# Algebraic entropy for amenable semigroup actions

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Let $A$ be an abelian group and $\phi : A \to A$ an endomorphism;
$\mathcal{P}_f(A) = \{F \subseteq A \mid F \neq \emptyset \text{ finite}\} \supseteq \mathcal{F}(A) = \{F \leq A \mid F \text{ finite}\}$.

For $F \in \mathcal{P}_f(A)$, $n > 0$, let $T_n(\phi, F) = F + \phi(F) + \ldots + \phi^{n-1}(F)$.

The *algebraic entropy of $\phi$ with respect to $F$* is

$$H_{alg}(\phi, F) = \lim_{n\to\infty} \frac{\log |T_n(\phi, F)|}{n}.$$

[Adler-Konheim-McAndrew, M.Weiss] The *algebraic entropy of $\phi$* is

$$\mathrm{ent}(\phi) = \sup\{H_{alg}(\phi, F) \mid F \in \mathcal{F}(A)\}.$$

[Peters, Dikranjan] The *algebraic entropy of $\phi$* is

$$h_{alg}(\phi) = \sup\{H_{alg}(\phi, F) \mid F \in \mathcal{P}_f(A)\}.$$

Clearly, $\boxed{\mathrm{ent}(\phi) = \mathrm{ent}(\phi \restriction_{t(A)}) = h_{alg}(\phi \restriction_{t(A)}) \leq h_{alg}(\phi).}$

[Dikranjan-Goldsmith-Salce-Zanardo for $\mathrm{ent}$, D-GB for $h_{alg}$]

**Theorem (Addition Theorem = Yuzvinski's addition formula)**

*If $B$ is a $\phi$-invariant subgroup of $A$, then*

$$h_{alg}(\phi) = h(\phi\restriction_B) + h(\phi_{A/B}),$$

*where $\phi_{A/B} : A/B \to A/B$ is induced by $\phi$.*

[Weiss for $\mathrm{ent}$, Peters, D-GB for $h_{alg}$]

**Theorem (Bridge Theorem)**

*Denote $\widehat{A}$ the Pontryagin dual of $A$ and $\widehat{\phi} : \widehat{A} \to \widehat{A}$ the dual of $\phi$. Then*

$$h_{alg}(\phi) = h_{top}(\widehat{\phi}).$$

Here $h_{top}$ denotes the topological entropy for continuous selfmaps of compact spaces [Adler-Konheim-McAndrew].

## Non-abelian case

Let $G$ be a group and $\phi : G \to G$ an endomorphism.
Let $\mathcal{P}_f(G) = \{F \subseteq G \mid F \neq \emptyset \text{ finite}\}$.

For $F \in \mathcal{P}_f(G)$, $n > 0$, let $T_n(\phi, F) = F \cdot \phi(F) \cdot \ \dots \ \cdot \phi^{n-1}(F)$.

The *algebraic entropy of $\phi$ with respect to $F$* is

$$H_{alg}(\phi, F) = \lim_{n\to\infty} \frac{\log |T_n(\phi, F)|}{n}.$$

[Dikranjan-GB] The *algebraic entropy of $\phi$* is

$$h_{alg}(\phi) = \sup\{H_{alg}(\phi, F) \mid F \in \mathcal{P}_f(G)\}.$$

$G = \langle X \rangle$ finitely generated group ($X \in \mathcal{P}_f(G)$).

For $g \in G \setminus \{1\}$, $\ell_X(g)$ is the length of the shortest word representing $g$ in $X \cup X^{-1}$, and $\ell_X(1) = 0$.

For $n \geq 0$, let $B_X(n) = \{g \in G \mid \ell_X(g) \leq n\}$.

The *growth function of G wrt X* is $\gamma_X : \mathbb{N} \to \mathbb{N},\ n \mapsto |B_X(n)|$.
The *growth rate of G wrt X* is $\lambda_X = \lim_{n\to\infty} \frac{\log \gamma_X(n)}{n}$.

For $\phi = id_G$ and $1 \in X$,

$$\boxed{T_n(id_G, X) = B_X(n) \text{ and } H_{alg}(id_G, X) = \lambda_X.}$$

[Milnor Problem, Grigorchuk group, Gromov Theorem]
There exists a group of intermediate growth.
$G$ has polynomial growth if and only if $G$ is virtually nilpotent.

