Timeline-Based Planning over Dense Temporal Domains with Trigger-less Rules is NP-Complete

ICTCS 2018—Urbino

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Point-based vs. interval-based MC

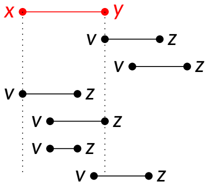
- Model checking (MC) is usually point-based:
  - properties express requirements over points (snapshots) of a computation (states of the state-transition system)
  - they are specified by means of point-based temporal logics such as LTL, CTL, and CTL* .

- Interval-based MC:
  - Interval-based properties express conditions on computation stretches
  - they are specified by means of interval temporal logics, which feature intervals as their basic ontological entities (e.g., HS)
    - ability to express: actions with duration, accomplishments, temporal aggregations
    - applied to computational linguistics, artificial intelligence, temporal databases, formal verification

The logic HS

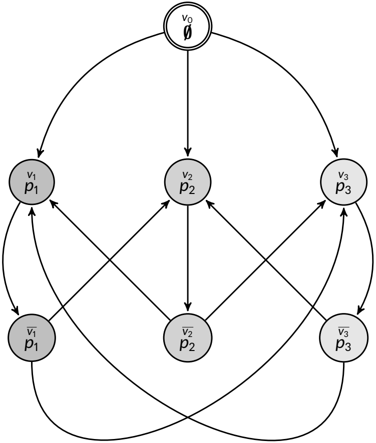
HS features a modality for each of the 13 Allen's ordering relations between pairs of intervals (except for equality)

Allen rel.	HS	Definition	Example
meets	$\langle A \rangle$	$[x, y]\mathcal{R}_A[v, z] \iff y = v$	
before	$\langle L \rangle$	$[x, y]\mathcal{R}_L[v, z] \iff y < v$	
started-by	$\langle B \rangle$	$[x, y]\mathcal{R}_B[v, z] \iff x = v \land z < y$	
finished-by	$\langle E \rangle$	$[x, y]\mathcal{R}_E[v, z] \iff y = z \land x < v$	
contains	$\langle D \rangle$	$[x, y]\mathcal{R}_D[v, z] \iff x < v \land z < y$	
overlaps	$\langle O \rangle$	$[x, y]\mathcal{R}_O[v, z] \iff x < v < y < z$	



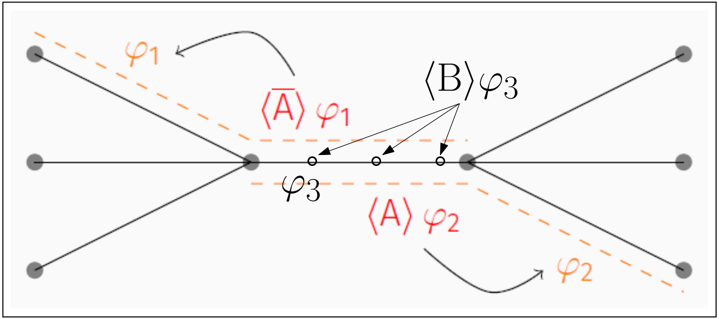
$$\boxed{\psi ::= p \mid \neg\psi \mid \psi \lor \psi \mid \langle X \rangle\psi \mid \langle \overline{X} \rangle\psi} \qquad X \in \{A, L, B, E, D, O\}.$$

Kripke structures



- HS formulas are interpreted over (finite) state-transition systems whose states are labeled with sets of proposition letters (Kripke structures)
- An interval is a trace (finite path) in a Kripke structure

HS (state-based) semantics



- Branching semantics of past/future operators

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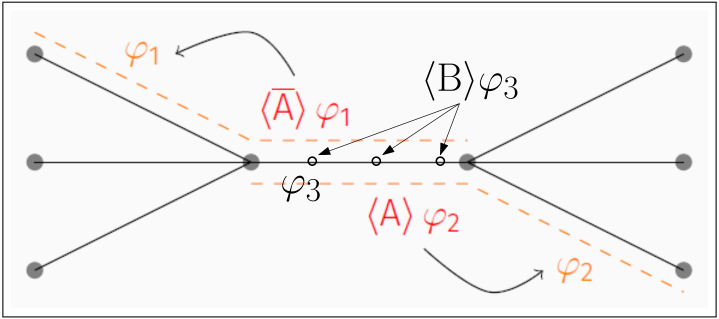

- Branching semantics of past/future operators

MC

$\mathcal{K} \models \psi \iff$ for all *initial* traces ρ of $\mathcal{K}$, it holds that $\mathcal{K}, \rho \models \psi$

Possibly infinitely many traces!

