# Probabilistic Logic Programming
# Review of sintax and semantics. Inference

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# Outline

# Logic Programs with Annotated Disjunctions (LPADs)

$$sneezing(X) : 0.7 \vee null : 0.3 \leftarrow flu(X).$$
$$sneezing(X) : 0.8 \vee null : 0.2 \leftarrow hay\_fever(X).$$
$$flu(bob).$$
$$hay\_fever(bob).$$

- **Distributions over the head of rules**
- *null* does not appear in the body of any rule, and is usually omitted

# ProbLog

$$sneezing(X) \leftarrow flu(X), flu\_sneezing(X).$$
$$sneezing(X) \leftarrow hay\_fever(X), hay\_fever\_sneezing(X).$$
$$flu(bob).$$
$$hay\_fever(bob).$$
$$0.7 :: flu\_sneezing(X).$$
$$0.8 :: hay\_fever\_sneezing(X).$$

- **Distributions over (probabilistic) facts**
- Worlds obtained by selecting or not every grounding of each probabilistic fact

This program has $2 \cdot 2 = 4$ possible worlds.

# Distribution Semantics

- We consider the case of programs without functions symbols
- An LPAD defines a **probability distribution over normal logic programs called worlds**
- A world is obtained from an LPAD by:
  1. grounding the program
  2. selecting a single head atom for each ground clause
  3. including in the world the clause with the selected head atom and the body
- The probability of a world is the product of the probabilities associated to the heads selected
- The probability of a ground atom (the query) is given by the sum of the probabilities of the worlds where the query is true
- If the LPAD contains function symbols, the definition is more complex

# Worlds for the LPAD Program

$sneezing(bob) \leftarrow flu(bob).$
$sneezing(bob) \leftarrow hay\_fever(bob).$
$flu(bob).$
$hay\_fever(bob).$
$\mathbf{P(w_1) = 0.7 \times 0.8}$

$null \leftarrow flu(bob).$
$sneezing(bob) \leftarrow hay\_fever(bob).$
$flu(bob).$
$hay\_fever(bob).$
$\mathbf{P(w_2) = 0.3 \times 0.8}$

$sneezing(bob) \leftarrow flu(bob).$
$null \leftarrow hay\_fever(bob).$
$flu(bob).$
$hay\_fever(bob).$
$\mathbf{P(w_3) = 0.7 \times 0.2}$

$null \leftarrow flu(bob).$
$null \leftarrow hay\_fever(bob).$
$flu(bob).$
$hay\_fever(bob).$
$\mathbf{P(w_4) = 0.3 \times 0.2}$

- $sneezing(bob)$ is true in 3 worlds out of 4
- $P(sneezing(bob)) = 0.7 \times 0.8 + 0.3 \times 0.8 + 0.7 \times 0.2 = 0.94$

# Worlds for the ProbLog Program

$sneezing(X) \leftarrow flu(X), flu\_sneezing(X).$
$sneezing(X) \leftarrow hay\_fever(X), hay\_fever\_sneezing(X).$
$flu(bob).$
$hay\_fever(bob).$

$flu\_sneezing(bob).$
$hay\_fever\_sneezing(bob).$
$P(w_1) = 0.7 \times 0.8$

$hay\_fever\_sneezing(bob).$
$P(w_2) = 0.3 \times 0.8$

$flu\_sneezing(bob).$
$P(w_3) = 0.7 \times 0.2$

$P(w_4) = 0.3 \times 0.2$

- $sneezing(bob)$ is true in 3 worlds out of 4
- $P(sneezing(bob)) = 0.7 \times 0.8 + 0.3 \times 0.8 + 0.7 \times 0.2 = 0.94$

# Reasoning Tasks

- **Inference**: we want to compute the probability of a query given the model and, possibly, some evidence
- **Weight learning**: we know the structural part of the model (the logic formulas) but not the numeric part (the weights) and we want to infer the weights from data
- **Structure learning**: we want to infer both the structure and the weights of the model from data

# Inference under the Distribution Semantics

- Let **At** be the set of all ground (probabilistic and derived) atoms in a given LPAD program
- Assume that we are given a set $\mathbf{E} \subset \mathbf{At}$ of observed atoms (*evidence* atoms)
  - a vector $\mathbf{e}$ with their observed truth values means that the evidence is $\mathbf{E} = \mathbf{e}$
- We are also given a set $\mathbf{Q} \subset \mathbf{At}$ of atoms of interest (*query* atoms)
- $q$ indicates a single ground atom of interest
- Different inference tasks are possible

# Inference under the Distribution Semantics

- Unconditional inference: computing the unconditional probability $P(\mathbf{e})$, the probability of evidence
- Conditional or marginal inference: computing the conditional probability distribution of every query atom given the evidence, i.e., computing $P(q|\mathbf{E} = \mathbf{e})$, for each $q \in \mathbf{Q}$
- Computing the probability distribution or density of the non-ground arguments of a conjunction of literals $\mathbf{Q} = \mathbf{q}$

# Inference under the Distribution Semantics

- MPE task, or *most probable explanation*: finds the most likely joint truth value of all query (unobserved) atoms given the evidence, i.e., solves the optimization problem $argmax_{\mathbf{q}} P(\mathbf{Q} = \mathbf{q} | \mathbf{E} = \mathbf{e})$
- MAP task, or *maximum a posteriori*: finds the most likely joint truth value of a set of query (unobserved) atoms given the evidence, i.e., solves the optimization problem $argmax_{\mathbf{q}} P(\mathbf{Q} = \mathbf{q} | \mathbf{E} = \mathbf{e})$

# Inference under the Distribution Semantics

## Example

```
heads(Coin):1/2; tails(Coin):1/2:- toss(Coin),\+biased(Coin).
heads(Coin):0.6; tails(Coin):0.4:- toss(Coin),biased(Coin).
fair(Coin):0.9; biased(Coin):0.1.
toss(coin).
```

- Unconditional inference: $P(heads(coin))$, $P(tails(coin))$
- Conditional inference: $P(heads(coin)|biased(coin))$

# Inference under the Distribution Semantics

## Example

- **MPE**: Most likely world where *heads*(*coin*) is true?
- $2 \times 2 \times 2 = 8$ possible worlds
- H=heads(coin), T=tails(coin), F=fair(coin), B=biased(coin)
- $HHF : 0.5 * 0.6 * 0.9 = \mathbf{0.27}$
  $HHB : 0.5 * 0.6 * 0.1 = 0.03$
  $HTF : 0.5 * 0.4 * 0.9 = 0.18$
  $HTB : 0.5 * 0.4 * 0.1 = 0.02$
  $THF : 0.5 * 0.6 * 0.9 = \mathbf{0.27}$
  $THB : 0.5 * 0.6 * 0.1 = 0.03$
  $TTF : 0.5 * 0.4 * 0.9 = 0.18$
  $TTB : 0.5 * 0.4 * 0.1 = 0.02$
- *heads*(*coin*), *heads*(*coin*), *fair*(*coin*)
- *tails*(*coin*), *heads*(*coin*), *fair*(*coin*)

# Inference under the Distribution Semantics

The inference problem is #**P-hard** and for large models is intractable. Possible **solutions**:

1. Exact inference
   - **Knowledge Compilation**: compile the probabilistic logic program to an *intermediate representation* and then compute the probability by Weighted Model Counting
   - **Bayesian Network (BN) based**: convert the probabilistic logic program into a BN and apply BN inference algorithms (Meert et al. (2010))
   - **Lifted** inference (Poole (2003)): exploits symmetries in the model to speed up inference
2. Approximate inference

# Knowledge Compilation

A probabilistic logic program can be compiled into the following intermediate representations:

- **Binary Decision Diagrams** (BDD): De Raedt et al. (2007), `cplint` (Riguzzi (2007),Riguzzi (2009)), `PITA` (Riguzzi and Swift (2010a))
- deterministic-Decomposable Negation Normal Form circuits **(d-DNNF)**: `ProbLog2`, Fierens et al. (2015)
- **Sentential Decision Diagrams** (SDD), Darwiche (2011)

