# Modal Event Calculi with Preconditions

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## Abstract

*Kowalski and Sergot's Event Calculus (EC) is a simple temporal formalism that, given a set of event occurrences, allows the derivation of the maximal validity intervals (MVIs) over which properties initiated or terminated by those events hold. The limited expressive power of EC is notably augmented by permitting events to initiate or terminate a property only if a given set of preconditions hold at their occurrence time. We define a semantic formalization of the Event Calculus with Preconditions. We gain further expressiveness by considering modal variants of this formalism, and show how to adapt our semantic characterization to encompass the additional operators. We discuss the complexity of MVI validation and describe examples showing that modal event calculi with preconditions can be successfully exploited to deal with real-world applications.*

**Keywords:** Reasoning about Actions and Events, Non-monotonic Reasoning,Modal Logic, Logic Programming.

## 1 Introduction

The *Event Calculus*, abbreviated *EC* [11], is a simple temporal formalism designed to model situations characterized by a set of *events*, whose occurrences have the effect of starting or terminating the validity of determined *properties*. Given a possibly incomplete description of when these events take place and of the properties they affect, *EC* is able to determine the *maximal validity intervals*, or *MVIs*, over which a property holds uninterruptedly. The algorithm *EC* relies on for the verification or calculation of MVIs is polynomial [6]. It can advantageously be implemented as a logic program. Indeed, the primitive operations of logic programming languages can be exploited to express boolean combinations of MVI computations and limited forms of quantification.

The range of queries that can be expressed in *EC* is too limited for modeling realistic situations, even when permitting boolean connectives. Expressiveness can be improved either by extending the representation capabilities of *EC* to encompass a wider spectrum of situations, or by enriching the query language of this formalism. In this paper, we explore both aspects relatively to a specific subclass of *EC* problems consisting of a fixed set of events that are known to have happened, but with incomplete information about the relative order of their occurrences [1, 2, 3, 7, 8, 12].

In many common situations, the occurrence of an event is no guaranty that a property is initiated or terminated. For example, turning the key in the ignition will start a car only if there is gasoline in the tank. In these situations, the effect of an event happening is tied to the validity of a number of properties, or *preconditions*, at its occurrence time. Computing MVIs in the presence of preconditions acquires a recursive flavor since an event initiates or terminates an MVI if and only if each of its preconditions is satisfied when it occurs, i.e. if and only if it occurs inside an MVI for each of its preconditions. Syntactic restrictions can however be imposed in order to ensure termination. In this case, the complexity of computing an MVI remains polynomial, but the exponent is a function of the nesting degree of the preconditions.

Even with the addition of preconditions, the MVIs derived by *EC* bear little relevance when only partial knowledge about event ordering is available. Indeed, in these situations, the acquisition of additional knowledge about the actual event ordering might both dismiss current MVIs and validate new MVIs [5]. It is instead critical to compute precise variability bounds for the MVIs of the (currently underspecified) actual ordering of events. Optimal bounds have been identified in the set of *necessary MVIs*, or $\Box$-*MVIs*, and the set of *possible MVIs*, or $\Diamond$-*MVIs*. They are the subset of the current MVIs that are not invalidated by the acquisition of new ordering information, and the set of intervals that are MVIs in at least one completion of the current ordering of events, respectively.

In [2], we defined a *Generalized Modal Event Calculus* (without preconditions), *GMEC*, that reduces the computation of $\Box$-MVIs and $\Diamond$-MVIs to the derivation of basic MVIs, mediated by the resolution of the operators $\Box$ and $\Diamond$ from the modal logic *K1.1*, a refinement of *S4* [15]. The query language of *GMEC* permits a free mixing of boolean connectives and modal operators, recovering the possibility of expressing a large number of common situations, but at the price of intractability: the resolution of a *GMEC* query is indeed an NP-hard problem [4, 8].

In this paper, we focus on the integration of modal operators and preconditions. This combination has received limited attention in the literature [7, 8], although the resulting calculus benefits from the added expressiveness of both features. Its computational complexity is however known to be beyond tractability.

The main contributions of this paper lie in the formalization for the first time of an *Event Calculus with Preconditions* (*PEC*), the extension of the resulting semantics to cope with modal operators (*PMEC* and *PGMEC*), and the formal analysis of the complexity of the calculi. We invite the interested reader to consult [8] for a more detailed discussion of the topics treated in this paper, and for the proofs of the statements we mention.

The paper is organized as follows. Section 2 first gives a formal account of *PEC* and of its semantics, and then extends it to *PGMEC* with a treatment of the modal operators and connectives of this formalism. Case studies drawn from the domains of medicine and fault diagnosis are described in Section 3. Section 4 proposes a complexity analysis for the calculi considered in this paper. Finally, Section 5 summarizes the main points of the paper and outlines directions of future work.