Decidability of HS MC

Theorem

The MC problem for full HS *over Kripke structures is decidable (with a non-elementary algorithm)*

Reference

A. Molinari, A. Montanari, A. Murano, G. Perelli, and A. Peron. Checking interval properties of computations.
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Theorem

The MC problem for BE over Kripke structures, under homogeneity, is **EXPSPACE**-*hard*

Reference

L. Bozzelli, A. Molinari, A. Montanari, A. Peron, and P. Sala. Interval Temporal Logic Model Checking: the Border Between Good and Bad HS Fragments.
In *IJCAR*, pages 389–405, 2016

Many other HS fragments studied (**PSPACE** $\leftrightsquigarrow$ **NP**).

Ongoing work

We are looking for possible replacements of Kripke structures by more expressive system models in interval-based MC:

- interval-based system models, that allow one to directly describe systems on the basis of their interval behavior/properties (e.g., timelines).
- visibly pushdown systems, that can encode recursive programs and infinite state systems;

Timelines

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- Timeline-based planning (TP for short) is a more declarative alternative to the classic action-based planning
- Temporal domain commonly assumed discrete.
- Gigante et al. showed that TP with bounded temporal relations and token durations, and no temporal horizon, is **EXPSPACE**-complete and expressive enough to capture action-based temporal planning. (**EXPSPACE**-completeness also with unbounded relations)

Timelines

State variable

$$x = (V_x, T_x, D_x)$$

where, e.g.,

- $V_x = \{a, b, c\}$,
- $T_x(a) = \{b, c\}$, $T_x(b) = \{a, b, c\}$, $T_x(c) = \{a, b\}$ and
- $D_x(a) = [5, 8]$, $D_x(b) = [1, 4]$, $D_x(c) = [2, \infty[$

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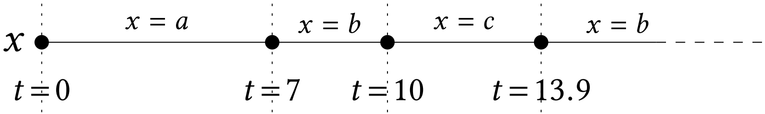
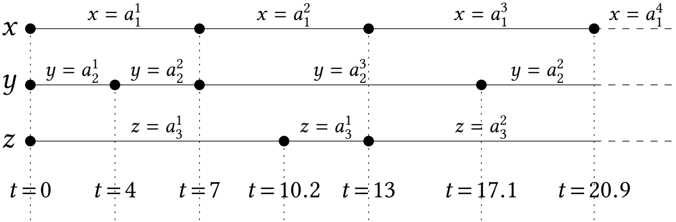
Example of timeline for x:



$$(a, 7)(b, 3)(c, 3.9)\cdots$$

Pairs of value/duration are called tokens.

Timelines


x
y
z
$x = a_1^1$
$x = a_1^2$
$x = a_1^3$
$x = a_1^4$
$y = a_2^1$
$y = a_2^2$
$y = a_2^3$
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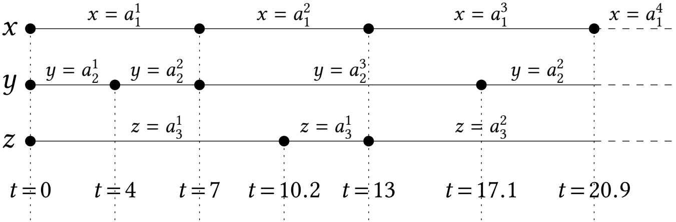
$z = a_3^1$
$z = a_3^1$
$z = a_3^2$
$t = 0$
$t = 4$
$t = 7$
$t = 10.2$
$t = 13$
$t = 17.1$
$t = 20.9$

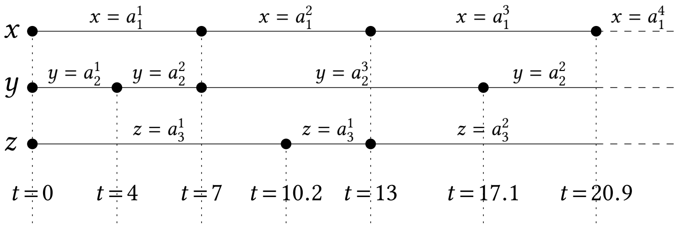
Timelines



Synchronization rules on timelines:

$$\forall o_0[x = a_1^1] \rightarrow \exists o_1[z = a_3^1]\exists o_2[y = a_2^2].\left(o_0 \leq^{e,s}_{[3,4]} o_1 \wedge o_0 \leq^{s,s}_{[5,\infty[} o_2\right)$$

Timelines



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Trigger-less rules.

