Interval Temporal Logic Model Checking: the Border Between Good and Bad HS Fragments

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Laura Bozzelli, Alberto Molinari, Angelo Montanari, Adriano Peron (*), Pietro Sala

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(*) University of Napoli "Federico II", Dept. of Electrical Engineering and Information Technologies (DIETI)

Model checking

- Model checking: the desired properties of a system are checked against a model of the system
  - the model is a (finite) state-transition graph
  - system properties are specified by a temporal logic (e.g., LTL, CTL, CTL*, . . .)

- Distinctive features of model checking:
  - exhaustive verification of all the possible behaviours
  - fully automatic process
  - a counterexample is produced for a violated property

Point-based vs. interval-based model checking

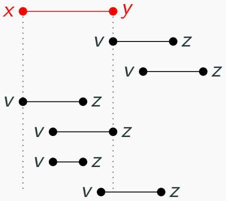
- Model checking is usually point-based:
  - properties express requirements over points (snapshots) of a computation (states of the state-transition system)
  - they are specified by means of point-based temporal logics such as LTL and CTL

- Interval-based model checking:
  - Interval-based properties express conditions on computation stretches: accomplishments, actions with duration, and temporal aggregations
  - they are specified by means of interval temporal logics, e.g., HS

The logic HS

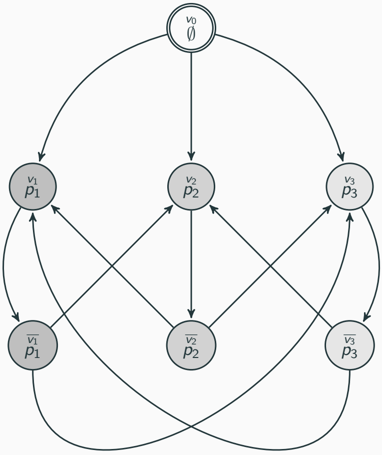
HS features a modality for any Allen ordering relation between pairs of intervals (except for equality)

Allen rel.	HS	Definition	Example
meets	$\langle A \rangle$	$[x,y]\mathcal{R}_A[v,z] \iff y = v$	
before	$\langle L \rangle$	$[x,y]\mathcal{R}_L[v,z] \iff y < v$	
started-by	$\langle B \rangle$	$[x,y]\mathcal{R}_B[v,z] \iff x = v \wedge z < y$	
finished-by	$\langle E \rangle$	$[x,y]\mathcal{R}_E[v,z] \iff y = z \wedge x < v$	
contains	$\langle D \rangle$	$[x,y]\mathcal{R}_D[v,z] \iff x < v \wedge z < y$	
overlaps	$\langle O \rangle$	$[x,y]\mathcal{R}_O[v,z] \iff x < v < y < z$	



All modalities can be expressed by means of $\langle A \rangle$, $\langle B \rangle$, $\langle E \rangle$ and their transposed modalities only

Kripke structures



An example of Kripke structure

- $\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$
- HS formulas are interpreted over (finite) state-transition systems whose states are labelled with sets of proposition letters (Kripke structures)

- An interval is a track (finite path) in a Kripke structure

HS semantics and model checking

Truth of a formula ψ over a track ρ of a Kripke structure $\mathcal{K}$:

- $\mathcal{K}, \rho \models p$ iff $p \in \bigcap_{w \in \mathsf{states}(\rho)} \mu(w)$, for any letter $p \in \mathcal{AP}$ (homogeneity assumption);
- negation, disjunction, and conjunction are standard;
- $\mathcal{K}, \rho \models \langle \mathsf{A} \rangle\, \psi$ iff there is a track ρ' s.t. $\mathsf{lst}(\rho) = \mathsf{fst}(\rho')$ and $\mathcal{K}, \rho' \models \psi$;
- $\mathcal{K}, \rho \models \langle \mathsf{B} \rangle\, \psi$ iff there is a prefix ρ' of ρ s.t. $\mathcal{K}, \rho' \models \psi$;
- $\mathcal{K}, \rho \models \langle \mathsf{E} \rangle\, \psi$ iff there is a suffix ρ' of ρ s.t. $\mathcal{K}, \rho' \models \psi$;
- the semantic clauses for $\langle \overline{\mathsf{A}} \rangle$, $\langle \overline{\mathsf{B}} \rangle$, and $\langle \overline{\mathsf{E}} \rangle$ are similar

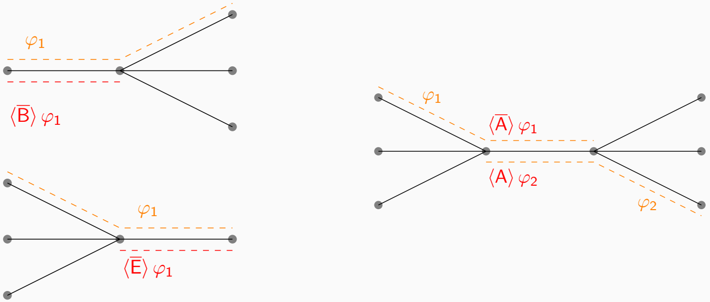
Model Checking

$\mathcal{K} \models \psi \iff$ for all *initial* tracks ρ of $\mathcal{K}$, it holds that $\mathcal{K}, \rho \models \psi$

Possibly infinitely many tracks!