## 2 Modal Event Calculi with Preconditions

In this section, we first give a formalization of the syntax and semantics of *PEC* and adapt it to encompass *PGMEC*; then we present some relevant properties of these formalisms. Implementations in the language of hereditary Harrop formulas [9] have been given in [8], together with formal proofs of soundness and completeness with respect to the specifications below. We will analyze the complexity of these calculi in Section 4.

The *Event Calculus with Preconditions* (*PEC*) and its modal variants aim at modeling situations that consist of a set of events, whose occurrences over time have the effect of initiating or terminating the validity of properties when given preconditions are met. We formalize the time-independent aspects of a situation by means of a *PEC-structure*, defined as follows.

**Definition 2.1** (*PEC-structure*)

*A* structure *for the* Event Calculus with Preconditions (PEC-structure for short) *is a quadruple* $\mathcal{H} = (E,\ P,\ [\cdot|\cdot\rangle,\ \langle\cdot|\cdot])$ *such that:*

- $E = \{e_1, \ldots, e_n\}$ *and* $P = \{p_1, \ldots, p_m\}$ *are finite sets of* events *and* properties*, respectively. Elements of* $\mathbf{2}^P$ *are called* contexts *and the properties in them are referred to as* preconditions.
- $[\cdot|\cdot\rangle : P \times \mathbf{2}^P \to \mathbf{2}^E$ *and* $\langle\cdot|\cdot] : P \times \mathbf{2}^P \to \mathbf{2}^E$ *are respectively the* initiating *and* terminating map *of* $\mathcal{H}$*. For every property* $p \in P$*,* $[p|C\rangle$ *and* $\langle p|C]$ *represent the set of events that initiate and terminate* $p$*, respectively, in case all preconditions in* $C$ *hold at their occurrence time.* $\Box$

Traditional formulations of the Event Calculus, *EC* [11], also prescribe an exclusivity relation, which specifies which properties are not supposed to be holding at the same time. The presence of preconditions in *PEC* permits an easy emulation of the exclusivity relation [8]. On the other hand, in the absence of incompatible properties, an *EC* problem is modeled by a degenerated *PEC*-structure where all contexts are empty.

Unlike the original presentation of *EC* [11], we focus our attention on situations where the occurrence time of events is unknown. Indeed, we only assume the availability of incomplete information about the relative order according to which these events have happened. Therefore, we formalize the time-dependent aspects of a *PEC* problem by providing a *strict partial order* for the involved event occurrences. We write $W_{\mathcal{H}}$ for the set of all such orders over a *PEC*-structure $\mathcal{H}$ and use the letter $w$ to denote individual orderings, or *knowledge states*, in $W_{\mathcal{H}}$. Given $w \in W_{\mathcal{H}}$, we will sometimes call a pair of events $(e_1, e_2) \in w$ an *interval*. For reasons of efficiency, implementations generally represent the current situation $w$ as a *quasi-order* $o$, from which $w$ can be recovered as the transitive closure $o^+$ of $o$. We denote with $O_{\mathcal{H}}$ the set of all quasi-orders over $\mathcal{H}$; clearly $W_{\mathcal{H}} \subseteq O_{\mathcal{H}}$. In the following, we will often work with *extensions* of an ordering $w$, defined as any element of $W_{\mathcal{H}}$ that contains $w$ as a subset (recall that an order is a relation, i.e. a set of pairs). We define a *completion* of $w$ as any extension of this knowledge state that is a total order. We denote with $\mathrm{Ext}_{\mathcal{H}}(w)$ and $\mathrm{Comp}_{\mathcal{H}}(w)$ the set of all extensions and the set of all completions of the ordering $w$ in $W_{\mathcal{H}}$, respectively. We will drop the subscript $_{\mathcal{H}}$ when clear from the context.

Given a structure $\mathcal{H}$ and a knowledge state $w$, *PEC* offers means to infer the *maximal validity intervals*, or *MVIs*, over which a property $p$ holds uninterruptedly. We represent an MVI for $p$ as $p(e_i, e_t)$, where $e_i$ and $e_t$ are the events that initiate and terminate the interval, respectively. Consequently, we adopt as the *query language* of *PEC* the set $\mathcal{A}_{\mathcal{H}}$ of all such property-labeled intervals over $\mathcal{H}$. We interpret the elements of $\mathcal{A}_{\mathcal{H}}$ as propositional letters and the task performed by *PEC* reduces to deciding which of these formulas are MVIs and which are not, with respect to the current partial order of events.

