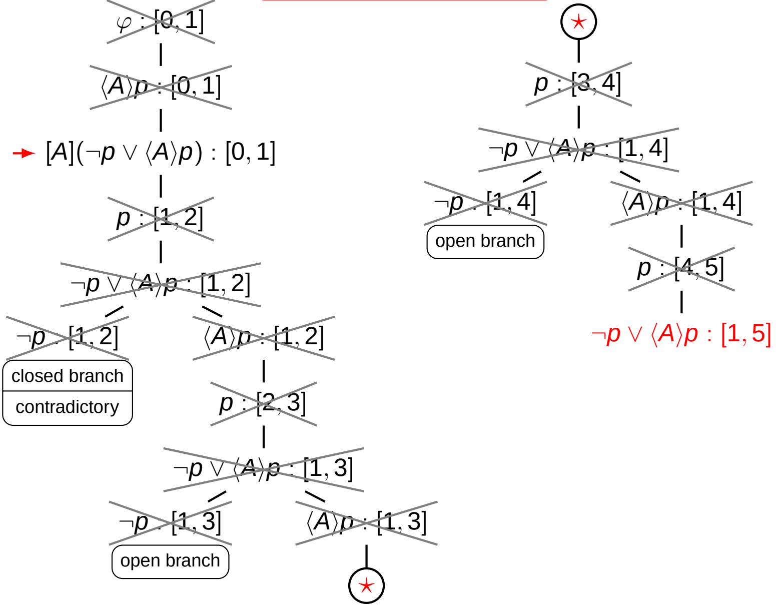

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closed branch

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open branch

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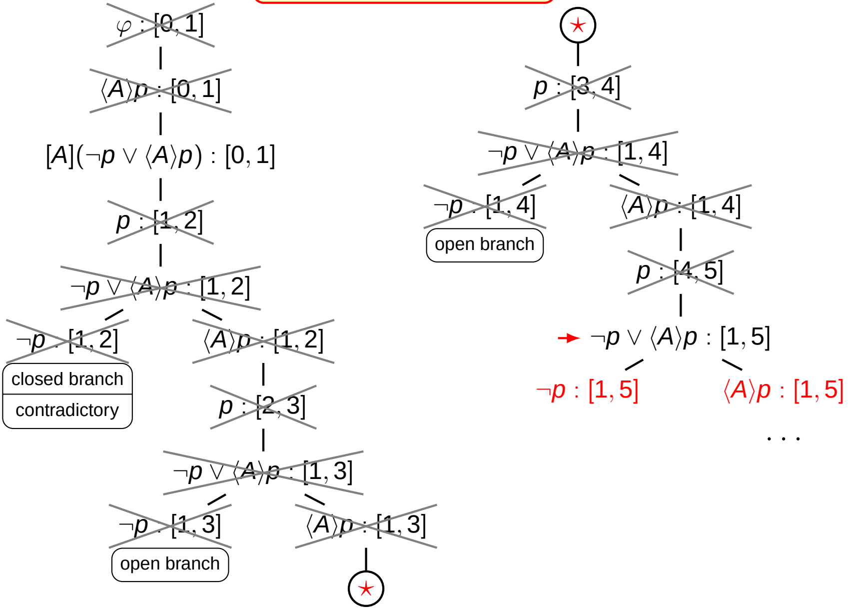