Let $G$ be a group, $\phi : G \to G$ an endomorphism and $X \in \mathcal{P}_f(G)$.
The *growth rate of* $\phi$ *wrt* $X$ is $\gamma_{\phi,X} : \mathbb{N}_+ \to \mathbb{N}_+,\ n \mapsto |T_n(\phi, X)|$.

If $G = \langle X \rangle$ with $1 \in X \in \mathcal{P}_f(G)$, then $\boxed{\gamma_X = \gamma_{id_G,X}.}$

- $\phi$ has polynomial growth if $\gamma_{\phi,X}$ is polynomial $\forall X \in \mathcal{P}_f(G)$;
- $\phi$ has exponential growth if $\exists\ F \in \mathcal{P}_f(G)$, $\gamma_{\phi,X}$ is exp.;
- $\phi$ has intermediate growth otherwise.

$\phi$ has exponential growth if and only if $h_{alg}(\phi) > 0$.

The Addition Theorem does not hold for $h_{alg}$: let $G = \mathbb{Z}^{(\mathbb{Z})} \rtimes_\beta \mathbb{Z}$;

- $G$ has exponential growth and so $h_{alg}(id_G) = \infty$;
- $\mathbb{Z}^{(\mathbb{Z})}$ and $\mathbb{Z}$ are abelian and hence $h_{alg}(id_{\mathbb{Z}^{(\mathbb{Z})}}) = 0 = h_{alg}(id_{\mathbb{Z}})$.

**Theorem ([GB-Spiga, Dikranjan-GB for abelian groups, Milnor-Wolf in the classical setting])**

*No endomorphism of a locally virtually soluble group has intermediate growth.*

Let $S$ be a cancellative semigroup.
$S$ is *right-amenable* if and only if $S$ admits a *right-Følner net*, i.e., a net $(F_i)_{i\in I}$ in $\mathcal{P}_f(S)$ such that $\lim_{i\in I} \frac{|F_i s\setminus F_i|}{|F_i|} = 0\ \forall s \in S$.
(analogously, left-amenable).

A map $f : \mathcal{P}_f(S) \to \mathbb{R}$ is:

1. *subadditive* if $f(F_1 \cup F_2) \leq f(F_1) + f(F_2)\ \forall F_1, F_2 \in \mathcal{P}_f(S)$;
2. *left-subinvariant* if $f(sF) \leq f(F)\ \forall s \in S\ \forall F \in \mathcal{P}_f(S)$;
3. *right-subinvariant* if $f(Fs) \leq f(F)\ \forall s \in S\ \forall F \in \mathcal{P}_f(S)$;
4. *unif. bounded on singletons* if $\exists M \geq 0,\ f(\{s\}) \leq M\ \forall s \in S$.

Let $\mathcal{L}(S) = \{f : \mathcal{P}_f(S) \to \mathbb{R} \mid (1), (2), (4) \text{ hold for } f\}$ and $\mathcal{R}(S) = \{f : \mathcal{P}_f(S) \to \mathbb{R} \mid (1), (3), (4) \text{ hold for } f\}$.

[Ceccherini Silberstein-Coornaert-Krieger, generalizing Ornstein-Weiss Theorem]

Let $S$ be a cancellative right-amenable (resp., left-amenable) semigroup. For every $f \in \mathcal{L}(S)$ (resp., $f \in \mathcal{R}(S)$) there exists $\lambda \in \mathbb{R}_{\geq 0}$ such that

$$\mathcal{H}_S(f) := \lim_{i \in I} \frac{f(F_i)}{|F_i|} = \lambda$$

for every right-Følner (resp., left-Følner) net $(F_i)_{i \in I}$ of $S$.

Let $S$ be a cancellative left-amenable semigroup, $X$ a compact space and $\mathrm{cov}(X)$ the family of all open covers of $X$.
For $\mathcal{U} \in \mathrm{cov}(X)$, let $N(\mathcal{U}) = \min\{|\mathcal{V}| \mid \mathcal{V} \subseteq \mathcal{U}\}$.

Consider a left action $S \overset{\gamma}{\curvearrowright} X$ by continuous maps.
For $\mathcal{U} \in \mathrm{cov}(X)$ and $F \in \mathcal{P}_f(S)$, let

$$\mathcal{U}_{\gamma,F} = \bigvee_{s \in F} \gamma(s)^{-1}(\mathcal{U}) \in \mathrm{cov}(X).$$

$$f_{\mathcal{U}} : \mathcal{P}_{fin}(S) \to \mathbb{R}, \quad F \mapsto \log N(\mathcal{U}_{\gamma,F}).$$

Then $f_{\mathcal{U}} \in \mathcal{R}(S)$.