In order for $p(e_1, e_2)$ to be an MVI relatively to the knowledge state $w$, $(e_1, e_2)$ must be an interval in $w$. Moreover, $e_1$ and $e_2$ must witness the validity of the property $p$ at the ends of this interval by initiating and terminating $p$, respectively, and by having all of their preconditions validated. These requirements are enforced by conditions $(i)$, $(ii)$ and $(iii)$, respectively, in the definition of valuation given below. The maximality requirement is caught by the meta-predicate $nb(p, e_1, e_2, w)$ in condition $(iv)$, which expresses the fact that the validity of an MVI must not be *broken* by any interrupting event. Any event $e$ which is known to have happened between $e_1$ and $e_2$ in $w$ and that initiates or terminates $p$ interrupts the validity of $p(e_1, e_2)$. These observations are formalized as follows.

**Definition 2.2** (*PEC-model*)

*Let $\mathcal{H} = (E,\ P,\ [\cdot|\cdot\rangle,\ \langle\cdot|\cdot])$ be a PEC-structure. An intended* PEC-model *of $\mathcal{H}$ is any propositional valuation $\upsilon_{\mathcal{H}} : W_{\mathcal{H}} \rightarrow 2^{\mathcal{A}_{\mathcal{H}}}$ defined in such a way that $p(e_1, e_2) \in \upsilon_{\mathcal{H}}(w)$ if and only if*

*i.* $(e_1, e_2) \in w$;

*ii.* $init(e_1, p, w)$, *where* $init(e_1, p, w)$ *iff*
$$\exists C \in \mathbf{2}^P.\ \forall q \in C.\ \exists e', e'' \in E.\quad e_1 \in [p|C\rangle \ \wedge\ q(e', e'') \in \upsilon_{\mathcal{H}}(w) \ \wedge\ (e', e_1) \in w \ \wedge\ ((e_1, e'') \in w \vee e_1 = e'')$$

*iii.* $term(e_2, p, w)$, *where* $term(e_2, p, w)$ *iff*
$$\exists C \in \mathbf{2}^P.\ \forall q \in C.\ \exists e', e'' \in E.\quad e_2 \in \langle p|C] \ \wedge\ q(e', e'') \in \upsilon_{\mathcal{H}}(w) \ \wedge\ (e', e_2) \in w \ \wedge\ ((e_2, e'') \in w \vee e_2 = e'');$$

*iv.* $nb(p, e_1, e_2, w)$, *where* $nb(p, e_1, e_2, w)$ *iff*
$$\neg\exists e \in E.\quad (e_1, e) \in w \ \wedge\ (e, e_2) \in w \ \wedge\ (init(e, p, w) \vee term(e, p, w)).\qquad \Box$$

Notice that the extremes of an interval are not treated symmetrically. This anomaly implements the intuition according to which a property does not hold yet when an event initiates it, while it must hold at the moment when a terminating event occurs.

The meta-predicates *init*, *term* and *nb* are mutually recursive in the above definition. In particular, an attempt at computing MVIs by simply unfolding their definition is non-terminating in pathological situations [8]. In general, a *PEC* problem can have zero or more models. However, most *PEC* problems encountered in practice satisfy syntactic conditions ensuring the termination of this procedure and the uniqueness of the model. This is particularly important since it permits the transcription of the above specification as a logic program that is guaranteed to terminate [8]. We need the following definition.

**Definition 2.3** (*Dependency Graph*)

*Let $\mathcal{H} = (E,\ P,\ [\cdot|\cdot\rangle,\ \langle\cdot|\cdot])$ be a PEC-structure. The* dependency graph *of $\mathcal{H}$, denoted by $G_{\mathcal{H}}$, consists of one node for each property in $P$, and contains the edge $(q, p)$ if and only if $\exists e \in E.\ \exists C \in 2^P.\ q \in C \wedge (e \in [p|C\rangle \vee e \in \langle p|C])$.* $\Box$

In the following, we will restrict our attention to those *PEC*-structures $\mathcal{H}$ such that $G_{\mathcal{H}}$ is acyclic. Under such an assumption, for every property $p \in P$, the length of the longest path to $p$ in $G_{\mathcal{H}}$ is finite. We denote it by $B_{\mathcal{H}}(p)$. Furthermore, we set $B_{\mathcal{H}} = \max_{p\in P} B_{\mathcal{H}}(p)$ and denote by $C_{\mathcal{H}}$ the cardinality of the largest context in $[\cdot|\cdot\rangle$ or $\langle\cdot|\cdot]$. It is worth noting that the above restriction ensures that the computation of any MVI on the basis of Definition 2.2 can never contain more than $B_{\mathcal{H}}$ embedded MVI calculations; therefore it always terminate, as formally stated in [8].