# Knowledge Compilation to BDD

1. **Assign Boolean random variables** (r.v.) to the probabilistic rules
2. Given a query $Q$, compute its **explanations**, i.e. assignments to the random variables that are sufficient for entailing the query
3. Let $K$ be the set of all possible explanations
4. Build a Boolean formula $f_K$ representing $K$
5. **Build a BDD** encoding $f_K$
6. **Compute the probability of evidence** from the BDD (unconditional inference)

# Knowledge Compilation to BDD

## (2) Computing explanations for $Q$

- **Atomic choice** $(C, \theta, k)$: selection of the $k$-th atom for a grounding $C\theta$ of a switch/clause $C$ and a substitution $\theta$: $(C_1, \{X/bob\}, 1)$ selects $sneezing(bob)$ in the LPAD
- Composite choice $\kappa$: consistent set of atomic choices $(C_i, \theta_j, k)$
- A composite choice $\kappa$ identifies a set of worlds $\omega_\kappa = \{w_\sigma | \kappa \subseteq \sigma\}$ whose selections $\sigma$ are a superset of $\kappa$
- A composite choice $\kappa$ is an explanation for a query $Q$ if $Q$ is true in every world of $\omega_\kappa$
- In practice, a *covering* set of explanations must be found
- A set of composite choices $K$ is covering with respect to $Q$ if every world $w$ in which $Q$ is true is such that $w \in \omega_K$ where $\omega_K = \bigcup_{\kappa \in K} \omega_\kappa$

# Knowledge Compilation to BDD

## (2) Computing explanations for $Q$

- Two composite choices $\kappa_1$ and $\kappa_1$ are exclusive if their union is inconsistent
- If $\kappa_1 = \{(C_1, X/bob, 1)\}, \kappa_2 = \{(C_1, X/bob, 2), (C_2, X/bob, 1)\}$ then $\kappa_1 \cup \kappa_2$ is inconsistent
- A set $K$ of composite choices is mutually exclusive if for all $\kappa_1 \in K, \kappa_2 \in K, \kappa_1 \neq \kappa_2 \Rightarrow \kappa_1$ and $\kappa_2$ are exclusive
- if $K$ is mutually exclusive, $P(K) = \sum_{\kappa \in K} P(\kappa)$

# Knowledge Compilation to BDD

## (3,4) Building a formula for the explanations

- $K$ = set of covering explanations found for $Q$
- Boolean formula representing $K$, over a set $\mathbf{X}$ of multi-valued r. v.

$$f_K(\mathbf{X}) = \bigvee_{\kappa \in K} \bigwedge_{(C_i,\theta_j,k)\in\kappa} (X_{ij} = k)$$

- $X_{ij}$: random variable whose domain is $1, ..., n_i$ (no of head atoms)
- $P(X_{ij} = k) = p(C_i, k)$
- $f_K(\mathbf{X}) = 1$ if the values of the variables correspond to an explanation for $Q$
- Equations for a single explanation are conjoined and the conjunctions for the different explanations are disjoined

# Knowledge Compilation to BDD

## (2,3,4) Example

- For the (LPAD) program:

$$sneezing(X) : 0.7 \leftarrow flu(X).$$
$$sneezing(X) : 0.8 \leftarrow hay\_fever(X).$$
$$flu(bob).$$
$$hay\_fever(bob).$$

  a set of covering explanations for $Q = sneezing(bob)$ is $K = \{\kappa_1, \kappa_2\}$

- $\kappa_1 = \{(C_1, \{X/bob\}, 1)\} \quad \kappa_2 = \{(C_2, \{X/bob\}, 1)\}$

# Knowledge Compilation to BDD

## (2,3,4) Example

- $\theta_1 = X/bob$
- $X_{C_1\theta_1} \rightarrow X_{11}$ From $\kappa_1, X_{11} = 1$
- $X_{C_2\theta_1} \rightarrow X_{21}$ From $\kappa_2, X_{21} = 1$
- $f_K(\mathbf{X}) = (X_{11} = 1) \vee (X_{21} = 1)$
- $P(Q) = P(f_K(\mathbf{X})) = P(X_{11} \vee X_{21}) = P(X_{11}) + P(X_{21}) - P(X_{11})P(X_{21})$

# Knowledge Compilation to BDD

## And Now What?

- Worlds are mutually exclusive, and in theory we could compute $P(Q)$ as $\sum_{w \in W_T : w \models Q} P(w)$,
  - BUT in practice, it is unfeasible to find all the worlds $w$ where the query is true
- It's easier to find explanations for the query
  - BUT explanations might not be mutually exclusive
  - **they must be made mutually exclusive in order to sum up probabilities**
  - Binary Decision Diagrams (BDD): they split paths on the basis of the values of binary variables, so **the branches are mutually disjoint**

# Knowledge Compilation to BDD

## (3,4) Building a formula for the explanations

- A BDD performs a *Shannon expansion* of $f_K(\mathbf{X})$
- $f_K(\mathbf{X}) = X_{11} \times f_K^{X_{11}}(\mathbf{X}) + \overline{X_{11}} \times f_K^{\overline{X_{11}}}(\mathbf{X})$
- The two disjuncts above are mutually exclusive
- $f_K^X(\mathbf{X})(f_K^{\overline{X}}(\mathbf{X}))$ is the formula obtained by $f_K(\mathbf{X})$ by setting $X$ to 1 (0)
- $P(f_K(\mathbf{X})) = P(X_{11})P(f_K^{X_{11}}(\mathbf{X})) + (1 - P(X_{11}))P(f_K^{\overline{X_{11}}}(\mathbf{X}))$
- $P(f_K(\mathbf{X})) = 0.7 \cdot P(f_K^{X_{11}}(\mathbf{X})) + 0.3 \cdot P(f_K^{\overline{X_{11}}}(\mathbf{X}))$

# Knowledge Compilation to BDD

## (3,4) Building a formula for the explanations

- In order to use BDD, a multi-valued random variable $X_{ij}$ with $n_i$ values must be converted into $n_i - 1$ Boolean variables $X_{ij1}, ..., X_{ijn_i-1}$
- $X_{ij} = k$ for $k = 1, ..., n_i - 1$ is represented by the conjunction $\overline{X_{ij1}} \wedge \ldots \wedge \overline{X_{ijk-1}} \wedge X_{ijk}$
- $X_{ij} = n_i$ by $\overline{X_{ij1}} \wedge \ldots \wedge \overline{X_{ijn_i-1}}$

# Knowledge Compilation to BDD

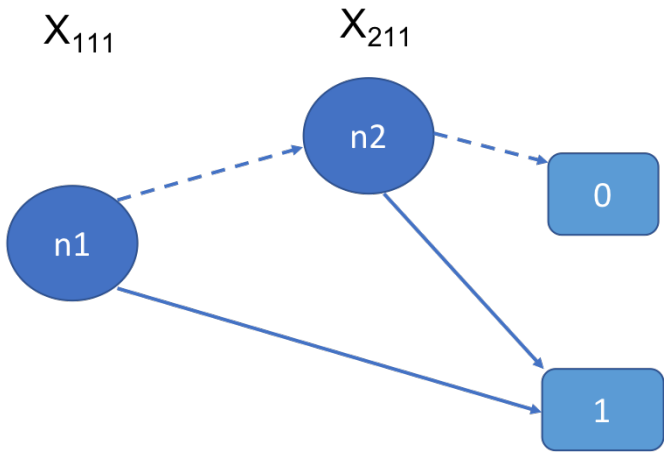
## (3,4) Example

- For both $C_1$ and $C_2$ : $n_i = 2$
- (multi-valued) $X_{11} = 1 \rightarrow$ (Boolean) $X_{111}$
- (multi-valued) $X_{11} = 2 \rightarrow$ (Boolean) $\overline{X_{111}}$
- (multi-valued) $X_{21} = 1 \rightarrow$ (Boolean) $X_{211}$
- (multi-valued) $X_{21} = 2 \rightarrow$ (Boolean) $\overline{X_{211}}$
- $f'_K(\mathbf{X}) = X_{111} \vee X_{211}$

# Knowledge Compilation to BDD

## (5) Building a BDD

- A BDD for a function of Boolean variables is a rooted graph that has one level for each Boolean variable
- A node $n$ has two children: one corresponding to the 1 value of the variable associated with $n$ (solid arc) and one corresponding the 0 value of the variable (dashed arc)
- The leaves store either 0 or 1: a path to a 1-leaf corresponds to an explanation for $Q$
- BDD for $f'_K(\mathbf{X}) = X_{111} \vee X_{211}$:



# Knowledge Compilation to BDD

## (5) Building a BDD

- BDDs can be built by combining simpler BDDs using Boolean operators
- While building BDDs, simplification operations can be applied that delete or merge nodes
- Merging is performed when the diagram contains two identical sub-diagrams
- Deletion is performed when both arcs from a node point to the same node
- A reduced BDD often has a much smaller number of nodes with respect to the original BDD

# Knowledge Compilation to BDD

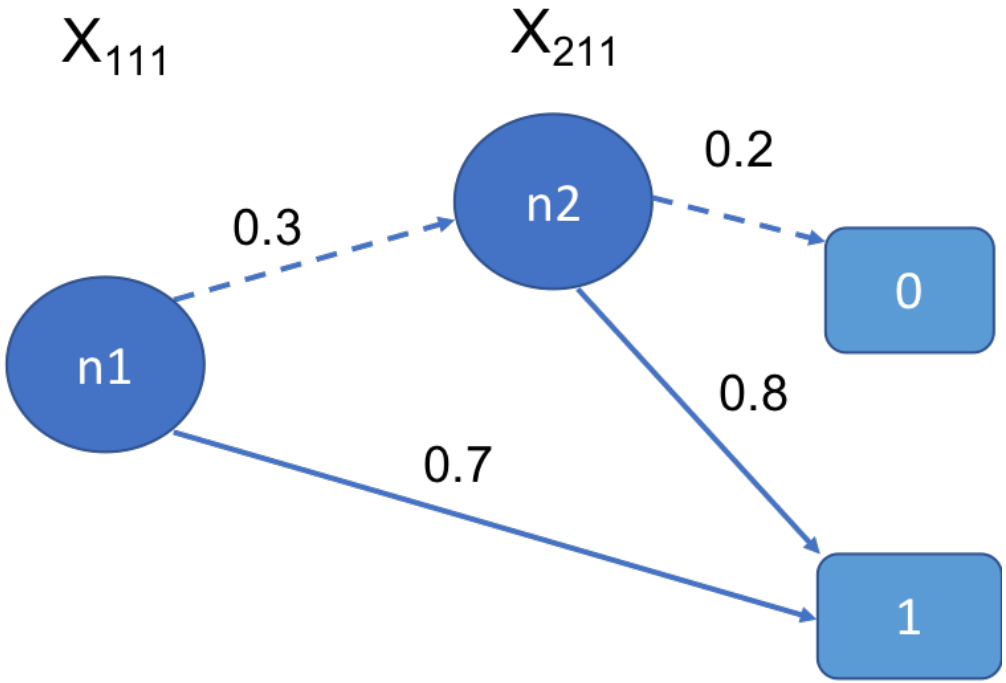
## (6) Computing the probability from a BDD

**Algorithm 4** Function PROB: Computation of the probability of a BDD.

```
1:  function PROB(node)
2:      if node is a terminal then
3:          return 1
4:      else
5:          if Table(node) ≠ null then
6:              return Table(node)
7:          else
8:              p0 ←PROB(child_0(node))
9:              p1 ←PROB(child_1(node))
10:             let π be the probability of being true of var(node)
11:             Res ← p1 · π + p0 · (1 − π)
12:             add node → Res to Table
13:             return Res
14:         end if
15:     end if
16: end function
```

# Knowledge Compilation to BDD

## (6) Example




$$P(n_2) = 0 \cdot 0.2 + 1 \cdot 0.8 = 0.8$$
$$P(n_1) = P(root) = 1 \cdot 0.7 + 0.3 \cdot 0.8 = 0.94 = P(Q) = P(sneezing(bob))$$

# MDDs vs BDDs

- A Multivalued Decision Diagrams (MDD) represents a function taking Boolean values on a set of **multivalued variables X**, each node is associated to the variable of its level and has one child for each possible value of the variable
- A BDD represents a function taking Boolean values on a set of **binary variables X**, each node is associated to the variable of its level and has 2 children
- Most packages for the manipulation of decision diagrams are restricted to work on Binary Decision Diagrams (see CUDD @`http://vlsicad.eecs.umich.edu/BK/Slots/cache/vlsi.colorado.edu/~fabio/`), as they offer several operators for handling BDDs

# Tabling-based Inference

- Idea: **maintain in a *table* both subgoals encountered in a query evaluation and answers to these subgoals**
- If a subgoal is encountered more than once, **the evaluation reuses information from the table** rather than re-performing resolution against program clauses
- Tabling can be used to evaluate programs with negation
- Tabling integrates closely with Prolog: a predicate p/n is evaluated using SLDNF by default; the predicate can be made to use tabling by the directive `:- table p/n` that is added by the user or compiler

# Tabling-based Inference

## PITA: *Probabilistic Inference with Tabling and Answer subsumption*

- Introduced by Riguzzi and Swift (2010b)
- PITA computes the probability of queries from LPADs with tabling
- PITA builds all explanations for every subgoal encountered during a derivation of the query
- Explanations for subgoals are stored with tabling
- Explanations are compactly represented using BDDs that also allow an efficient computation of the probability
- Subgoals (ground atoms) have an extra argument storing a BDD that represents the explanations for their answers
- When an answer $q(\mathbf{x}, bdd)$ is found, $bdd$ represents the explanations for $q(\mathbf{x})$

# Tabling-based Inference

## Answer Subsumption

- A feature of tabling in XSB and SWI Prolog
- **Combine different answers for the same goal**
- E.g `:-table path(X,Y,lattice(or/3))` means that, if two explanations `path(a,b,bdd0)` and `path(a,b,bdd1)` are found, the single answer `path(a,b,bdd)` will be stored in the table where `or(bdd0,bdd1,bdd)`

# Tabling-based Inference

## PITA: Program Transformation

- The first step of the PITA algorithm is to apply a program transformation to an LPAD to create a normal logic program that contains calls for manipulating BDDs
- Prolog interface to the CUDD C library with the predicates:
  - *init*, *end*: allocation and deallocation of a BDD manager
  - $zero(-BDD)$, $one(-BDD)$, $and(+BDD1, +BDD2, -BDDO)$, $or(+BDD1, +BDD2, -BDDO)$, $not(+BDDI, -BDDO)$: BDD operations
  - $add\_var(+N\_Val, +Probs, -Var)$: adds a new random variable *Var* associated to a new instantiation of a rule with $N\_Val$ head atoms and parameters list *Probs*
  - $equality(+Var, +Value, -BDD)$: BDD represents Var=Value
  - $ret\_prob(+BDD, -P)$: returns the probability of the formula encoded by BDD

# Tabling-based Inference

## PITA: Program Transformation

- $get\_var\_n(R, C, Probs, Var)$ wraps $add\_var/3$ and avoids adding a new variable when one already exists for an instantiation
- $R$ is an identifier for the LPAD rule, $C$ is a list of constants, one for each variables of the clause, and $Var$ is a integer that identifies the random variable associated with clause $R$ under a particular grounding; $Probs$ is a list of floats that stores the parameters in the head of rule $R$

$$
\begin{aligned}
get\_var\_n(R, C, Probs, Var) :- \quad & (var(R, C, Var) \rightarrow true; \\
& length(Probs, L), \\
& add\_var(L, Probs, Var), \\
& assert(var(R, C, Var))).
\end{aligned}
$$

# Tabling-based Inference

## PITA: Program Transformation

The disjunctive clause $C_r = H_1 : \alpha_1 \vee \ldots \vee H_n : \alpha_n \leftarrow L_1, \ldots, L_m.$
is transformed into the set of clauses $PITA(C_r)$

$$PITA(C_r, 1) = PITA(H_1) \leftarrow \begin{array}{l} one(BB_0), \\ PITA(L_1), and(BB_0, B_1, BB_1), \\ \ldots, \\ PITA(L_m), and(BB_{m-1}, B_m, BB_m), \\ get\_var\_n(r, C, [\alpha_1, \ldots, \alpha_n], Var), \\ equality(Var, 1, BB), and(BB_m, BB, BDD). \end{array}$$

$$\ldots$$

$$PITA(C_r, n) = PITA(H_n) \leftarrow \begin{array}{l} one(BB_0), \\ PITA(L_1), and(BB_0, B_1, BB_1), \\ \ldots, \\ PITA(L_m), and(BB_{m-1}, B_m, BB_m), \\ get\_var\_n(r, C, [\alpha_1, \ldots, \alpha_n], Var), \\ equality(Var, n, BB), and(BB_m, BB, BDD). \end{array}$$