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open branch

$\xcancel{\langle A \rangle p : [1,3]}$

$\star$

$\star$

$\xcancel{p : [3,4]}$

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open branch

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$\dots$

Example

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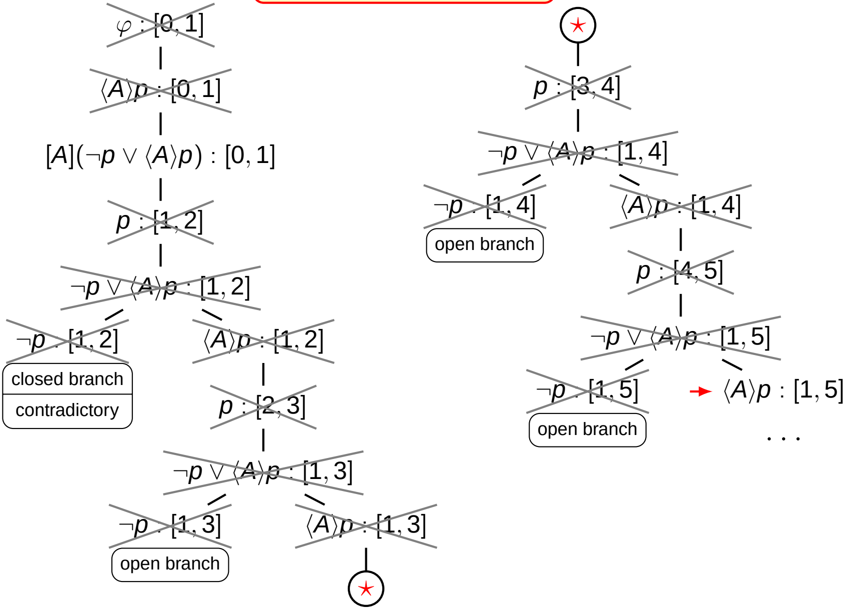

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. . .

Example

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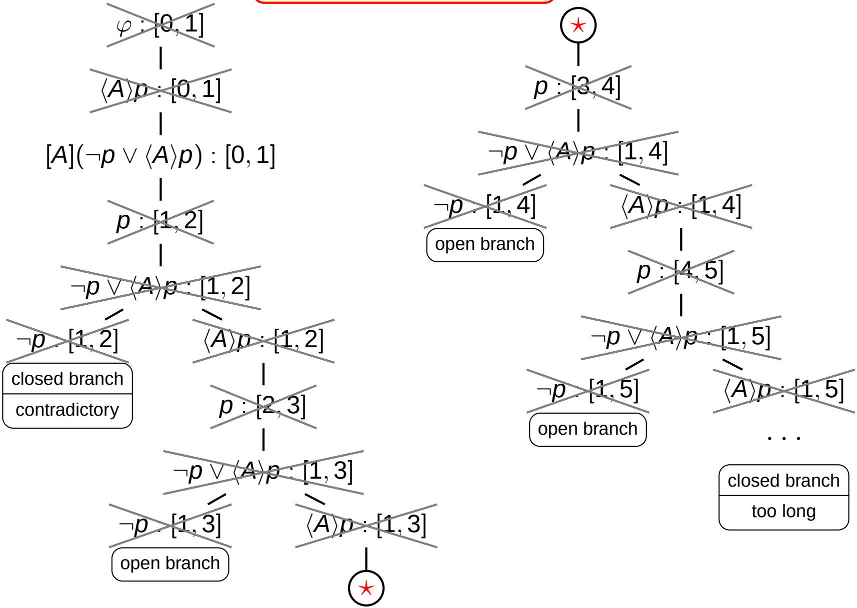

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open branch

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$\ldots$

closed branch

too long

Benchmark

Tests on benchmarks:

- *combinatorics* (to test scalability)
  - the n-th combinatorial problem
    - n conjuncts $\langle A \rangle p_i$ $(0 \leq i \leq n)$
    - $\frac{n(n+1)}{2}$ formulas $[A]\neg(p_i \wedge p_j)$ $(i \neq j)$
- *randomized* (to simulate the behaviour in real cases)
  - 36 completely random problems