The set of MVIs of a *PEC* problem, defined as a pair $(\mathcal{H}, w)$, is not stable with respect to the acquisition of new ordering information. Indeed, as we move to an extension of $w$, current MVIs might become invalid and new MVIs can emerge [3]. The *Generalized Modal Event Calculus with Preconditions*, or *PGMEC*, extends the language of *PEC* with the possibility of enquiring about which MVIs will remain valid in every extension of the current knowledge state, and about which intervals might become MVIs in some extension of it. We call intervals of these two types *necessary MVIs* and *possible MVIs*, respectively. *PGMEC* interprets a necessary MVI $\Box p(e_1, e_2)$ and a possible MVI $\Diamond p(e_1, e_2)$ as the application of the operators $\Box$ and $\Diamond$, respectively, from an appropriate modal logic to the MVI $p(e_1, e_2)$. Boolean connectives are permitted as well. More formally, the query language of *PGMEC* is defined as follows.

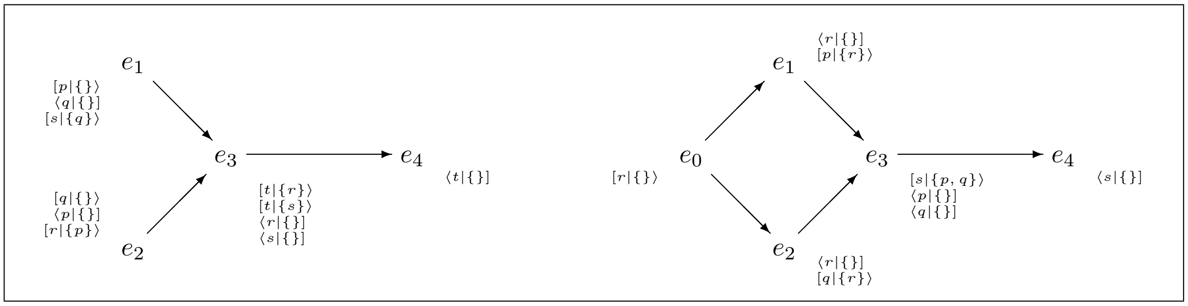


**Figure 1. Reasoning with Extensions versus Reasoning with Completions**

**Definition 2.4** (*PGMEC-language*)

*Let $\mathcal{H} = (E,\ P,\ [\cdot|\cdot\rangle,\ \langle\cdot|\cdot])$ be a PEC-structure. Given the PEC-language $\mathcal{A}_\mathcal{H} = \{p(e_1, e_2) : p \in P$ and $e_1, e_2 \in E\}$, the* PGMEC-language *of $\mathcal{H}$, denoted by $\mathcal{L}_\mathcal{H}$, is the modal language with propositional letters in $\mathcal{A}_\mathcal{H}$ and logical operators in $\{\neg, \wedge, \vee, \Box, \Diamond\}$.* □

In order to provide *PGMEC* with a semantics, we must shift the focus from the current knowledge state $w$ to all knowledge states that are reachable from $w$, i.e. $\text{Ext}_\mathcal{H}(w)$, and more generally to $W_\mathcal{H}$. Now, by definition, $w'$ is an extension of $w$ if $w \subseteq w'$. Since $\subseteq$ is a non-strict order, $(W_\mathcal{H}, \subseteq)$ can be naturally viewed as a finite, reflexive, transitive and antisymmetric modal frame. If we consider this frame together with the straightforward modal extension of the valuation $\upsilon_\mathcal{H}$ to an arbitrary knowledge state, we obtain a modal model for *PGMEC*.

**Definition 2.5** (*PGMEC-model*)

*Let $\mathcal{H} = (E,\ P,\ [\cdot|\cdot\rangle,\ \langle\cdot|\cdot])$ be a PEC-structure. The PGMEC*-frame *$\mathcal{F}_\mathcal{H}$ of $\mathcal{H}$ is the frame $(W_\mathcal{H}, \subseteq)$. The* intended *PGMEC*-model *of $\mathcal{H}$ is the modal model $\mathcal{I}_\mathcal{H} = (W_\mathcal{H}, \subseteq, \upsilon_\mathcal{H})$, where the propositional valuation $\upsilon_\mathcal{H} : W_\mathcal{H} \to 2^{\mathcal{A}_\mathcal{H}}$ is defined as in Definition 2.2. Given $w \in W_\mathcal{H}$ and $\varphi \in \mathcal{L}_\mathcal{H}$, the truth of $\varphi$ at $w$ with respect to $\mathcal{I}_\mathcal{H}$, denoted by $\mathcal{I}_\mathcal{H}; w \models \varphi$, is defined as follows:*