# Tabling-based Inference

## PITA Example

```
:- table path(X,Y,lattice(or/3)),edge(X,Y,lattice(or/3)).
```

LPAD

```
path(X,X).
path(X,Y):- path(X,Z),edge(Z,Y).
edge(a,b):0.3.
....
```

PITA Transformation

```
path(X,X,One):- one(One).
path(X,Y,BDD):- one(One),path(X,Z,BDD0),and(One,BDD0,BDD1),
                edge(Z,Y,BDD2),and(BDD1,BDD2,BDD).
edge(a,b,BDD):- one(One),get_var_n(3,[],[0.3,0.7],Var),
                equality(Var,1,BDD0),and(BDD0,One,BDD).
...
```

# Tabling-based Inference

## PITA Example

Query: `path(a,b)`

```
:- init,
   path(?HGNC_620?,?HGNC_983?,BDD),
   ret_prob(BDD,P),
   end.
```

# Tabling-based Inference

## PITA Results

- PITA was compared with CVE (Meert et al. (2010)) and ProbLog1 (Kimmig et al. (2008a)).
  - CVE transforms an LPAD into an equivalent Bayesian network and then performs inference on the network using the variable elimination algorithm
  - ProbLog employs BDDs for efficient inference
- The algorithm was able to successfully solve more complex queries than the other algorithms in most cases and it was also almost always faster

# Inference Systems based on BDDs: `cplint`

- Suite of programs for reasoning with LPADs
- Inference and learning
- Versions for Yap Prolog and SWI-Prolog
- Distributed as a pack of SWI-Prolog. To install it, use
  `?- pack_install(cplint).`
- Available in the web application *cplint on SWISH*: http://cplint.eu/
- Exact Inference: module `PITA`

# Inference Systems based on BDDs: cplint

- *Input*: Prolog file where you must
  - load the inference module (pita)
  - initialize it with a directive (`:- pita.`)
  - enclose the LPAD clauses in `:-begin_lpad.` and `:-end_lpad.`
- *Example*: coin.pl

```
:- use_module(library(pita)).
:- pita.
:- begin_lpad.
 heads(Coin):1/2; tails(Coin):1/2:- toss(Coin),\+biased(Coin).
 heads(Coin):0.6; tails(Coin):0.4:- toss(Coin),biased(Coin).
 fair(Coin):0.9; biased(Coin):0.1.
 toss(coin).
:- end_lpad.
```

# Inference Systems based on BDDs: cplint

- You can have also (non-probabilistic) clauses outside `:-begin/end_lpad.`, considered as database clauses.
- Subgoals in the body of probabilistic clauses can query them by enclosing the query in `db/1`.
- *Example*

```
:- use_module(library(pita)).
:- pita.
:- begin_lpad.
    sampled_male(X):0.5:- db(male(X)).
:- end_lpad.
male(john).
male(david).
```

# Hands-on: Coin example

- http://cplint.eu/example/inference/coin.pl
- Unconditional inference:
  - What is the probability that *coin* lands heads? `prob(heads(coin),Prob).`
  - What is the probability that *coin* lands tails? `prob(tails(coin),Prob).`
- Conditional inference:
  - What is the probability that *coin* lands heads, given that I know it is biased? `prob(heads(coin),biased(coin),Prob).`

# Hands-on: Sneezing example

- http://cplint.eu/p/sneezing_simple.pl
- Unconditional inference:
  - What's the probability of Bob sneezing?
    `prob(sneezing(bob),Prob).`
- We should expect `Prob` = ?

# Knowledge Compilation to d-DNNF

- A tractable logical form known as Deterministic, Decomposable Negation Normal Form, which **permits some generally intractable logical queries to be computed in time polynomial** in the form size (Darwiche (2004))
- Superset of OBDDs (Ordered BDD): BDDs with a defined variable ordering
- Allows to perform unconditional, conditional and MPE inference in **ProbLog2**

# Knowledge Compilation to d-DNNF

- A negation normal form (NNF) is a rooted directed acyclic graph in which each leaf node is labeled with a literal, *true* or *false*, and each internal node is labeled with a conjunction or disjunction

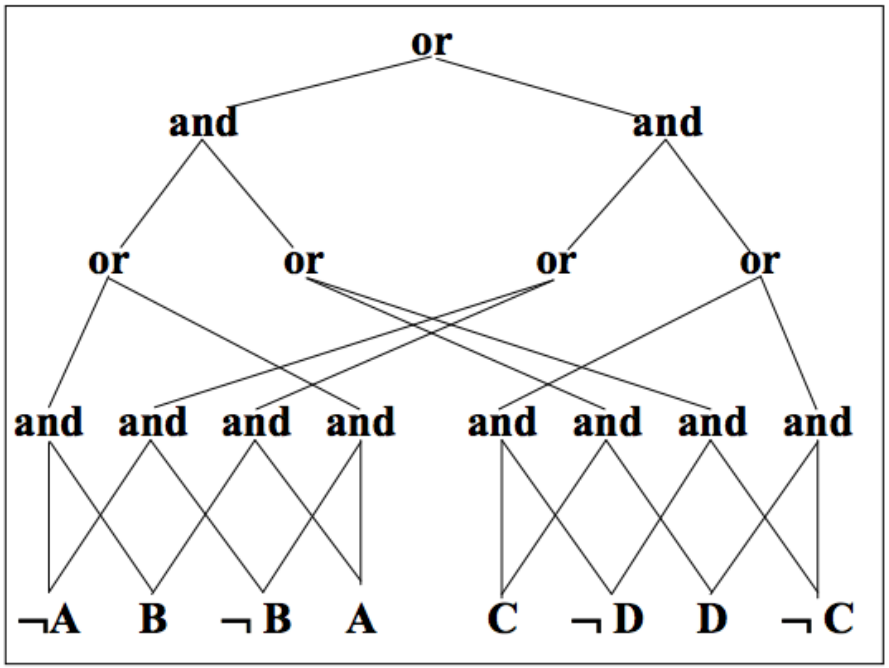



**Figure 1.** A negation normal form.

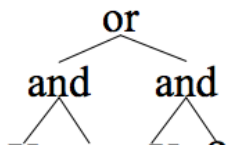
# Knowledge Compilation to d-DNNF

- For any node $n$, $Vars(n)$ denotes all propositional variables that appear in the subgraph rooted at $n$, and $\Delta(n)$ denotes the formula represented by $n$ and its descendants.

## Properties

- **Decomposability**: $Vars(n_i) \cap Vars(n_j) = \emptyset$ for any two children $n_i$ and $n_j$ of an and-node $n$. The NNF in the figure is decomposable.
- **Determinism**: $\Delta(n_i) \wedge \Delta(n_j)$ is logically inconsistent for any two children $n_i$ and $n_j$ of an or-node $n$. The NNF in the figure is deterministic.

# Knowledge Compilation to d-DNNF

- **Decision**: holds when the root node of the NNF graph is a *decision node*. A decision node is a node labeled with true, false, or is an or-node having the form 

where $X$ is a variable, $\alpha$ and $\beta$ are decision nodes. The NNF in the figure does not satisfy the decision property since its root is not a decision node.
- **Ordering**: only for NNFs that satisfy the decision property. Ordering holds when decision variables appear in the same order along any path from the root to any leaf.