Timelines as system models

- We study timeline-based planning (TP) over dense domains (no recourse to discretization).
  1. Why TP before MC? Timelines will be our system models. TP is a necessary condition for MC (feasibility check of the system description).
  2. Why dense domains? To avoid discreteness in system descriptions $\Rightarrow$ abstraction at a higher level, neglecting unnecessary details, and paving the way for a more general interval-based MC;
- Both (i) the system model and (ii) the specifications (temporal formulas) can be translated into a common formalism (timed automata)

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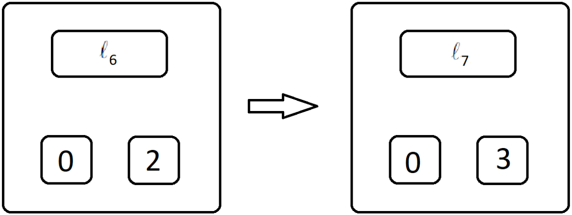

$\ell_6 : c_2 := c_2 + 1; \text{goto } \ell_7$

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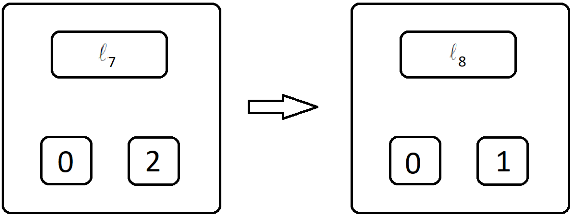
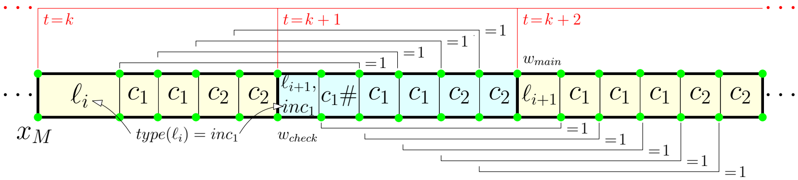


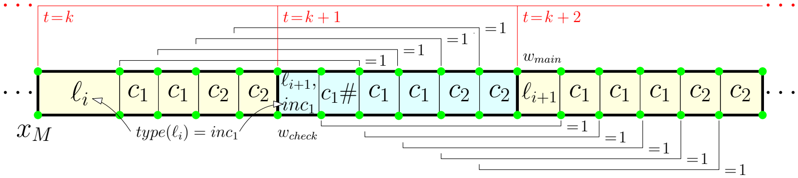
ℓ_7: if $c_2 > 0$ then $c_2 := c_2 - 1$; goto ℓ_8 else goto ℓ_{12}

Undecidability of TP over dense domains



- Exactly one occurrence of $\ell_{\mathtt{init}}$ and $\ell_{\mathtt{halt}}$ (transition function);

Undecidability of TP over dense domains



- Exactly one occurrence of $\ell_{\mathtt{init}}$ and $\ell_{\mathtt{halt}}$ (transition function);
- For each $v \in V_{\mathtt{Ctrl}} \setminus \{\ell_{\mathtt{halt}}\}$,

$$o[x_M = v] \rightarrow \bigvee_{u \in V_{\mathtt{Ctrl}}} \exists\, o'[x_M = u].\, o \leq^{s,s}_{[1,1]} o'.$$

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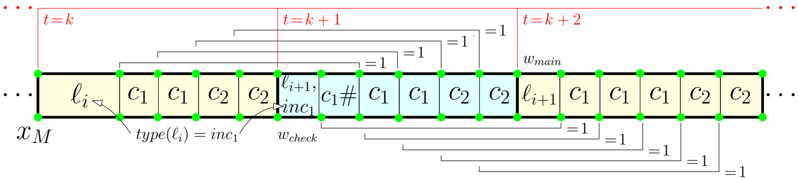