$\mathcal{I}_\mathcal{H}; w \models p(e_1, e_2)$ *iff* $p(e_1, e_2) \in \upsilon_\mathcal{H}(w)$;
$\mathcal{I}_\mathcal{H}; w \models \neg\varphi$ *iff* $\mathcal{I}_\mathcal{H}; w \not\models \varphi$;
$\mathcal{I}_\mathcal{H}; w \models \varphi_1 \wedge \varphi_2$ *iff* $\mathcal{I}_\mathcal{H}; w \models \varphi_1$ *and* $\mathcal{I}_\mathcal{H}; w \models \varphi_2$;
$\mathcal{I}_\mathcal{H}; w \models \varphi_1 \vee \varphi_2$ *iff* $\mathcal{I}_\mathcal{H}; w \models \varphi_1$ *or* $\mathcal{I}_\mathcal{H}; w \models \varphi_2$;
$\mathcal{I}_\mathcal{H}; w \models \Box\varphi$ *iff* $\forall w' \in \text{Ext}_\mathcal{H}(w).\ \mathcal{I}_\mathcal{H}; w' \models \varphi$;
$\mathcal{I}_\mathcal{H}; w \models \Diamond\varphi$ *iff* $\exists w' \in \text{Ext}_\mathcal{H}(w).\ \mathcal{I}_\mathcal{H}; w' \models \varphi$.

*A PGMEC-formula $\varphi$ is* valid *in $\mathcal{I}_\mathcal{H}$, written $\mathcal{I}_\mathcal{H} \models \varphi$, if $\mathcal{I}_\mathcal{H}; w \models \varphi$ for all $w \in W_\mathcal{H}$.* □

We will drop the subscripts $_\mathcal{H}$ whenever this does not lead to ambiguities. Moreover, given a knowledge state $w$ in $W_\mathcal{H}$ and a *PGMEC*-formula $\varphi$ over $\mathcal{H}$, we write $w \models \varphi$ for $\mathcal{I}_\mathcal{H}; w \models \varphi$. Similarly, we abbreviate $\mathcal{I}_\mathcal{H} \models \varphi$ as $\models \varphi$.

In the following, we will also consider a simple linguistic fragment of *PGMEC*, called *PMEC*, consisting of the class of formulas $B_\mathcal{H} = \{\varphi, \Box\varphi, \Diamond\varphi : \varphi \in A_\mathcal{H}\}$, that we will show to be sufficiently expressive to model significant application domains.

We conclude this section by showing that reasoning about the extensions of a given partial order is not the same as reasoning about its completions. However, it is possible to show that completions can be modally defined in terms of extensions [8]. This result will be exploited in Section 4 to prove the complexity of the proposed calculi.

Consider the following example (Figure 1, left). Let $e_1$, $e_2$, $e_3$ and $e_4$ be four events, and $p$, $q$, $r$, $s$, $t$ be five properties. Suppose that $e_1$ initiates $p$ and terminates $q$ without preconditions, while it initiates $s$ with precondition $q$; $e_2$ initiates $q$ and terminates $p$, while it initiates $r$ with precondition $p$; $e_3$ terminates both $r$ and $s$ and initiates $t$ if at least one among $r$ and $s$ holds. Finally, $e_4$ terminates $t$. Consider a scenario $o$ according to which $e_1$ precedes $e_3$, $e_2$ precedes $e_3$, the relative order of $e_1$ and $e_2$ is unknown and $e_3$ precedes $e_4$. Under these hypotheses, $t(e_3, e_4)$ holds in every completion of $o$, but it does not hold in $o$ itself. Thus it does not hold in every extension of the current state.

The next example (Figure 1, right) describes a dual situation using a similar symbology. Here, $s(e_3, e_4)$ holds in the current state of knowledge $o$, but it does not hold in any of its completion. This means that there exists one extension ($o$ itself) in which $s(e_3, e_4)$ holds, while there exist no completion in which it holds.

## 3 Modeling Real-World Examples

In this section, we consider two examples, taken from the domains of hardware and medical diagnosis, respectively, that show how modal event calculi with preconditions can be successfully exploited to deal with real-world applications.