HS state semantics

- The semantics features branching both in the past and in the future.



- HS with state semantics is not comparable w.r.t. to *LTL*, *CTL* and *CTL**.

Decidability of HS model checking

Theorem

The model checking problem for full HS on Kripke structures is decidable (with a non-elementary algorithm)

Reference

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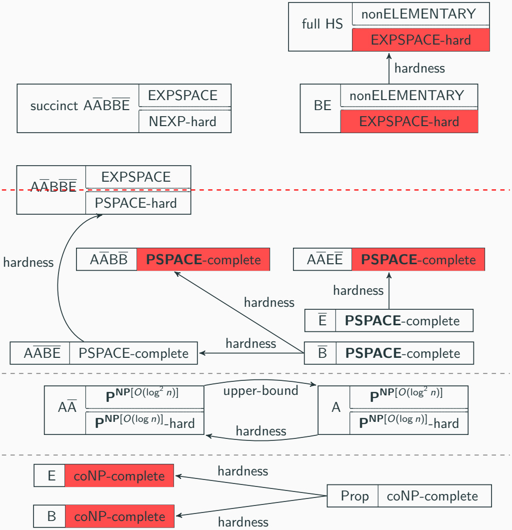
Accepted for publication

Complexity picture



The fragment BE

Theorem

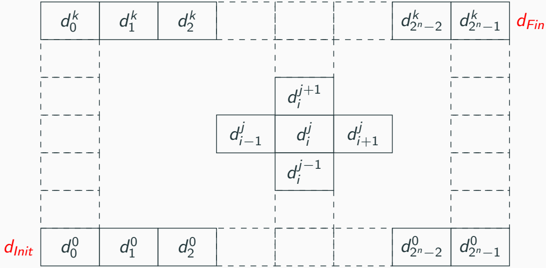
The model checking problem for BE *on Kripke structures is EXPSPACE-hard*

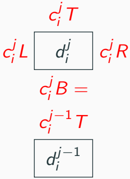
Proved by a polynomial-time reduction from a domino-tiling problem for grids with rows of single exponential length:

- For an instance $\mathcal{I}$ of the problem we build in polynomial time a Kripke structure $\mathcal{K}_{\mathcal{I}}$ and a BE formula $\varphi_{\mathcal{I}}$;
- there exists an initial track of $\mathcal{K}_{\mathcal{I}}$ satisfying $\varphi_{\mathcal{I}}$ if and only if there exists a tiling of $\mathcal{I}$;
- Hence, $\mathcal{K}_{\mathcal{I}} \models \neg\varphi_{\mathcal{I}}$ iff there does not exist a tiling of $\mathcal{I}$.

BE hardness: Encoding of a tiling

Instance of tiling problem: $(C, \Delta, n, d_{init}, d_{final})$, where C is a finite set of colors, $\Delta \subseteq C^4$ is a set of tuples (c_B, c_L, c_T, c_R)





String encoding of a tiling

row 0								row 1							
d_0^0	$0\cdots00$	d_1^0	$1\cdots00$	$\cdots$	$d_{2^n-1}^0$	$1\cdots11$	\$	d_0^1	$0\cdots00$	d_1^1	$1\cdots00$	$\cdots$	$d_{2^n-1}^1$	$1\cdots11$	\$
column 0		column 1			column 2^n-1			column 0		column 1			column 2^n-1		

BE hardness: $\mathcal{K}_\mathcal{I}$ and $\varphi_\mathcal{I}$

- For $\mathcal{AP} = \Delta \cup \{\$\} \cup \{0, 1\}$, the Kripke structure $\mathcal{K}_\mathcal{I}$ is defined as $\mathcal{K}_\mathcal{I} = (\mathcal{AP}, \mathcal{AP}, \mathcal{AP} \times \mathcal{AP}, \mu, d_{init})$, where $\mu(p) = \{p\}$, for any $p \in \mathcal{AP}$.
- The formula $\varphi_\mathcal{I}$ checks that an initial track of $\mathcal{K}_\mathcal{I}$ is a correct encoding of a tiling.
  It exploits the following features of BE:
  - Measuring the length of a track: the formula $length_i$ characterizes the tracks of length i.