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- For each $i = 1, 2$, $v \in (U_{c_i} \cap V_{\text{main}}) \setminus V_{\texttt{halt}}$ (forward):

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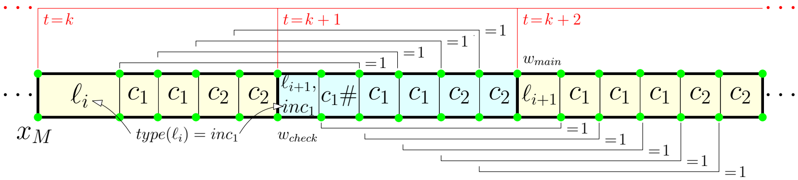

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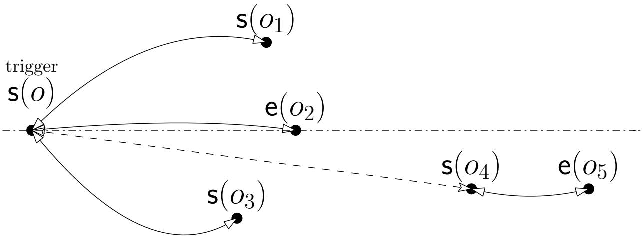
- For each $i = 1, 2$, $v \in (U_{c_i} \cap V_{\text{check}}) \setminus V_{\mathtt{init}}$ (backward):

$$o[x_M = v] \to \bigvee_{u \in U_{c_i}} \exists\, o'[x_M = u].\, o' \leq^{s,s}_{[1,1]} o.$$

What happens if we restrict to future?

- Future: the token triggering a rule can only “refer” to other tokens in the future (i.e., starting after it).



Theorem

Future TP is non-primitive recursive-hard, even with a single state variable.

- Reduction from the halting problem for Gainy counter machines, known to be non-primitive recursive
- Only forward constraint can be expressed by future rules!

Decidability—(1) Translating rules

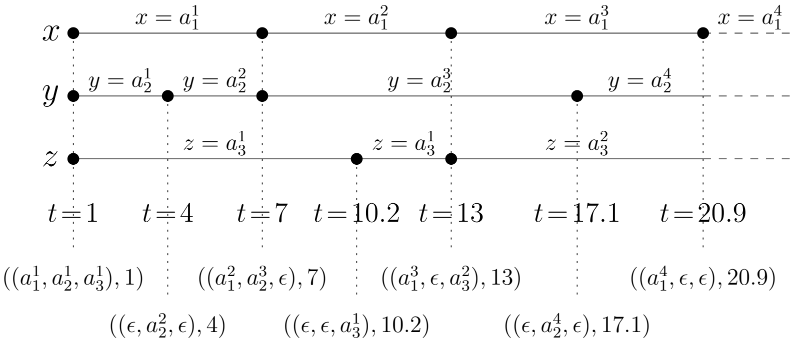
- Decidability of future TP with arbitrary trigger rules is open.
- We restrict to simple trigger rules:
  all existentially quantified tokens (but not the trigger!) occur just once in the rule.
- Decidability can be recovered if rules are simple and future.

Translation into MTL/MITL

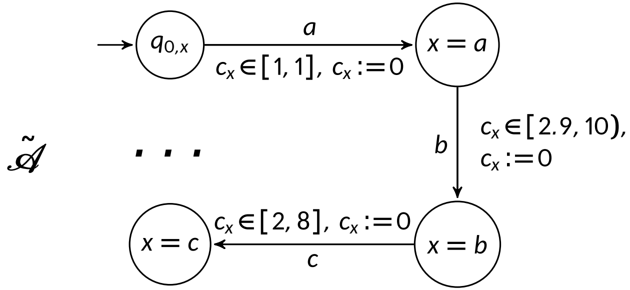
The simple form allows translation into MTL/MITL (future only+finite w!):

$$\varphi ::= \top \mid p \mid \varphi \vee \varphi \mid \neg\varphi \mid \varphi U_I \varphi$$

with $p \in \mathcal{AP}$, $I \in$ `Intv`, U_I is the standard *strict timed until* MTL modality



