Propositional interval temporal logics: some promising paths

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Interval temporal logics

There exists a **broad and multidisciplinary interest** in interval temporal logic:

- philosophy
- linguistics
- artificial intelligence
- theoretical computer science

Halpern and Shoham's HS

HS features **four basic unary operators**: $\langle B \rangle$ (*begins*) and $\langle E \rangle$ (*ends*), and their transposes $\langle \overline{B} \rangle$ (*begun by*) and $\langle \overline{E} \rangle$ (*ended by*)

Given a formula ϕ and an interval $[d_0, d_1]$, $\langle B \rangle \phi$ holds over $[d_0, d_1]$ if ϕ holds over $[d_0, d_2]$, for some $d_0 \leq d_2 < d_1$, and $\langle E \rangle \phi$ holds over $[d_0, d_1]$ if ϕ holds over $[d_2, d_1]$, for some $d_0 < d_2 \leq d_1$

Many **other operators** can be derived from the basic ones

Halpern and Shoham have shown that the validity/satisfiability problem for HS over various classes of linear orders is (highly) **undecidable** by a suitable encoding of the halting problem

Later, Lodaya showed that some undecidability proofs for HS can actually be tailored to its $\langle B \rangle \langle E \rangle$ **fragment**

Venema's CDT

Venema's CDT logic has **three binary operators** C (*chop*), D, and T, which correspond to the ternary interval relations occurring when an extra point is added in one of the three possible distinct positions with respect to the two endpoints of the current interval (*between, before*, and *after*), plus a modal constant π which holds over a given interval if and only if it is a point-interval

Since HS can be embedded into CDT, **undecidability** results for the latter follow from those for the former

Moszkowski's PITL

Moszkowski PITL features the **two modalities** $\bigcirc$ (*next*) and C (the specialization of the *chop* operator for discrete structures)

Given two formulas ϕ, ψ and a (finite) interval $d_0, \ldots, d_n$, $\bigcirc\phi$ holds over $d_0, \ldots, d_n$ if and only if ϕ holds over $d_1, \ldots, d_n$, while $\phi C \psi$ holds over $d_0, \ldots, d_n$ if and only if there exists i, with $0 \leq i \leq n$, such that ϕ holds over $d_0, \ldots, d_i$ and ψ holds over $d_i, \ldots, d_n$

PITL has been proved to be **undecidable** by a reduction from the problem of testing the emptiness of the intersection of two grammars in Greibach form

Goranko, Montanari, and Sciavicco's PNL

Goranko, Montanari, and Sciavicco's PNL has **two unary modalities** for right and left interval neighborhoods $\langle A \rangle$ and $\langle \overline{A} \rangle$

Given a formula ϕ and an interval $[d_0, d_1]$, $\langle A \rangle \phi$ holds over $[d_0, d_1]$ if ϕ holds over $[d_1, d_2]$, for some $d_2 > d_1$, and $\langle \overline{A} \rangle \phi$ holds over $[d_0, d_1]$ if ϕ holds over $[d_2, d_0]$, for some $d_2 < d_0$ (strict)

While the **undecidability** of **first-order Neighborhood Logic** (NL) can be easily proved by embedding HS in it, the **decidability problem** for its propositional fragments, which can be embedded into HS, is still **open**

Interval temporal logics are very expressive

Propositional interval temporal logics are **very expressive** temporal logics, with simple syntax and semantics, which allow one to naturally express statements that refer to time intervals and continuous processes.

It can be shown that interval logics such as HS and CDT are strictly **more expressive than** every point-based temporal logic on linear orderings

Interval temporal logics are (highly) undecidable

We have that the validity problem for HS, interpreted over any class of ordered structures with an infinitely ascending sequence, is **at least r.e.-hard** In the case of Dedekind-complete ordered structures having an infinitely ascending sequence, it becomes Π^1_1**-hard** (this means that the validity problem for HS over natural numbers, integers, or reals is not recursively axiomatizable)

As a matter of fact, it is possible to show that undecidability occurs even without existence of infinitely ascending sequences

The problem of finding fragments which are **expressive enough** to express meaningful statements about time intervals **and decidable** has been recognized as a fundamental one by several authors

A simple path to decidability

In propositional interval temporal logics undecidability is the **rule** and decidability the **exception**

Interval logics make it possible to express properties of **pairs** of time points (think of intervals as constructed out of points), rather than **single** time points. In most cases, this feature prevents one from the possibility of reducing interval-based temporal logics to point-based ones

However, there are a few exceptions where the logic satisfies suitable **syntactic and/or semantic restrictions**, and such a reduction can be defined, thus allowing one to benefit from the good computational properties of point-based logics

Case 1: constraining interval modalities

This is the case with the $\langle B\rangle\langle\overline{B}\rangle$ and $\langle E\rangle\langle\overline{E}\rangle$ fragments of HS.