$$length_i := (\underbrace{\langle \mathrm{B} \rangle \dots \langle \mathrm{B} \rangle}_{i-1} \top) \ \wedge \ (\underbrace{[B] \dots [B]}_{i} \bot).$$

  - Constraining arbitrary subtracks with the derived operator $\langle \mathrm{G} \rangle$ and its dual $[G]$, which allow us to select arbitrary subtracks of a given track:

$$\langle \mathrm{G} \rangle \, \psi := \psi \vee \langle \mathrm{B} \rangle \, \psi \vee \langle \mathrm{E} \rangle \, \psi \vee \langle \mathrm{B} \rangle \, \langle \mathrm{E} \rangle \, \psi.$$

The fragments $\mathsf{A}\overline{\mathsf{A}}\mathsf{E}\overline{\mathsf{E}}$ and $\mathsf{A}\overline{\mathsf{A}}\mathsf{B}\overline{\mathsf{B}}$

- A **PSPACE** MC algorithm for $\mathsf{A}\overline{\mathsf{A}}\mathsf{E}\overline{\mathsf{E}}$ and $\mathsf{A}\overline{\mathsf{A}}\mathsf{B}\overline{\mathsf{B}}$ can be devised by exploiting a polynomial size model-track property.
- Polynomial size model-track property: if a track ρ of a Kripke structure $\mathcal{K}$ satisfies a formula φ, then there is a track π, whose length is polynomial in the sizes of φ and $\mathcal{K}$, that satisfies φ.
- The MC algorithm to decide $\mathcal{K} \models \varphi$ searches a counterexample, i.e. a track ρ such that $|\rho| \leq |W| \cdot (2|\varphi| + 3)^2$ and $\mathcal{K}, \rho \models \neg\varphi$.

Algorithm 1 `ModCheck`$(\mathcal{K}, \varphi)$

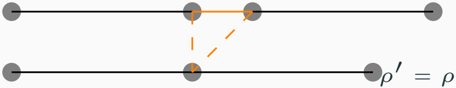
for all initial $\tilde{\rho} \in \mathsf{Trk}_{\mathcal{K}}$ s.t. $|\tilde{\rho}| \leq |W| \cdot (2|\varphi| + 3)^2$ **do**
    if `Check`$(\mathcal{K}, \varphi, \tilde{\rho}) = 0$ **then**
        return 0: "$\mathcal{K}, \tilde{\rho} \not\models \varphi$" $\triangleleft$ *Counterexample found*
return 1: "$\mathcal{K} \models \varphi$"

The fragment $A\bar{A}E\bar{E}$: Polynomial-size model-track property

- Contraction technique (permitted by homogeneity assumption):

$Pattern(\rho, i) = Pattern(\rho, j)$ $\quad Pattern(\rho, k) = \{p \in \mathcal{AP} : \mathcal{K}, \rho^{k[} \models p\}$

$\rho(i) = \rho(j)$



ρ

$\rho' = \rho(0, i) \cdot \rho^{(j+1)[}$

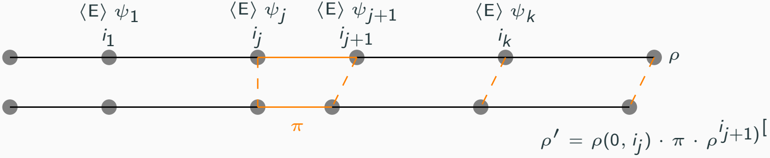
- $\forall i \leq h \leq j, \forall p \in \mathcal{AP} : \mathcal{K}, \rho^{h[} \models p$ iff $\mathcal{K}, \rho^{j[} \models p$;
- ρ' is well-formed w.r. to ρ.

Proposition

For any track ρ of $\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$, there exists a track π of $\mathcal{K}$, which is well-formed with respect to ρ, such that $|\pi| \leq |W| \cdot (|\mathcal{AP}| + 1)$.

The fragment $\mathsf{A\overline{A}E\overline{E}}$: Polynomial-size model-track property

- Contraction technique by well-formedness contraction:



- $\langle \mathsf{E} \rangle\, \psi_j$ is a subformula of φ s.t. $\mathcal{K}, \rho \models \langle \mathsf{E} \rangle\, \psi_j$ for $1 \leq j \leq k$;
- i_j is the greatest index of ρ such that $\mathcal{K}, \rho^{i_j[} \models \psi_j$;
- π is well-formed w.r.t. to $\rho(i_j, i_{j+1})$ and $|\pi| \leq |W| \cdot (|\mathcal{AP}| + 1)$;
- $\rho' \models \varphi$.