[Ceccherini-Silberstein-Coornaert-Krieger, gen. Moulin Ollagnier]
The *topological entropy of* $\gamma$ *with respect to* $\mathcal{U}$ is

$$H_{top}(\gamma, \mathcal{U}) = \mathcal{H}_S(f_{\mathcal{U}}).$$

The *topological entropy of* $\gamma$ is

$$h_{top}(\gamma) = \sup\{H_{top}(\gamma, \mathcal{U}) \mid \mathcal{U} \in \mathrm{cov}(X)\}.$$

Let $S$ be a cancellative right-amenable semigroup.
Let $A$ be an abelian group and
consider a left action $S \overset{\alpha}{\curvearrowright} A$ by endomorphisms.

For $X \in \mathcal{P}_f(A)$ and $F \in \mathcal{P}_f(S)$, let

$$T_F(\alpha, X) = \sum_{s \in F} \alpha(s)(X) \in \mathcal{P}_f(A).$$

$$f_X : \mathcal{P}_{fin}(S) \to \mathbb{R}, \quad F \mapsto \log |T_F(\alpha, X)|.$$

Then $f_X \in \mathcal{L}(S)$.
The *algebraic entropy of* $\alpha$ *with respect to* $X$ is

$$H_{alg}(\alpha, X) = \mathcal{H}_S(f_X).$$

[Fornasiero-GB-Dikranjan, Virili for groups]
The *algebraic entropy of* $\alpha$ is

$$h_{alg}(\alpha) = \sup\{H_{alg}(\alpha, X) \mid X \in \mathcal{P}_f(A)\}.$$

Moreover, $\mathrm{ent}(\alpha) = \sup\{H_{alg}(\alpha, X) \mid X \in \mathcal{F}(A)\}$.

Let $S$ be a cancellative right-amenable semigroup.
Let $A$ be an abelian group and
consider a left action $S \overset{\alpha}{\curvearrowright} A$ by endomorphisms.

**Theorem (Addition Theorem)**

*If $A$ is torsion and $B$ is an $\alpha$-invariant subgroup of $A$, then*

$$h_{alg}(\alpha) = h_{alg}(\alpha_B) + h_{alg}(\alpha_{A/B}),$$

*where $S \overset{\alpha_B}{\curvearrowright} B$ and $S \overset{\alpha_{B/A}}{\curvearrowright} B/A$ are induced by $\alpha$.*

Let $S$ be a cancellative left-amenable semigroup.
Let $K$ be a compact abelian group and
consider a left action $S \overset{\gamma}{\curvearrowright} K$ by continuous endomorphisms.

$\gamma$ induces a right action $\widehat{K} \overset{\widehat{\gamma}}{\curvearrowleft} S$, defined by

$$\widehat{\gamma}(s) = \widehat{\gamma(s)} : \widehat{K} \to \widehat{K} \quad \text{for every } s \in S;$$

$\widehat{\gamma}$ is the *dual action of* $\gamma$.

Denote by $\widehat{\gamma}^{op}$ the left action $S^{op} \overset{\widehat{\gamma}}{\curvearrowright} \widehat{K}$ associated to $\widehat{\gamma}$ of the cancellative right-amenable semigroup $S^{op}$.

**Theorem (Bridge Theorem)**

*If $K$ is totally disconnected (i.e., $A$ is torsion), then*

$$h_{top}(\gamma) = h_{alg}(\widehat{\gamma}^{op}).$$

[Virili for amenable group actions on locally compact abelian groups]

Let $S$ be a cancellative left-amenable semigroup.
Let $K$ be a compact abelian group and
consider a left action $S \overset{\gamma}{\curvearrowright} K$ by continuous endomorphisms.

**Corollary (Addition Theorem)**

*If $K$ is totally disconnected and $L$ is a $\gamma$-invariant subgroup of $K$, then*

$$h_{top}(\gamma) = h_{top}(\gamma_L) + h_{top}(\gamma_{K/L}),$$

*where $S \overset{\gamma_L}{\curvearrowright} L$ and $S \overset{\gamma_{K/L}}{\curvearrowright} K/L$ are induced by $\gamma$.*

Known in the case of compact groups for:

- $\mathbb{Z}^d$-actions on compact groups [Lind-Schmidt-Ward];
- actions of countable amenable groups on compact metrizable groups [Li].