Consider the case of $\langle B\rangle\langle\overline{B}\rangle$ (the case of $\langle E\rangle\langle\overline{E}\rangle$ is similar). The decidability of $\langle B\rangle\langle\overline{B}\rangle$ can be obtained by **embedding** it into the propositional temporal logic of linear time LTL[F,P] with temporal modalities F (sometime in the future) and P (sometime in the past)

The formulas of $\langle B\rangle\langle\overline{B}\rangle$ are simply translated into formulas of LTL[F,P] by a mapping that replaces $\langle B\rangle$ by P and $\langle\overline{B}\rangle$ by F. LTL[F,P] has the finite model property and is **decidable**

Case 2: constraining temporal structures

This is the case with the so-called Split Logics (SLs)

SLs are propositional interval logics equipped with operators borrowed from HS and CDT, but interpreted over specific structures, called **split structures**. The distinctive feature of split structures is that every interval can be 'chopped' in at most one way

The **decidability** of various SLs has been proved by embedding them into the first-order fragments of monadic second-order decidable theories of time granularity (which are proper extensions of the well-known monadic second-order theory of one successor S1S)

Case 3: constraining semantic interpretations

Another possibility is to constrain the relation between the truth value of a formula over an interval and its truth value over the subintervals of that interval

This is the case with the decidable fragment of PITL extended with quantification over propositional variables (QPITL) which has been obtained by imposing a suitable **locality** constraint which states that a propositional variable is true over an interval if and only if it is true at its starting point (point-interval)

By exploiting such a constraint, **decidability** of QPITL can be proved by embedding it into Quantified LTL (QLTL)

Some promising alternative paths

In view of the previous analysis, a (maybe the) major challenge in the area of interval temporal logics is to identify expressive enough, yet decidable, fragments and/or logics which are **genuinely** interval-based

A logic is **genuinely** interval-based if it cannot be directly translated into point-based logics and it does not invoke locality, or other semantic restrictions, reducing the interval-based semantics to the point-based one

Path 1: tableau-based decision procedures

By combining syntactic restrictions (temporal operators: no past operators) and semantic ones (temporal structure: natural numbers), we recently succeeded in devising a **tableau-based decision procedure** for the future fragment of PNL, interpreted over natural numbers

Unlike the case of the $\langle B \rangle \langle \overline{B} \rangle$ and $\langle E \rangle \langle \overline{E} \rangle$ fragments, in such a case we **cannot abstract way** from the left endpoint of intervals: there can be contradictory formulas that hold over intervals that have the same right endpoint, but a different left one

Path 1: tableau-based decision procedures (con't)

The proposed tableau method partly resembles the tableau-based decision procedure for LTL

However, while the latter takes advantage of the so-called fix-point definition of temporal operators which makes it possible to proceed stepwise by splitting every temporal formula into a (possibly empty) part related to the current state and a part related to the next state, the former must also **keep track** of universal and (pending) existential requests coming from the past.

Path 2: restrict attention to finite models

There are contexts where interpretations in which infinitely many statements (events) hold (occur) in a finite space of time are of no interest. Examples can be found in computational linguistics

Pratt developed a decidable interval logic of temporal prepositions which is interpreted over **finite models** (notice that, under such an assumption, any formula with only infinite models turns out to be unsatisfiable)

Since the restriction to finite models is neither a necessary nor a sufficient condition for decidability, it would be interesting to investigate the effects of imposing it to other interval logics

Path 3: connections with other decidable logics

Research on interval temporal logics can benefit from interesting **connections** that there seem to be between them and other decidable logics

Examples

Montanari and Sciavicco are investigating the relations between full PNL and **two variable first-order logic over ordered domains**

Shapirovsky and Shehtman explored the relations between the logics of subintervals, the so-called D logics, and the **logics of Minkowski spacetime**