Theorem

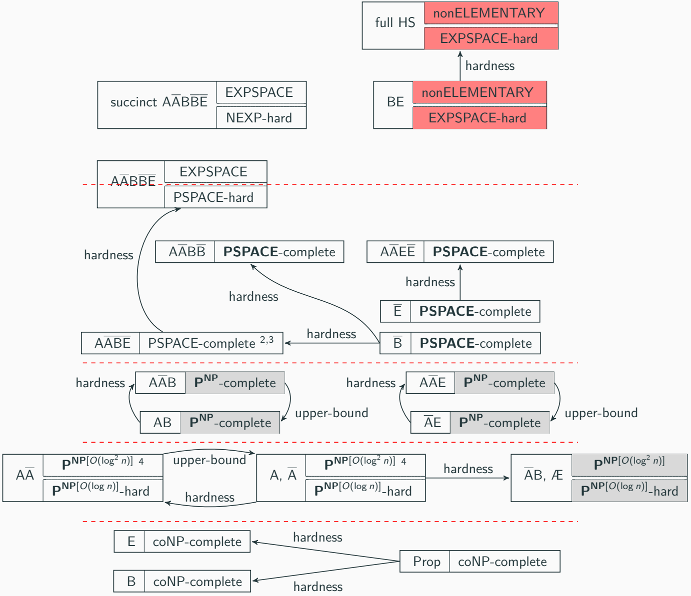
Let $\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$, $\rho \in \mathsf{Trk}_{\mathcal{K}}$, and φ be an $\mathsf{A\overline{A}E\overline{E}}$ formula (in NNF) such that $\mathcal{K}, \rho \models \varphi$. Then, there is $\overline{\rho} \in \mathsf{Trk}_{\mathcal{K}}$, induced by ρ, such that $\mathcal{K}, \overline{\rho} \models \varphi$ and $|\overline{\rho}| \leq |W| \cdot (|\varphi| + 1)^2$.

Model checking routine

Algorithm 2 $\texttt{Check}(\mathcal{K}, \psi, \tilde{\rho})$

if $\psi = p$, for $p \in \mathcal{AP}$ **then**
    if $p \in \bigcap_{s \in \texttt{states}(\tilde{\rho})} \mu(s)$ **then**
        return 1 **else return** 0
else if $\psi = \varphi_1 \wedge \varphi_2$ **then**
    if $\texttt{Check}(\mathcal{K}, \varphi_1, \tilde{\rho}) = 0$ **then**
        return 0
    else
        return $\texttt{Check}(\mathcal{K}, \varphi_2, \tilde{\rho})$
else if $\psi = \langle \mathrm{A} \rangle \varphi$ **then**
    for all $\rho \in \mathrm{Trk}_{\mathcal{K}}$ such that $\mathrm{fst}(\rho) = \mathrm{lst}(\tilde{\rho})$, and $|\rho| \leq |W| \cdot (2|\varphi| + 1)^2$ **do**
        if $\texttt{Check}(\mathcal{K}, \varphi, \rho) = 1$ **then**
            return 1
    return 0
else if $\psi = \langle \mathrm{E} \rangle \varphi$ **then**
    for each $\overline{\rho}$ suffix of $\tilde{\rho}$ **do**
        if $\texttt{Check}(\mathcal{K}, \varphi, \overline{\rho}) = 1$ **then**
            return 1
    return 0
else if $\psi = \langle \overline{\mathrm{E}} \rangle \varphi$ **then**
    for all $\rho \in \mathrm{Trk}_{\mathcal{K}}$ such that $\mathrm{lst}(\rho) = \mathrm{fst}(\tilde{\rho})$, and $2 \leq |\rho| \leq |W| \cdot (2|\varphi| + 1)^2$ **do**
        if $\texttt{Check}(\mathcal{K}, \varphi, \rho \star \tilde{\rho}) = 1$ **then**
            return 1
    return 0
else if $\psi = \neg\varphi$ **then**
    return $1 - \texttt{Check}(\mathcal{K}, \varphi, \tilde{\rho})$
… $\triangleleft$ $\psi = \langle \overline{\mathrm{A}} \rangle \varphi$ is analogous to $\psi = \langle \mathrm{A} \rangle \varphi$

Complexity picture: future work



Future work

- Determining the precise complexity of full HS (automata-based approach)
- $A\overline{A}B$ and $A\overline{A}E$ are $\mathbf{P}^{NP}$-complete;
- Investigating other possible HS semantics, in particular to compare HS with LTL/CTL (in particular, linear track semantics, computation tree semantics);
- Relaxing the homogeneity assumption.

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