## Restriction and quotient actions

Let $G$ be an amenable group, $A$ an abelian group, $G \overset{\alpha}{\curvearrowright} A$.
For $H \leq G$ consider $H \overset{\alpha\restriction_H}{\curvearrowright} A$.

- If $[G:H] = k \in \mathbb{N}$, then $h_{alg}(\alpha \restriction_H) = k \cdot h_{alg}(\alpha)$.
- If $H$ is normal, then $h_{alg}(\alpha) \leq h_{alg}(\alpha \restriction_H)$.

For $N \leq G$ normal with $N \subseteq \ker\alpha$, consider $G/N \overset{\bar{\alpha}_{G/N}}{\curvearrowright} A$.

- $h_{alg}(\alpha) = \begin{cases} 0 & \text{if } N \text{ is infinite,} \\ \frac{h_{alg}(\overline{\alpha}_{G/N})}{|N|} & \text{if } N \text{ is finite.} \end{cases}$

**Corollary**

*If $h_{alg}(\alpha) > 0$, then $\ker\alpha$ is finite and $h_{alg}(\alpha) = \frac{h_{alg}(\overline{\alpha}_{G/\ker\alpha})}{|\ker\alpha|}$.*

So: reduction to faithful actions.

## Generalized shifts

Let $S$ be a semigroup, $Y$ a non-empty set and $A$ an abelian group.

- For a right action $Y \overset{\gamma}{\curvearrowleft} S$, the *generalized backward $S$-shift* is $S \overset{\beta_{A,\gamma}}{\curvearrowright} A^{(Y)}$ defined by
  $$\boxed{\beta_{A,\gamma}(s)(f) = f \circ \gamma(s)} \ \forall s \in S, \forall f \in A^{(Y)}.$$
- For a left action $S \overset{\eta}{\curvearrowright} Y$, such that each $\gamma(s)$ has finite fibers, the *generalized forward $S$-shift* is $S \overset{\sigma_{A,\eta}}{\curvearrowright} A^{(Y)}$ defined by
  $$\boxed{\sigma_{A,\eta}(s)(f)(y) = \sum_{\eta(s)(z)=y} f(z)} \ \forall s \in S, \forall f \in A^{(Y)}, \forall y \in Y.$$

If $S = Y = \mathbb{N}$, and $\mathbb{N} \overset{\rho}{\curvearrowright} \mathbb{N}$ is given by $\rho(1) : n \mapsto n+1$, then
$\beta_{A,\rho}(1) : A^{(\mathbb{N})} \to A^{(\mathbb{N})}, \ (x_0, x_1, x_2, \ldots) \mapsto (x_1, x_2, x_3 \ldots)$ and
$\sigma_{A,\rho}(1) : A^{(\mathbb{N})} \to A^{(\mathbb{N})}, \ (x_0, x_1, x_2, \ldots) \mapsto (0, x_0, x_1, \ldots)$.

Let $S$ be a cancellative right-amenable monoid and $A$ an abelian group.

Consider $S \overset{\rho}{\curvearrowleft} S$ defined by $\rho(s)(x) = xs\ \forall s \in S,\ \forall x \in S$, and $S \overset{\beta_{A,\lambda}}{\curvearrowright} A^{(S)}$;

$$\mathrm{ent}(\beta_{A,\rho}) = \begin{cases} \log|t(A)| & \text{if } S \text{ is a group,} \\ 0 & \text{if } S \text{ is not a group.} \end{cases}$$

Consider $S \overset{\lambda}{\curvearrowright} S$ defined by $\lambda(s)(x) = sx\ \forall s \in S,\ \forall x \in S$, and $S \overset{\sigma_{A,\lambda}}{\curvearrowright} A^{(S)}$;

$$h_{alg}(\sigma_{A,\lambda}) = \begin{cases} \log|A| & \text{if } S \text{ is infinite,} \\ \frac{\log|A|}{|S|} & \text{if } S \text{ is finite.} \end{cases}$$

## Set-theoretic entropy

Let $S$ be a cancellative right-amenable monoid.

Let $Y$ be a non-empty set and consider a left action $S \overset{\eta}{\curvearrowright} Y$.