# Knowledge Compilation to d-DNNF

## Procedure

1. Ground the probabilistic logic program and convert it into an equivalent Boolean formula $\phi_r$
2. Build $\phi_e$, the Boolean formula for the evidence
3. Rewrite $\phi = \phi_r \wedge \phi_e$ in CNF
4. Construct a *weighted* Boolean formula for $\phi$
5. CNF formula $\rightarrow$ d-DNNF formula (#P hard step)
6. d-DNNF formula $\rightarrow$ aritmetic circuit (AC)
7. Compute the probability of evidence from the AC

# Knowledge Compilation to d-DNNF

## (1) Ground the PL program and convert it into a Boolean formula $\phi_r$

- **ProbLog program**

$$
\begin{aligned}
&0.1 :: burglary.\\
&0.2 :: earthquake.\\
&0.7 :: hears\_alarm(X) \leftarrow person(X).\\
&alarm \leftarrow burglary.\\
&alarm \leftarrow earthquake.\\
&calls(X) \leftarrow alarm, hears\_alarm(X).\\
&person(mary).\\
&person(john).
\end{aligned}
$$

# Knowledge Compilation to d-DNNF

## (1) Ground the PL program and convert it into a Boolean formula $\phi_r$

- Suppose $e = calls(john)$
- **Relevant Ground Program (RGP)** (taking into account $e$ (and eventually $q$), in order to consider only the part of the program that is relevant to the (query given the) evidence):

$$0.1 :: burglary.$$
$$0.2 :: earthquake.$$
$$0.7 :: hears\_alarm(john).$$
$$alarm \leftarrow burglary.$$
$$alarm \leftarrow earthquake.$$
$$calls(john) \leftarrow alarm, hears\_alarm(john).$$

# Knowledge Compilation to d-DNNF

## (1-2) Ground the PL program and convert it into a Boolean formula $\phi_r$. Build $\phi_e$

- Apply specific transformation rules (Lloyd (1987); Janhunen (2004); Mantadelis and Janssens (2010)) to generate a Boolean formula $\phi_r$
- $\phi_e = calls(john)$
- In the ProbLog example, $\phi$ is the conjunction of the following three sub-formulas (the first two for $\phi_r$, the last one for $\phi_e$)

$$alarm \leftrightarrow burglary \vee earthquake$$
$$calls(john) \leftrightarrow alarm \wedge hears\_alarm(john)$$
$$calls(john)$$

# Knowledge Compilation to d-DNNF

## (3) Rewrite $\phi$ in CNF

- CNF:

$$l_{11} \vee \ldots \vee l_{1m_1} \wedge \ldots \wedge l_{n1} \vee \ldots \vee l_{nm_n}$$

  where each $l_{ij}$ is a literal
- $alarm \leftrightarrow burglary \vee earthquake$ becomes $(alarm \vee \neg burglary) \wedge (alarm \vee \neg earthquake) \wedge (\neg alarm \vee burglary \vee earthquake)$
- $calls(john) \leftrightarrow alarm \wedge hears\_alarm(john)$ becomes $(\neg calls(john) \vee alarm) \wedge (\neg calls(john) \vee hears\_alarm(john)) \wedge (\neg alarm \vee \neg hears\_alarm(john) \vee calls(john))$
- $\phi$ in CNF: $(alarm \vee \neg burglary) \wedge (alarm \vee \neg earthquake) \wedge (\neg alarm \vee burglary \vee earthquake) \wedge (\neg calls(john) \vee alarm) \wedge (\neg calls(john) \vee hears\_alarm(john)) \wedge (\neg alarm \vee \neg hears\_alarm(john) \vee calls(john)) \wedge calls(john)$

# Knowledge Compilation to d-DNNF

## (4) Construct a weighted Boolean formula

- Define the weight function for all literals in $\phi$
- The weight of a probabilistic literal is derived from the probabilistic facts in the program
- If the RGP contains a probabilistic fact $p :: f$, then we assign weight $p$ to $f$ and weight $1 - p$ to $\neg f$
- For literals not occurring in a probabilistic fact (derived literals), the weight is 1

# Knowledge Compilation to d-DNNF

## (4) Construct a weighted Boolean formula

$burglary \mapsto 0.1$ $\quad$ $\neg burglary \mapsto 0.9$

$earthquake \mapsto 0.2$ $\quad$ $\neg earthquake \mapsto 0.8$

$hears\_alarm(john) \mapsto 0.7$ $\quad$ $\neg hears\_alarm(john) \mapsto 0.3$

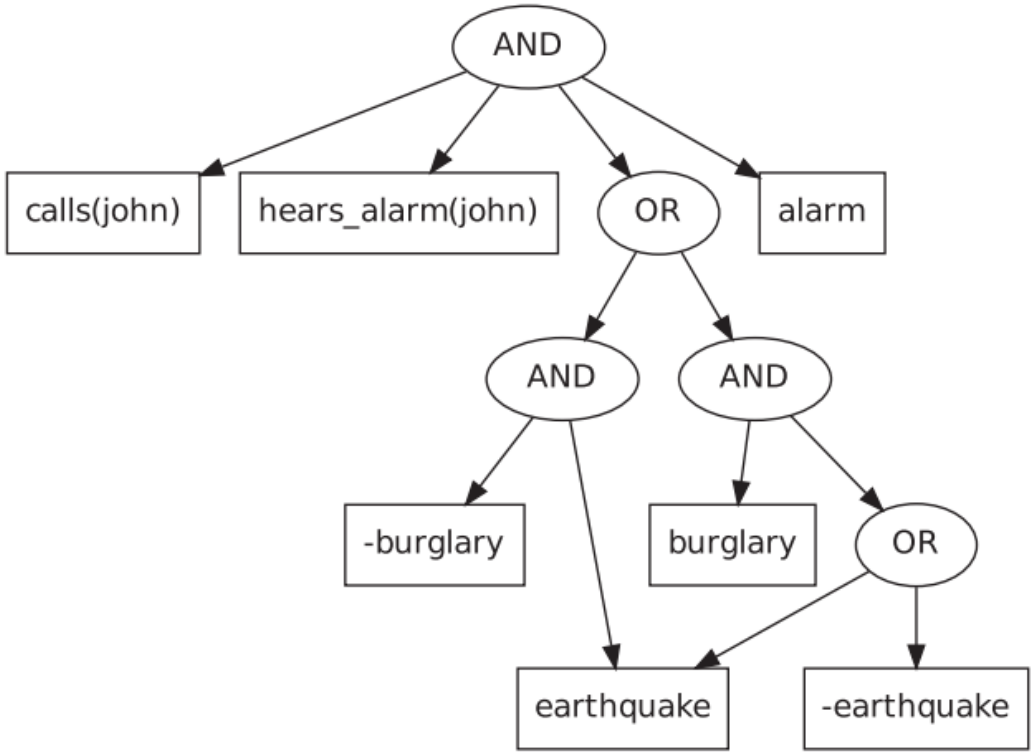
$alarm \mapsto 1$ $\quad$ $\neg alarm \mapsto 1$

$calls(john) \mapsto 1$ $\quad$ $\neg calls(john) \mapsto 1$

# Knowledge Compilation to d-DNNF

## (5) Convert the CNF into a d-DNNF

- Conversion is made by compilers: c2d (Darwiche (2004)), DSHARP (Muise et al. (2012)), irrespective of the weighting function