Decidability—(2) Translating state variables



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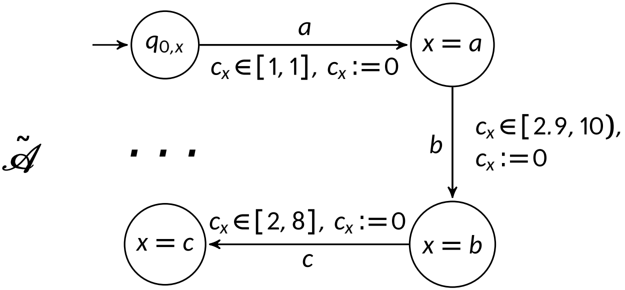

Theorem

Future TP with simple trigger rules is decidable (in non-primitive recursive time). If the intervals in atoms of the trigger rules are non-singular (resp., belong to $\mathtt{Intv}_{(0,\infty)}$), then it is in **EXPSPACE** *(resp., in* **PSPACE***).*

EXPSPACE-completeness (resp., **PSPACE**-completeness) holds.

Timeline-based planning and MC: results

System model:

$$x_{\text{temp}} = (V_{\text{temp}}, T_{\text{temp}}, D_{\text{temp}})$$

$$x_{\text{proc}} = (V_{\text{proc}}, T_{\text{proc}}, D_{\text{proc}})$$

$$x_{\text{transm}} = (V_{\text{transm}}, T_{\text{transm}}, D_{\text{transm}})$$

$$+$$

Property specification:

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$$\forall o[x_{\mathtt{proc}} = \mathtt{reading}_1] \rightarrow$$
$$(\exists o_1[x_{\mathtt{proc}} = \mathtt{read}_0].o \leq^{e,s}_{[0,1]} o_1) \vee$$
$$(\exists o_2[x_{\mathtt{proc}} = \mathtt{read}_1] \exists o_3[x_{\mathtt{temp}} = \mathtt{ready}].o \leq^{e,s}_{[0,1]} o_2 \wedge o_3 \leq^{e,e}_{[0,+\infty[} o).$$

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Property specification:

$$F_{\leq 8}\ \psi(s, \texttt{read}_1) \qquad F_{\geq 0}\big(\psi(s, \texttt{ready}) \wedge (\top\, U_{>0}\ \psi(s, \texttt{ready}))\big)$$

Timeline-based planning and MC: results

Given a system model P (state vars + rules), it is possible to build a TA $\tilde{\mathscr{A}}$ that accepts all and only the (timed words encoding) computations of P.

Definition (Timeline-based model checking)

Given a system model (in the form of) $\tilde{\mathscr{A}}$ and a MITL formula φ, to decide if

$$\mathscr{L}_T(\tilde{\mathscr{A}}) \subseteq \mathscr{L}_T(\varphi).$$

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We make a product between $\tilde{\mathscr{A}}$ and $\mathscr{A}_{\neg\varphi}$ and check for emptiness.

Theorem

The MC problem for MITL formulas over timelines, with simple future trigger rules and non-singular intervals, is in **EXPSPACE**.
The MC problem for $\mathsf{MITL}_{(0,\infty)}$ *formulas over timelines, with simple future trigger rules and intervals in* $\mathtt{Intv}_{(0,\infty)}$*, is in* **PSPACE**.

Matching lower bounds derive from TP.

Timelines with trigger-less rules only

- Trigger-less synchronization rules can be directly translated into a timed automaton (no need to translate into MTL)
- Timed automaton of exponential size: it gives us an exponential bound (*) on the number of tokens and on the horizon

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- We observe that:
    1. timelines for different variables evolve independently, and
    2. each trigger-less rule enforces at most one "synchronization point" among timelines.

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How we deal with 1. and 2.:

1. timeline evolutions are enforced by a linear program (where constants are exponential (*)), resting on results on Eulerian multi-graphs (thanks G. Woeginger!)
2. we non-deterministically position tokens (those to which rules refer) along timelines (their start/end times can be generated in polynomial time (*)) and check satisfaction of rules

Thanks!

Publications

[1] L. Bozzelli, A. Molinari, A. Montanari, and A. Peron.
An in-depth investigation of interval temporal logic model checking with regular expressions.
In *SEFM*, pages 104–119, 2017.

[2] L. Bozzelli, A. Molinari, A. Montanari, A. Peron, and P. Sala.
Interval Temporal Logic Model Checking: the Border Between Good and Bad HS Fragments.
In *IJCAR*, pages 389–405, 2016.