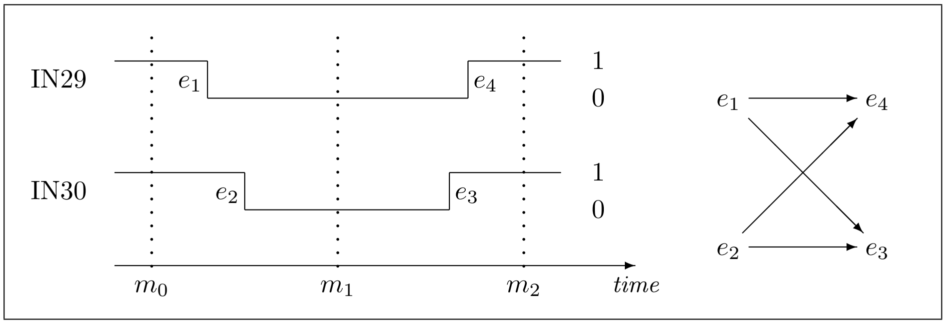


**Figure 2. Expected Register Behavior, Measurements and Resulting Event Ordering**

### 3.1 Diagnosis of a faulty CNCC

We focus our attention on the representation and processing of information about fault symptoms that is spread out over periods of time and for which current expert system technology is particularly deficient [14]. Consider the following example, which diagnoses a fault in a computerized numerical control center (CNCC) for a production chain.

> *A possible cause for an undefined position of a tool magazine is a faulty limit switch $S$. This cause can be ruled out if the status registers IN29 and IN30 of the control system show the following behavior: at the beginning both registers contain the value* 1*. Then IN29 drops to* 0*, followed by IN30. Finally, both return to their original values in the reverse order.*

Figure 2 describes a possible sequence of transitions, for *IN29* and *IN30*, that excludes the eventuality of $S$ being faulty. In order to verify this behavior, the contents of the status registers must be monitored over time. Typically, measurements are made at fixed intervals, asynchronously with respect to the update of status registers. While measurements can be taken frequently enough to guarantee that signal transitions are not lost, it is generally impossible to exactly locate the instants at which a register changes its value. Consequently, it is possible that several transitions take place between two measurements, making it impossible to recover their relative order. In the case of our example, the situation is depicted in Figure 2 (left): dotted lines indicate measurements. Moreover, we have given names to the individual transitions of state of the different registers. From the values found at measurements $m_0$ and $m_1$, we can conclude that both *IN29* and *IN30* were reset during this interval (transitions $e_1$ and $e_2$, respectively), but we have no information about their relative ordering. Similarly, measurement $m_2$ informs us that the registers assumed again the value 1 (transitions $e_3$ and $e_4$), but we do not know which was set first. The available ordering information is reported on the right-hand side of Figure 2.

The situation displayed in Figure 2 can be represented by the *PMEC*-structure $\mathcal{H} = (E,\ P,\ [\cdot|\cdot\rangle,\ \langle\cdot|\cdot])$, whose components are defined as follows:

- $E = \{e_1, e_2, e_3, e_4\}$;
- $P = \{one29, zero29, one30, zero30\}$;
- $\{e_1\} = [zero29|\{\}\rangle,\ \{e_2\} = [zero30|\{zero29\}\rangle,$ $\{e_3\} = [one30|\{\}\rangle,\quad \{e_4\} = [one29|\{\}\rangle$;
- $\{e_1\} = \langle one29|\{\}],\qquad \{e_2\} = \langle one30|\{\}],$ $\{e_3\} = \langle zero30|\{zero29\}],\ \{e_4\} = \langle zero29|\{\}]$.

We have represented transitions as events with the same name, and used mnemonic names for the properties corresponding to the two different values of *IN29* and *IN30*. It is easy to check that the dependency graph for $\mathcal{H}$ does not contain any loop.

It is worth noting that, in general, preconditions do not imply physical sequentiality. As an example, we state that the event $e_2$ initiates the property $zero30$ only if the property $zero29$ holds to express the fact that we are only interested in those situations where *IN30* is reset while *IN29* holds the value 0. In such a way, we are able to a priori eliminate a number of incorrect behaviors.

The partial order of transitions, described in Figure 2 (right), is captured by the following (current) knowledge state:

$$o = \{(e_1, e_3), (e_1, e_4), (e_2, e_3), (e_2, e_4)\}.$$

Let us consider the PEC-formula:

$$\varphi = zero30(e_2, e_3).$$

In order to verify that the switch $S$ is not faulty, we must ensure that the registers $IN29$ and $IN30$ display the expected behavior in all refinements of the current knowledge state $o$. With our encoding, this amounts to proving that the *PMEC*-formula $\Box\varphi$ holds in $o$. If