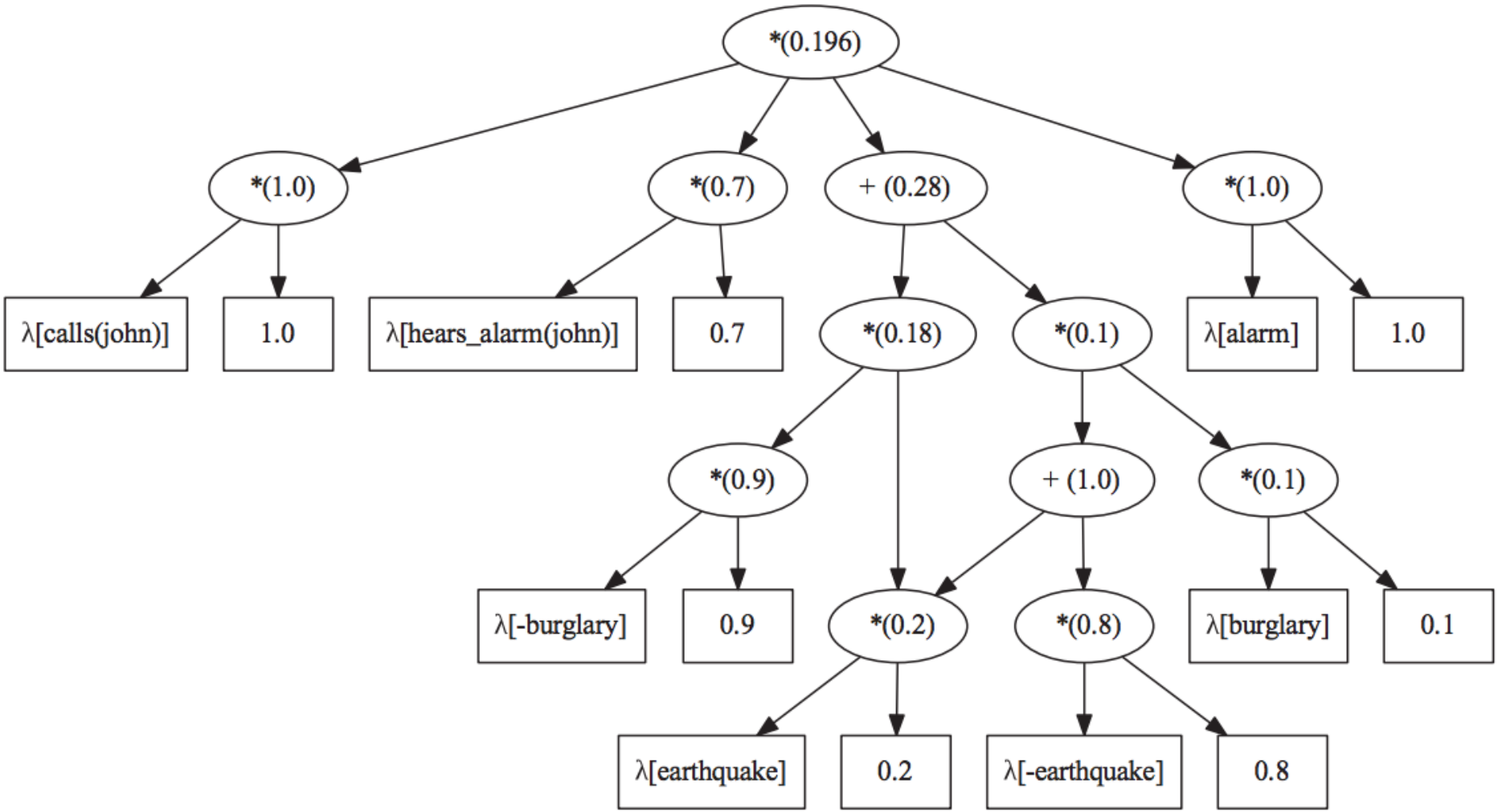
# Knowledge Compilation to d-DNNF

## (6) Convert the d-DNNF formula into an aritmetic circuit

1. Replace all conjunctions in the internal nodes of the d-DNNF by multiplications, and all disjunctions by summations
2. Replace every leaf node involving a literal $l$ by a subtree consisting of a multiplication node having two children, namely, a leaf node with an indicator variable for the literal $l$ and a leaf node with the weight of $l$ according to the weighted formula
3. Numbers in parentheses represent results of the intermediate computations

# Knowledge Compilation to d-DNNF

## (6) Convert the d-DNNF formula into an aritmetic circuit



# Knowledge Compilation to d-DNNF

## (7) Compute the probability from the AC

- Evaluate the circuit bottom-up after having assigned the value 1 to all the indicator variables
- $P(root) = P(\mathbf{e}) = P(calls(john)) = 0.196$
- $P(\mathbf{e})$ is called weighted model count (WMC)
- **Indicator variables** allow us to add **further evidence**, provided that it extends the initial evidence for which the circuit has been built
- To compute $P(\mathbf{e}, l_1 \dots l_n)$ for any conjunction of literals $l_1, \dots, l_n$ it is enough to set the indicator variables as $\lambda(l_i) = 1$, $\lambda(\neg l_i) = 0$ and $\lambda(l) = 1$ for the other literals $l$, and evaluate the circuit
  - Ex.: $P(calls(john) = true \wedge earthquake = true)$

# Knowledge Compilation to d-DNNF

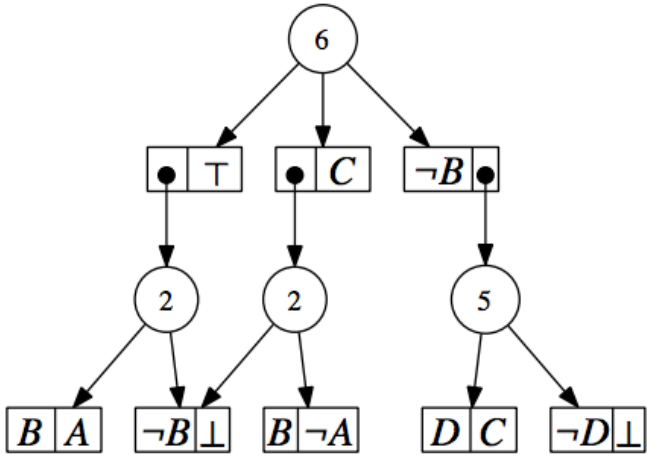
## BDD vs d-DNNF

- A BDD is a special kind of d-DNNF, namely, one that satisfies the additional properties of ordering and decision
- In the approach seen earlier, we can replace a d-DNNF compiler with a BDD compiler
- Computing the probability of evidence can then be done by either operating directly on the BDD, or by converting the BDD to an arithmetic circuit and evaluating the circuit
- d-DNNFs outperform BDDs (Darwiche (2004))

# Knowledge Compilation by SDD

- An SDD (Vlasselaer et al. (2014); Darwiche (2011)) contains two types of nodes
- *Decision nodes* (circles): **disjunctions** over mutually exclusive sentences
- *Elements* (paired boxes [p|s]): **conjunctions** between the two children
  - p: "prime"; s: "sub"
  - Elements are decision nodes' children and each box in an element can contain a pointer to a decision node or a terminal node, either a literal or the constants 0 or 1
- A decision node with children $[p_1|s_1], \ldots, [p_n|s_n]$ represents the function $(p_1 \wedge s_1) \vee \ldots \vee (p_n \wedge s_n)$
- Primes must form a partition: $p_i \neq 0$ (primes are consistent), $p_i \wedge p_j = 0$ for $i \neq j$ (every pair of distinct primes are mutually exclusive), and $p_1 \vee \ldots \vee p_n = 1$ (the disjunction of all primes is valid)

# Knowledge Compilation by SDD




(b) Graphical depiction of an SDD

$$f = (A \wedge B) \vee (B \wedge C) \vee (C \wedge D).$$

The top level decomposition has three elements, with primes representing $A \wedge B, \neg A \wedge B, \neg B$ and corresponding subs representing *true*, $C$, and $C \wedge D$.

# Knowledge Compilation by SDD

## Procedure

1. Ground the probabilistic logic program and convert it into an equivalent propositional formula
2. **Compile directly the formula into an SDD** OR convert it into a CNF Boolean formula to be compiled into an SDD
3. Compute the probability of evidence from the SDD