[3] L. Bozzelli, A. Molinari, A. Montanari, A. Peron, and P. Sala.
Interval vs. Point Temporal Logic Model Checking: an Expressiveness Comparison.
In *FSTTCS*, 2016.

[4] A. Molinari, A. Montanari, A. Murano, G. Perelli, and A. Peron.
Checking interval properties of computations.
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HS (state-based) semantics and MC

Truth of a formula ψ over a trace ρ of a Kripke structure $\mathcal{K} = (\mathcal{AP}, W, \delta, \mu, w_0)$:

- $\mathcal{K}, \rho \models p$ iff p labels all states of $\mathcal{K}$ composing ρ, for any $p \in \mathcal{AP}$ (homogeneity assumption);

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- negation, disjunction, and conjunction are standard;
- $\mathcal{K}, \rho \models \langle \mathrm{A} \rangle\, \psi \ldots;$
- $\mathcal{K}, \rho \models \langle \mathrm{B} \rangle\, \psi \ldots;$
- $\mathcal{K}, \rho \models \langle \mathrm{E} \rangle\, \psi \ldots;$
- inverse operators $\langle \overline{\mathrm{A}} \rangle$, $\langle \overline{\mathrm{B}} \rangle$, $\langle \overline{\mathrm{E}} \rangle$

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$\mathcal{K} \models \psi \iff$ for all *initial* traces ρ of $\mathcal{K}$, it holds that $\mathcal{K}, \rho \models \psi$

Possibly infinitely many traces!

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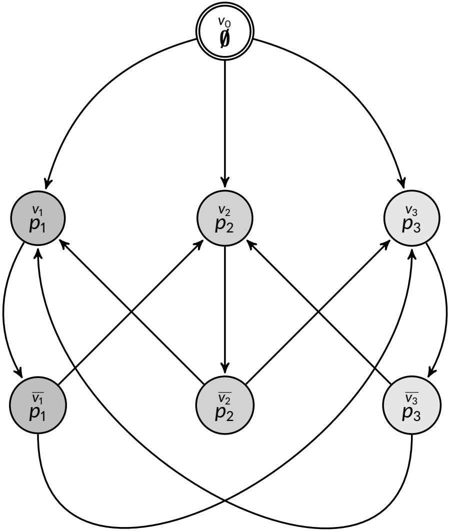
The Kripke structure $\mathcal{K}_{Sched}$ for a simple scheduler



A short account of $\mathcal{K}_{Sched}$

$\mathcal{K}_{Sched}$ models the behaviour of a scheduler serving 3 processes which are continuously requesting the use of a common resource
(easily generalizable to an arbitrary number of processes)

Initial state: v_0 (no process is served in that state)

In v_i and $\overline{v}_i$ the i-th process is served (p_i holds in those states)

The scheduler cannot serve the same process twice in two successive rounds:

- process i is served in state v_i, then, after "some time", a transition u_i from v_i to $\overline{v}_i$ is taken; subsequently, process i cannot be served again immediately, as v_i is not directly reachable from $\overline{v}_i$
- a transition r_j, with $j \neq i$, from $\overline{v}_i$ to v_j is then taken and process j is served

Some properties to be checked over $\mathcal{K}_{Sched}$

Validity of properties over all reachable computation intervals can be forced by modality $[E]$ (they are suffixes of at least one initial trace).

- In any computation interval of length at least 4, at least 2 processes are witnessed (YES: no process can be executed twice in a row)

  $$\mathcal{K}_{Sched} \models [E]\big(\langle \mathrm{E}\rangle^3 \top \rightarrow (\chi(p_1, p_2) \vee \chi(p_1, p_3) \vee \chi(p_2, p_3))\big),$$

  where $\chi(p, q) = \langle \mathrm{E}\rangle \langle \overline{\mathrm{A}}\rangle p \wedge \langle \mathrm{E}\rangle \langle \overline{\mathrm{A}}\rangle q$.