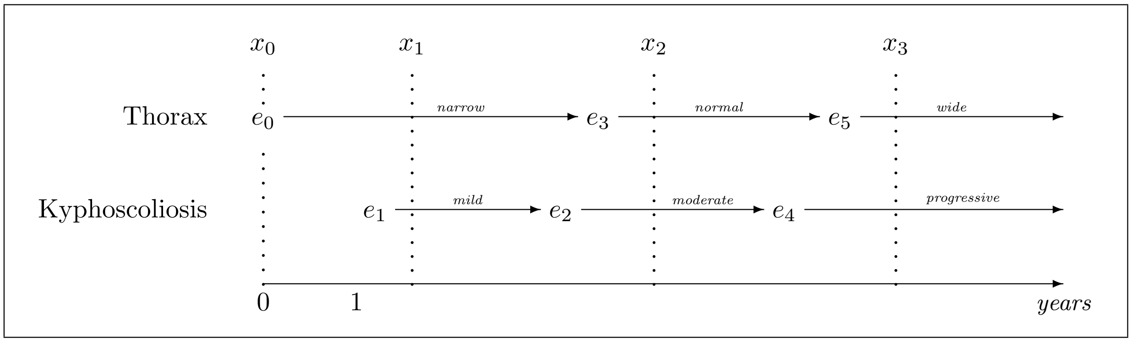


**Figure 3. Expected Symptom Evolution for Metatropic Dwarfism**

this is the case, the fault is to be excluded. If we want to determine the existence of at least one extension of $o$ where the registers behave correctly, we must verify the satisfiability of the *PMEC*-formula $\Diamond\varphi$ in $o$. If this is not the case, the fault is certain. Since we have that $o^+ \models \Diamond\varphi$ and $o^+ \not\models \Box\varphi$, the knowledge available in $o$ entitles us to assert that the fault is possible, but not certain. Assume that, unlike in the actual situation of Figure 2, we extend $o$ with the pair $(e_2, e_1)$. Let us denote the resulting state by $o_1$. It holds that $o_1^+ \not\models \Diamond\varphi$, and thus the switch $S$ is certainly faulty. On the other hand, if we refine $o$ with the pairs $(e_1, e_2)$ and $(e_3, e_4)$, calling $o_2$ the resulting state, we have that $o_2^+ \models \Box\varphi$. In this case the fault can be excluded.

### 3.2 Diagnosis of the Metatropic Dwarfism

As a second example, consider the following situation of illnesses taken from the domain of diagnosis of skeletal dysplasias [10].

> *The model of the Metatropic Dwarfism specifies that at birth the thorax is narrow and after the first year of age a mild kyphoscoliosis occurs. If the severity of the kyphoscoliosis is relatively mild then the thorax will continue to be narrow. If the severity of the kyphoscoliosis increases then there be a period during which the thorax is perceived as relatively normal but when the kyphoscoliosis is progressive the thorax becomes wide. Metatropic Dwarfism can be excluded if the symptoms do not comply to this model.*

Figure 3 schematizes the evolution of a patient to be diagnosed with Metatropic Dwarfism. Both kyphoscoliosis severity and thorax width are continuous attributes, but radiologists are only interested in a finite set of discrete qualitative values (*narrow*, *normal*, and *wide* for the thorax; *mild*, *moderate*, and *progressive* for the scoliosis), and hence only the events which mark the transitions from one qualitative value to the next one are significant. In order to verify this model, the width of the thorax and the severity of the kyphoscoliosis must be checked over time. However, as in the case of measurements of status registers, while the radiological examinations can be done frequently enough to guarantee that qualitative value transitions are not lost, it is generally impossible to exactly locate the instants at which these transitions happen. Consequently, it is possible that several transitions take place between two examinations making it impossible to recover their relative order. In the case of our example, the situation is depicted in Figure 3. Exams $x_0$ and $x_1$ tell us respectively that at birth the thorax was narrow and that after the first year a mild kyphoscoliosis had developed. We denote with $e_0$ and $e_1$ the corresponding events. With exam $x_2$, we observe that the thorax is now normal and the kyphoscoliosis has become moderate. We write $e_3$ and $e_2$ for the corresponding events. We know that they have occurred after $e_1$, but we have no information about their relative ordering. Finally, exam $x_3$ informs us that the thorax has successively become wide and the kyphoscoliosis progressive. Let $e_5$ and $e_4$ be the corresponding causing events. Again, we know they have happened after $e_2$ and $e_3$, however we are not able to order them.

The situation displayed in Figure 3 can be represented by the *PGMEC*-structure $\mathcal{H} = (E,\ P,\ [\cdot|\cdot\rangle,\ \langle\cdot|\cdot])$, whose components are defined as follows:

- $E = \{e_0, e_1, e_2, e_3, e_4, e_5, e_6\}$;
- $P = \{narrow, normal, wide,$
  $mild, moderate, progressive\}$;
- $\{e_0\} = [narrow|\{\}\rangle$, $\{e_1\} = [mild|\{\}\rangle$,
  $\{e_2\} = [moderate|\{\}\rangle$,
  $\{e_3\} = [normal|\{moderate\}\rangle$,
  $\{e_4\} = [progressive|\{\}\rangle$,
  $\{e_5\} = [wide|\{progressive\}\rangle$;

- $\{e_2\} = \langle mild|\{\}]$, $\{e_3\} = \langle narrow|\{\}]$, $\{e_4\} = \langle moderate|\{\}]$, $\{e_5\} = \langle normal|\{\}]$, $\{e_6\} = \langle wide|\{\}] = \langle progressive|\{\}]$.