Experimental Results - combinatorics

COMBINATORICS

	Policy (sec)					Outcome
n	FIFO	SDF	LDF	SAN	GAN	(size)
1	0.004	0.004	0.004	0.004	0.004	4
2	0.004	0.008	0.004	0.004	0.008	5
3	0.008	0.15	0.03	0.008	0.03	6
4	0.01	–	30.07	0.01	30.29	7
5	0.012	–	–	0.012	–	8
6	0.02	–	–	0.03	–	9
7	0.07	–	–	0.07	–	10
8	0.15	–	–	0.16	–	11
9	0.3	–	–	0.32	–	12
10	0.56	–	–	0.59	–	13
11	0.99	–	–	1.06	–	14
12	1.67	–	–	1.79	–	15
13	2.73	–	–	2.94	–	16
14	4.25	–	–	4.55	–	17
15	6.56	–	–	7.08	–	18
16	9.77	–	–	10.82	–	19
17	14.42	–	–	15.40	–	20
18	20.79	–	–	22.20	–	21
19	29.28	–	–	32.11	–	22
20	40.91	–	–	44.09	–	23
21	–	–	–	–	–	–
22	–	–	–	–	–	–

Experimental Results - randomized

RANDOMIZED

	Policy (sec)					Outcome
n	FIFO	SDF	LDF	SAN	GAN	(size)
1	0.004	0.004	0.004	0.004	0.004	4
2	0.004	0.004	0.004	0.004	0.004	4
3	0.004	0.004	0.004	0.004	0.004	4
4	0.004	0.004	0.004	0.004	0.004	4
5	0.004	0.004	0.004	0.004	0.004	4
6	0.004	0.004	0.004	0.004	0.004	4
7	0.07	0.23	0.004	0.18	0.004	3 / 4
8	0.004	0.004	0.004	0.004	0.004	4
9	0.004	0.004	0.004	0.004	0.004	4
10	0.004	0.004	0.004	0.004	0.004	4
11	0.004	0.004	0.004	0.004	0.004	4
12	0.004	0.004	0.004	0.004	0.004	4
13	0.01	0.04	0.004	0.02	0.004	4
14	0.004	0.004	0.004	0.004	0.004	4
15	0.004	0.004	0.004	0.004	0.004	4
16	0.004	1.37	0.004	0.01	0.004	4
17	0.004	0.004	0.004	0.004	0.004	4
18	0.004	0.004	0.004	0.004	0.004	3

	Policy (sec)					Outcome
n	FIFO	SDF	LDF	SAN	GAN	(size)
19	1.66	45.43	0.68	1.91	0.02	3 / 4
20	0.02	0.004	0.03	0.03	0.004	2 / 4
21	0.004	0.004	0.004	0.004	0.004	4
22	0.74	14.08	0.004	1.04	0.004	4
23	0.004	0.004	0.004	0.004	0.004	4
24	0.004	0.004	0.004	0.004	0.004	4
25	–	–	–	–	–	–
26	0.004	0.004	0.004	0.004	0.004	4
27	0.004	–	0.004	0.01	–	3 / 4
28	0.004	0.004	0.004	0.004	0.004	4
29	0.004	–	0.004	0.004	0.004	4
30	0.14	0.08	0.04	0.19	0.01	2 / 4
31	0.004	0.004	0.004	0.004	0.004	unsat
32	0.25	–	0.02	0.31	0.004	2 / 4
33	0.004	0.004	0.004	0.004	0.004	4
34	–	–	0.02	0.004	0.02	2 / 4
35	0.004	–	0.004	–	0.004	2 / 4
36	–	–	–	–	1.2	3

Outline

1. Introduction on Interval Temporal Logic
2. Halpern-Shoham’s modal logic HS
3. The tableau method for the logic A
4. Conclusions

Conclusions

Interval temporal logics

- a lot of theoretical results in the last 10 years
    - decidability, undecidability, expressiveness issues
- a first step towards an implementation
- the system is available at the page

`http://www.di.unisa.it/dottorandi/dario.dellamonica/tableaux/`

Future directions

- more expressive languages
- different classes of linear orders
- generation of a proper benchmark
- use of generic theorem prover (Mettel?)

The end

Thank you