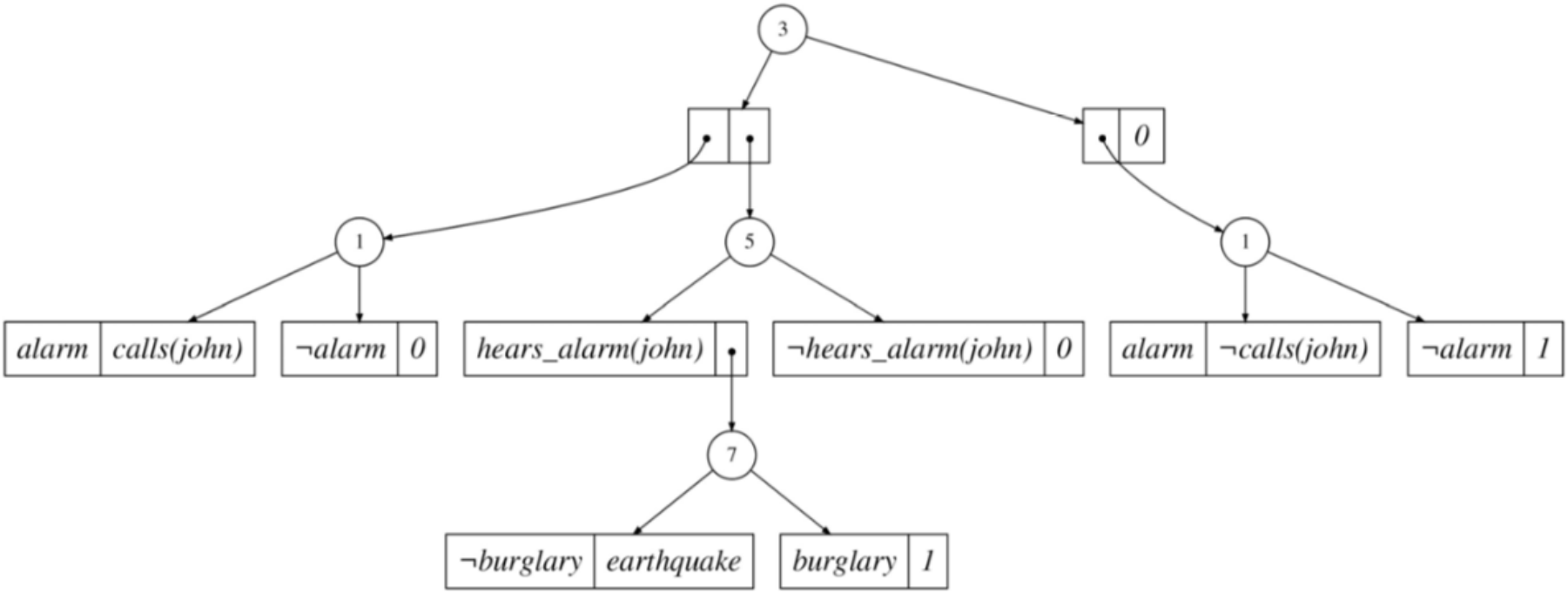
# Knowledge Compilation by SDD

## (1) Ground the probabilistic logic program and convert it into an equivalent propositional formula

Already done

# Knowledge Compilation by SDD

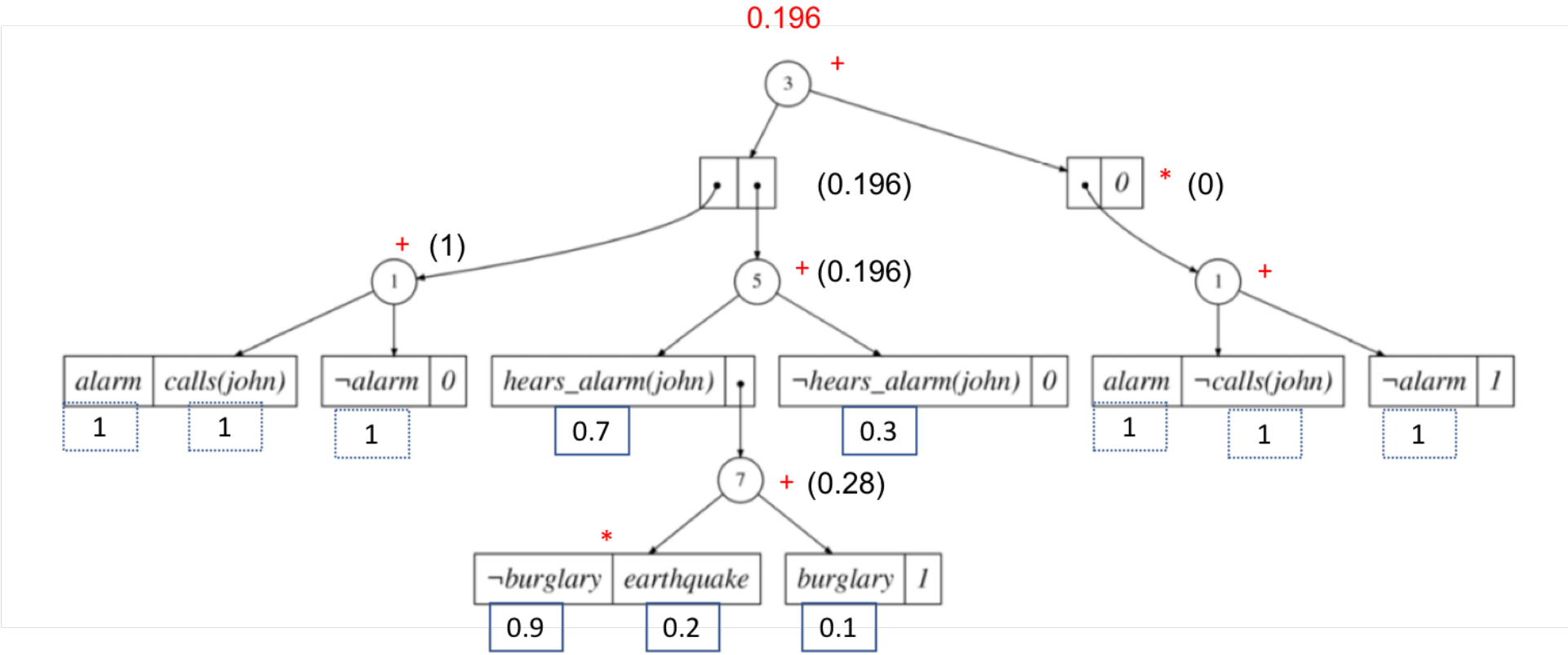
## (2) Compile directly the formula into an SDD




- The compilation is made by a compiler (`http://reasoning.cs.ucla.edu/`)

# Knowledge Compilation by SDD

## (3) Compute the probability of evidence from the SDD




- $P(root) = P(\mathbf{e}) = P(calls(john))$

# Knowledge Compilation by SDD

## Comparison of languages

- Succinctness: size of the smallest compiled circuit for every Boolean formula
  - $d - DNNF < SDD \leq OBDD$
  - There exists a Boolean formula whose smallest SDD representation is exponentially larger than its smallest d-DNNF representation, but the smallest OBDD for any formula is at least as big as its smallest SDD
- SDDs are special cases of d-DNNFs: if one replaces circle-nodes with or-nodes, and paired-boxes with and-nodes, one obtains a d-DNNF, with additional properties (structured decomposability and strong determinism)
- SDDs outperform d-DNNFs

# Knowledge Compilation by SDD

## Comparison of languages

- **Strongly deterministic decomposition**: An $(\mathbf{X}, \mathbf{Y})$-*decomposition* of a function $f(\mathbf{X}, \mathbf{Y})$ over non-overlapping variables $\mathbf{X}$ and $\mathbf{Y}$ is a set $\{(p_1, s_1), \ldots, (p_n, s_n)\}$ such that

$$f = p_1(\mathbf{X}) \wedge s_1(\mathbf{Y}) \vee \ldots \vee p_n(\mathbf{X}) \wedge s_n(\mathbf{Y})$$

  If $p_i \wedge p_j = 0$ for $i \neq j$, the decomposition is said to be *strongly deterministic*

- BDDs are a special case of SDDs where decompositions are all Shannon: formula $f$ is decomposed into $\{(X, f^X), (\neg X, f^{\neg X})\}$.
- SDDs generalize BDDs by considering non-binary decisions based on the value of primes

# Approximate Inference

- Approximate inference aims at computing the results of inference (probability of evidence $P(\mathbf{e})$) in an approximate way so that the process is cheaper than the exact computation of the results
- Two approaches: those that modify an exact inference algorithm and those based on sampling

- Iterative deepening
- *k-best*
- Monte Carlo

# Approximate Inference

## Iterative deepening

- De Raedt et al. (2007); Kimmig et al. (2008b)
- Input: an error bound $\epsilon$, a depth bound $d$, and a query $q$
- Construct an SLD tree for $q$ up to depth $d$
- Build two sets of explanations
  - $K_l$: set of composite choices corresponding to the successful proofs present in the tree
  - $K_u$: set of composite choices corresponding to the successful and still open proofs present in the tree
- $P(K_l)$: lower bound / $P(K_u)$: upper bound of the exact probability

# Approximate Inference

## Iterative deepening

- If the difference $P(K_u) - P(K_l)$ is smaller than $\epsilon$, this means that a solution with a satisfying accuracy has been found and the interval $[P(K_l), P(K_u)]$ is returned
- Otherwise, the depth bound is increased and a new SLD tree is built up to the new depth bound
- Stops when the difference $\leq \epsilon$