- In any computation interval of length at least 11, process 3 is executed at least once (NO: the scheduler can postpone the execution of a process ad libitum—starvation)

  $$\mathcal{K}_{Sched} \not\models [E](\langle \mathrm{E}\rangle^{10} \top \rightarrow \langle \mathrm{E}\rangle \langle \overline{\mathrm{A}}\rangle p_3).$$

- In any computation interval of length at least 6, all processes are witnessed (NO: the scheduler should be forced to execute them in a strictly periodic manner, which is not the case)

  $$\mathcal{K}_{Sched} \not\models [E](\langle \mathrm{E}\rangle^5 \rightarrow (\langle \mathrm{E}\rangle \langle \overline{\mathrm{A}}\rangle p_1 \wedge \langle \mathrm{E}\rangle \langle \overline{\mathrm{A}}\rangle p_2 \wedge \langle \mathrm{E}\rangle \langle \overline{\mathrm{A}}\rangle p_3)).$$

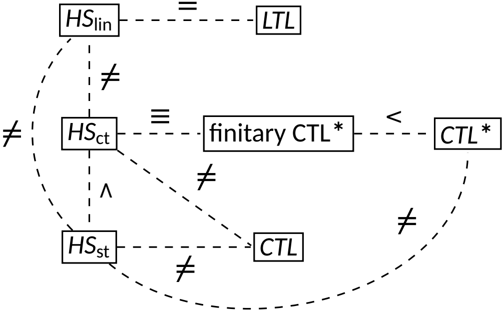
Complexity results

	Homogeneity
Full *HS*, BE	non-elementary **EXPSPACE**-hard
$A\bar{A}B\bar{B}\bar{E}$, $A\bar{A}E\bar{B}\bar{E}$	$\in$ **AEXP**$_{\text{Pol}}$ **PSPACE**-hard
$A\bar{A}\bar{B}\bar{E}$	**PSPACE**-complete
$A\bar{A}B\bar{B}$, $B\bar{B}$, $\bar{B}$, $A\bar{A}E\bar{E}$, $E\bar{E}$, $\bar{E}$	**PSPACE**-complete
$A\bar{A}B$, $A\bar{A}E$, AB, $\bar{A}E$	$\mathbf{P}^{\mathbf{NP}}$-complete
$A\bar{A}$, $\bar{A}B$, AE, A, $\bar{A}$	$\in \mathbf{P}^{\mathbf{NP}[O(\log^2 n)]}$ $\mathbf{P}^{\mathbf{NP}[O(\log n)]}$-hard
Prop, B, E	**co-NP**-complete

Complexity results

	Homogeneity	Regular expressions
Full *HS*, BE	non-elementary **EXPSPACE**-hard	non-elementary **EXPSPACE**-hard
$A\overline{A}B\overline{B}\overline{E}$, $A\overline{A}E\overline{B}\overline{E}$	$\in$ **AEXP**$_{\mathrm{Pol}}$ **PSPACE**-hard	**AEXP**$_{\mathrm{Pol}}$-complete
$A\overline{A}\overline{B}\overline{E}$	**PSPACE**-complete	$\in$ **AEXP**$_{\mathrm{Pol}}$ **PSPACE**-hard
$A\overline{A}B\overline{B}$, $B\overline{B}$, $\overline{B}$, $A\overline{A}E\overline{E}$, $E\overline{E}$, $\overline{E}$	**PSPACE**-complete	**PSPACE**-complete
$A\overline{A}B$, $A\overline{A}E$, AB, $\overline{A}E$	$\mathbf{P}^{\mathbf{NP}}$-complete	**PSPACE**-complete
$A\overline{A}$, $\overline{A}B$, AE, A, $\overline{A}$	$\in \mathbf{P}^{\mathbf{NP}[O(\log^2 n)]}$ $\mathbf{P}^{\mathbf{NP}[O(\log n)]}$-hard	**PSPACE**-complete
Prop, B, E	**co-NP**-complete	**PSPACE**-complete

Expressiveness results (under homogeneity)



Reference

L. Bozzelli, A. Molinari, A. Montanari, A. Peron, and P. Sala. Interval vs. Point Temporal Logic Model Checking: an Expressiveness Comparison. In *FSTTCS*, 2016

Sketch of **PSPACE**-hardness

- Reduction from the **PSPACE**-complete problem Periodic SAT
- We are given a Boolean formula $\varphi(x_1, \ldots, x_n, x_1^{+1}, \ldots, x_n^{+1})$ in CNF
- φ^j is φ in which we replace each x_i by a fresh x_i^j, and x_i^{+1} by x_i^{j+1}.
- Decide the satisfiability of the infinite-length formula

$$\Phi = \bigwedge_{j \geq 1} \varphi^j$$

(actually equivalent to $\Phi_f = \bigwedge_{j=1}^{2^{2n}+1} \varphi^j$).