For $X \in \mathcal{P}_f(Y)$ and $F \in P_f(S)$, let

$$F \cdot X = \alpha(F)(X) = \{\alpha(g)(x) \mid g \in F, x \in Y\}.$$

$$l_X : \mathcal{P}_f(S) \to \mathbb{R}, \quad F \mapsto |F \cdot X|.$$

Then $l_X \in \mathcal{L}(S)$.

The *set-theoretic entropy of* $\eta$ *with respect to* $X$ *is*

$$H_{set}(\eta, X) = \mathcal{H}_S(l_X).$$

The *set-theoretic entropy of* $\eta$ *is*

$$h_{set}(\eta) = \sup\{H_{set}(\eta, X) \mid X \in \mathcal{P}_f(Y)\}.$$

[For $\mathbb{N}$-actions this entropy was defined by Dikranjan-Shirazi, with applications towards the computation of the topological entropy of selfmaps $K^Y \to K^Y$, where $K$ is compact.]

Let $G$ be an amenable group, $Y$ a non-empty set and $G \overset{\eta}{\curvearrowright} Y$.
For $y \in Y$, let $\mathrm{Stab}_y = \{g \in G \mid \eta(g)(y) = y\}$ and $O_y = G \cdot \{y\}$.
The transitive action $G \overset{\eta}{\curvearrowright} O_y$ is isomorphic (with $H = \mathrm{Stab}_y$) to the canonical action $G \overset{\varrho_{G/H}}{\curvearrowright} G/H$ on the set $G/H$ given by

$$\varrho_{G/H}(g)(fH) = (gf)H \ \forall f, g \in G.$$

## Theorem

*If $H$ is a subgroup of $G$, then $h_{set}(\varrho_{G/H}) = \frac{1}{|H|}$.*
*So, if $\{O_{y_i} \mid i \in I\}$ are the orbits of $\eta$, then $h_{set}(\eta) = \sum_{i \in I} \frac{1}{|Stab_{y_i}|}$.*

Let $\mathfrak{s}(G) = \sup\{|F| \mid F \leq G \text{ finite}\}$. If $G$ is locally nilpotent then $t(G)$ is a normal subgroup of $G$, and so $\mathfrak{s}(G) = |t(G)|$.

## Corollary

*If $\mathfrak{s}(G)$ is finite, then*
*either $h_{set}(\eta) = \infty$, or $h_{set}(\eta) = \frac{m}{|\mathfrak{s}(G)|}$ for some $m \in \mathbb{N}$.*

Let $S$ be an infinite cancellative right-amenable monoid, $Y$ a non-empty set and $A$ an abelian group.

Consider $S \overset{\eta}{\curvearrowright} Y$, such that each $\gamma(s)$ has finite fibers, and $S \overset{\sigma_{A,\eta}}{\curvearrowright} A^{(Y)}$ defined by

$$\sigma_{A,\eta}(s)(f)(y) = \sum_{\eta(s)(z)=y} f(z)$$

$\forall s \in S, \forall f \in A^{(Y)}, \forall y \in Y$.

**Theorem**

$h_{alg}(\sigma_{A,\eta}) = h_{set}(\eta) \cdot \log |A|$.

Since $S \overset{\lambda}{\curvearrowright} S$ with $\lambda(s)(x) = sx \ \forall s \in S, \ \forall x \in S$, has $h_{set}(\lambda) = 1$, as a corollary we obtain the previous result: $h_{alg}(\sigma_{A,\lambda}) = \log |A|$.

## Entropy and Lehmer Problem

For a primitive polynomial $f(x) = sx^n + a_1 x^{n-1} \ldots + a_n \in \mathbb{Z}[x]$ with (complex) roots $\lambda_1, \ldots, \lambda_n$, the *Mahler measure* of $f$ is

$$m(f) = \log s + \sum_{|\lambda_i|>1} \log |\lambda_i|.$$

Let

$$\mathfrak{L} = \{m(f(x)) \mid f(x) \in \mathbb{Z}[x]\} \text{ and } \lambda = \inf(\mathfrak{L} \setminus \{0\}).$$

**Problem ([Lehmer 1933])**

*Is $\lambda > 0$?*

Algebraic Yuzvinski Formula: If $\phi : \mathbb{Q}^n \to \mathbb{Q}^n$ is an endomorphism, then

$$h_{alg}(\phi) = \log m(f(x)),$$

where $f(x)$ is the integer characteristic polynomial of $\phi$.
[Lind-Schmidt-Ward for $\mathbb{Z}^d$-actions and $h_{top}$;
Deninger, Li-Thom, Li in more general cases.]