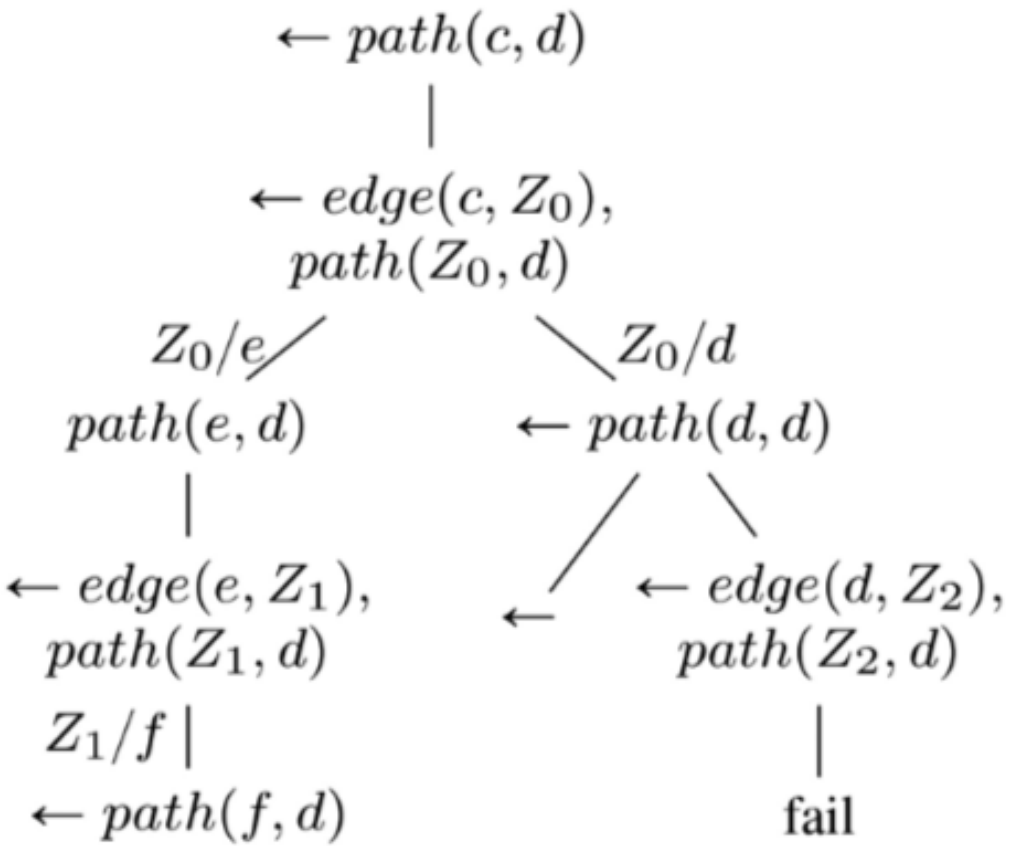
# Approximate Inference

## Iterative deepening

$$path(X, X).$$
$$path(X, Y) \leftarrow edge(X, Z), path(Z, Y).$$
$$0.8 :: edge(a, c).$$
$$0.7 :: edge(a, b).$$
$$0.8 :: edge(c, e).$$
$$0.6 :: edge(b, c).$$
$$0.9 :: edge(c, d).$$
$$0.625 :: edge(e, f).$$
$$0.8 :: edge(f, d).$$

# Approximate Inference

## Iterative deepening




SLD tree up to depth 4 for the query $path(c, d)$ from the program

# Approximate Inference

*k-best*

- Uses a fixed number $k$ of proofs to obtain a lower bound of the probability of the query: the larger the k, the better the bound
- Given $k$, the best $k$ proofs are found, corresponding to the set of best $k$ explanations $K_k$
- $P(K_k)$ is a (lower) estimate of the probability of the query
- **Best** is intended in terms of probability: an explanation is better than another if its probability is higher
- **Branch and bound** approach: prune a derivation if its probability *falls below* that of the k-th best explanation

# Approximate Inference

## Monte Carlo

- Iterative procedure, until convergence
  1. Sample a world, by sampling each ground probabilistic fact/clause in turn
  2. Check whether the query is true in the world
  3. Compute the probability $\hat{p}$ of the query as the fraction of samples where the query is true
- Convergence is reached when the size of the confidence interval of $\hat{p}$ drops below a user-defined threshold $\delta$
- The binomial proportion confidence interval of $\hat{p}$ is used

$$\hat{p} \pm z_{1-\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

  where $n$ is the number of samples and $z_{1-\alpha/2}$ is the $1-\alpha/2$ percentile of a standard normal distribution with $\alpha = 0.05$ usually
- If the width of the interval $< \delta$, it stops and returns $\hat{p}$

# Approximate Inference

## Monte Carlo

- The approach is not efficient on large programs, as proofs are often short while the generation of a world requires sampling many probabilistic facts
- Idea: generate samples lazily, by sampling probabilistic facts/clauses only when required by a proof
- In fact, it is not necessary to sample facts not needed by a proof, as any value for them would do

# Approximate Inference

## Monte Carlo Implementations

- **ProbLog1**: (Kimmig et al. (2011))
- **MCINTYRE** (Monte Carlo INference wiTh Yap REcord) (Riguzzi (2013)): applies the Monte Carlo approach of ProbLog1 to LPADs using the YAP internal database for storing all samples and using tabling for speeding up inference
- Also available in SWI-Prolog (included in the cplint suite)

# Approximate Inference

## MCINTYRE: *Monte Carlo INference wiTh Yap REcord*

- The LPAD clause $C_r = H_1 : \alpha_1 \vee \ldots \vee H_n : \alpha_n \leftarrow L_1, \ldots, L_m.$ is transformed into the set of clauses $MC(C_r)$:

$$MC(C_r, 1) = H_1 \leftarrow L_1, \ldots, L_m, sample\_head(ParList, r, VC, NH), NH = 1.$$
$$\ldots$$
$$MC(C_r, n) = H_n \leftarrow L_1, \ldots, L_m, sample\_head(ParList, r, VC, NH), NH = n.$$

  where $VC$ is a list containing each variable appearing in $C_r$ and $ParList$ is $[\Pi_{r1}, \ldots, \Pi_{rn_r}]$. If the parameters do not sum up to 1 the last clause (the one for *null*) is omitted

- It creates a clause for each head and samples a head index $NH$ with `sample_head/4`.
- If this index coincides with the index of the head disjunct, the derivation succeeds, otherwise it fails

# Approximate Inference

## MCINTYRE: *Monte Carlo INference wiTh Yap REcord*

*Example*: The following LPAD models the development of an epidemic or a pandemic:

$$
\begin{aligned}
C_1 &= epidemic : 0.6; pandemic : 0.3 : -flu(X), cold.\\
C_2 &= cold : 0.7.\\
C_3 &= flu(david).\\
C_4 &= flu(robert).
\end{aligned}
$$

Clause $C_1$ is transformed in:

$$
\begin{aligned}
MC(C_1, 1) = \; & epidemic : -flu(X), cold,\\
& sample\_head([0.6, 0.3, 0.1], 1, [X], NH), NH = 1.\\
MC(C_1, 2) = \; & pandemic : -flu(X), cold,\\
& sample\_head([0.6, 0.3, 0.1], 1, [X], NH), NH = 2.
\end{aligned}
$$

# Approximate Inference

## MCINTYRE: *Monte Carlo INference wiTh Yap REcord*

- If $Q = epidemic$, resolution matches the goal with the head of clause $MC(C_1, 1)$.
- Suppose $flu(X)$ succeeds with $X/david$ and $cold$ succeeds as well.
- Then

$$sample\_head([0.6, 0.3, 0.1], 1, [david], NH)$$

  is called.
- Since clause 1 with $X$ replaced by $david$ was not yet sampled, a number between 1 and 3 is sampled according to the distribution $[0.6, 0.3, 0.1]$ and stored in $NH$.
- If $NH = 1$, the derivation succeeds and the goal is true in the sample, if $NH = 2$ or $NH = 3$ then the derivation fails and backtracking is performed.

# Approximate Inference

## MCINTYRE: *Monte Carlo INference wiTh Yap REcord*

- This involves finding the solution $X/robert$ for $flu(X)$. $cold$ was sampled as true before so it succeeds again.
- Then

$$sample\_head([0.6, 0.3, 0.1], 1, [robert], NH)$$

is called to take another sample.

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