Sketch of **PSPACE**-hardness

$\cdots$	$\$$	$x_1^\top$	$x_2^\top$	$x_3^\bot$	$x_4^\top$	$\tilde{\$}$	$\tilde{x_1}^\top$	$\tilde{x_2}^\bot$	$\tilde{x_3}^\top$	$\tilde{x_4}^\bot$	$\$$	$x_1^\bot$	$x_2^\top$	$x_3^\bot$	$x_4^\top$	$\tilde{\$}$	$\cdots$

φ^{j+1}

φ^j

For the t-th conjunct of φ,

$$
\begin{aligned}
o[y = \tilde{\$}] \rightarrow \Big(\bigvee_{x_i \in \Gamma \cap L_t^+} \exists o'[y = \tilde{x}_i^{\top}].o \leq^{\mathsf{e,s}}_{[0,4n]} o' \Big) \vee \\
\Big(\bigvee_{x_i^{+1} \in \Gamma^{+1} \cap L_t^+} \exists o'[y = x_i^{\top}].o \leq^{\mathsf{e,s}}_{[0,4n]} o' \Big) \vee \\
\Big(\bigvee_{x_i \in \Gamma \cap L_t^-} \exists o'[y = \tilde{x}_i^{\bot}].o \leq^{\mathsf{e,s}}_{[0,4n]} o' \Big) \vee \\
\Big(\bigvee_{x_i^{+1} \in \Gamma^{+1} \cap L_t^-} \exists o'[y = x_i^{\bot}].o \leq^{\mathsf{e,s}}_{[0,4n]} o' \Big) \vee \\
\exists o''[y = stop].o \leq^{\mathsf{e,s}}_{[0,2n]} o''.
\end{aligned}
$$

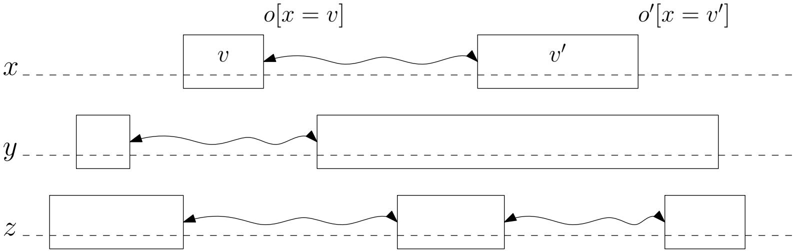
NP-completeness of the trigger-less case

- Timed automata give us (i) an exponential bound on the number of tokens of any plan and (ii) an exponential bound on the horizon.
- We start by reducing to integers all the rational values occurring in the instance.
- For every quantifier $o_i[x_i = v_i]$ in the rules, the algorithm guesses
    1. the integer part of the start and end time of the token for x_i to which o_i is mapped,
    2. an order of all fractional parts of such start/end times.

If we change the start/end time of (some of the) tokens associated with quantifiers, but we leave unchanged (i) *all the integer parts,* (ii) *zeroness/non-zeroness of fractional parts, and* (iii) *the fractional parts' order, satisfaction of atoms in the rules does not change.*

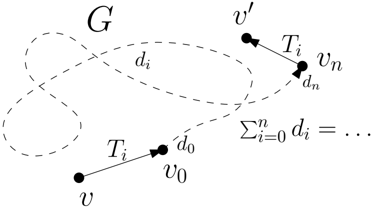
NP-completeness of the trigger-less case

- Now we have to check that there exists a legal timeline evolution "connecting" each pair of adjacent guessed tokens over the same variable



NP-completeness of the trigger-less case

- We interpret each state variable $x_i = (V_i, T_i, D_i)$ as a directed graph $G = (V_i, T_i)$ where D_i associates each $v \in V_i$ with a duration interval.
- For a pair of adjacent guessed tokens (x_i, v, d) and (x_i, v', d'):



- To this aim we guess a set of integers $\{\alpha_{u,v} \mid (u, v) \in T_i\}$ where $\alpha_{u,v}$ is the number of times the path traverses (u, v) and check that they specify a directed Eulerian path (in a multi-graph) $v_0 \rightsquigarrow v_n$.
- To check all this, we solve a linear problem. *(thanks G. Woeginger!)*

Theorem

TP with trigger-less rules is ***NP****-complete.*

NP-hardness: from existence of a Hamiltonian path in directed graph.