Let $\mathcal{E}_{alg} = \{h_{alg}(f) \mid f \in \mathrm{End}(G), G \text{ abelian group}\}$.

**Theorem ([Dikranjan-GB])**

- $\inf(\mathcal{E}_{alg} \setminus \{0\}) = \lambda$;
- $\lambda = 0$ *if and only if* $\mathcal{E}_{alg} = \mathbb{R}_{\geq 0} \cup \{\infty\}$;
- $\lambda > 0$ *if and only if* $\mathcal{E}_{alg}$ *is countable.*

Counterpart of [Lind-Schmidt-Ward, Theorem 4.6] for $\mathbb{Z}^d$-actions on compact groups.

Let $S$ be a cancellative right-amenable semigroup. Define:

- $\mathcal{E}_{set}(S) = \{h_{set}(\eta) \mid \eta \text{ action of } S \text{ on a set}\}$;
- $\mathcal{E}_{alg}(S) = \{h_{alg}(\alpha) \mid \alpha \text{ action of } S \text{ on an abelian group}\}$.

(Clearly, $\mathcal{E}_{alg} = \mathcal{E}_{alg}(\mathbb{N})$.)
By [Lawton, Lind-Schmidt-Ward] and the Bridge Theorem [Virili],
$\inf(\mathcal{E}_{alg}(\mathbb{N}) \setminus \{0\}) = \inf(\mathcal{E}_{alg}(\mathbb{Z}) \setminus \{0\}) = \inf(\mathcal{E}_{alg}(\mathbb{Z}^d) \setminus \{0\}) = \lambda$.

**Problem**

*Describe $\mathcal{E}_{set}(S)$ and $\mathcal{E}_{alg}(S)$.*

**Theorem**

*Let $G$ be an amenable group. Then*

$$\mathcal{E}_{set}(G) = \begin{cases} \mathbb{R}_{\geq 0} \cup \{\infty\} & \text{if } \mathfrak{s}(G) \text{ is infinite,} \\ \frac{1}{|\mathfrak{s}(G)|}\mathbb{N} \cup \{\infty\} & \text{if } \mathfrak{s}(G) \text{ is finite.} \end{cases}$$

In particular, $\mathcal{E}_{set}(G) = \mathbb{N} \cup \{\infty\}$ if $G$ is torsion-free.

Let $G$ be an amenable group. Then

$$(\log k)\mathcal{E}_{set}(G) \subseteq \mathcal{E}_{alg}(G) \text{ for every } k > 1.$$

In fact, if $r \in \mathcal{E}_{set}(G)$, that is, $r = h_{set}(\eta)$ for some $G \overset{\eta}{\curvearrowright} X$, then, for every finite abelian group $A$ of size $k > 1$, $h_{alg}(\sigma_{A,\eta}) = r \log k$.

**Theorem**

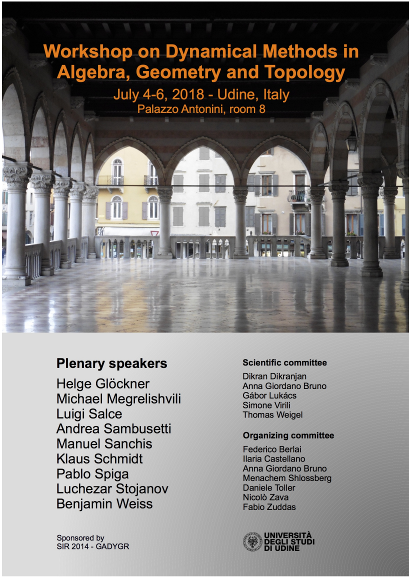
*If $\mathfrak{s}(G)$ is infinite, then $\mathcal{E}_{alg}(G) = \mathbb{R}_{\geq 0} \cup \{\infty\}$.*

Therefore, $\mathcal{E}_{alg}(G) = \mathbb{R}_{\geq 0} \cup \{\infty\}$ for every locally nilpotent group with infinite $t(G)$.
Yet $\mathcal{E}_{alg}(G)$ is unclear for arbitrary torson-free (abelian) groups.

**Problem**

*How do the sets $\mathcal{E}_{alg}(\mathbb{Q}), \mathcal{E}_{alg}(\mathbb{Q}^2), \mathcal{E}_{alg}(\mathbb{Z}^{\mathbb{N}})$ look like?*
*Are they countable?*

Thank you for your attention!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