We have added the event $e_6$ in order to terminate the validity of the properties *wide* and *progressive*; it corresponds to the death of the patient. As in the previous example, our use of preconditions is instrumental to the inferences we want to achieve. Finally, observe that the dependency graph for $\mathcal{H}$ does not contain loops. The partial order of transitions, described in Figure 3, is captured by the following (current) knowledge state:

$$o = \{(e_0, e_1), (e_1, e_2), (e_1, e_3), (e_2, e_4), (e_2, e_5), (e_3, e_4), (e_3, e_5), (e_4, e_6), (e_5, e_6)\}.$$

Consider the *PGMEC*-formula:

$$\varphi = normal(e_3, e_5) \wedge wide(e_5, e_6).$$

In order to verify that the diagnosis of the dysplasia is certain, we must ensure that the *PGMEC*-formula $\Box\varphi$ is satisfiable in $o$. If we want to determine if it is possible to diagnose the dysplasia, we must verify the satisfiability of the *PGMEC*-formula $\Diamond\varphi$ in $o$. Since we have that $o^+ \models \Diamond\varphi$ and $o^+ \not\models \Box\varphi$, the knowledge contained in $o$ entitles us to assert that the diagnosis of the dysplasia is possible, but not certain. Assume that, unlike the actual situation of Figure 3, we extend $o$ with the pair $(e_3, e_2)$. Let us denote the resulting state with $o_1$. It is easy to prove that $o_1^+ \not\models \Diamond\varphi$, and thus that the dysplasia can be excluded. On the other hand, if we refine $o$ with the pairs $(e_2, e_3)$ and $(e_4, e_5)$, calling $o_2$ the resulting state, we have that $o_2^+ \models \Box\varphi$. In this case, the dysplasia is certain.

## 4 Complexity Analysis

In this section, we study the complexity of the event calculi presented in Section 2. We model our analysis around the satisfiability relation given in Definitions 2.2 and 2.5. We measure the complexity of testing whether $\mathcal{I}_\mathcal{H}; w \models \varphi$ holds in terms of the size of the input structure (e.g. the number $n$ of events it includes). It is worth noting that, although possible in principle, it is disadvantageous in practice to implement knowledge states so that the test $(e_1, e_2) \in w$ has constant cost. We instead maintain a quasi-order $o$ on events whose transitive closure $o^+$ is $w$. Verifying whether $(e_1, e_2) \in w$ holds becomes a reachability problem in $o$ and it can be solved in quadratic time $O(n^2)$ [6].

Given a *PEC*-structure $\mathcal{H}$, a knowledge state $w \in W_\mathcal{H}$ and a *PEC*-formula (resp. *PGMEC*-formula) $\varphi$, we want to characterize the complexity of the problem of establishing whether $\mathcal{I}_\mathcal{H}; w \models \varphi$ is valid, an instance of the general problem of model checking. We call the triple $(\mathcal{H}, w, \varphi)$ an *instance* and generally prefix this term with the name of the calculus we are considering. In the following, we will show that, given an instance $(\mathcal{H}, w, \varphi)$, the satisfiability test for $\varphi$ is polynomial in *EC* and *PEC*, while it is NP-hard in *PGMEC*.

Given an *EC*-instance $(\mathcal{H}, w, \varphi)$, the cost of the test $w \models \varphi$ can be derived to be $O(n^3)$ directly from the relevant parts of Definition 2.2, as proved in [6]. In particular, we assume that verifying the validity of the propositions $e \in [p|C\rangle$ and $e \in \langle p|C]$ when the context $C$ is empty (as it is in basic *EC*) has constant cost $O(1)$, for given event $e$ and property $p$. This is not true anymore in *PEC*. Let $(\mathcal{H}, w, \varphi)$ be a *PEC*-instance such that the dependency graph of $\mathcal{H}$ is acyclic. The cost of the test $w \models \varphi$ is still polynomial in the number $n$ of events, but depends on $B_\mathcal{H}$ (see Definition 2.3).

**Theorem 4.1** (*Cost of model checking in PEC*)

*Given a PEC-instance* $(\mathcal{H}, w, \varphi)$*, the test* $w \models \varphi$ *has cost* $O(n^{3\cdot(B_\mathcal{H}+1)})$.

**Proof.** We proceed by induction on the value of $B_\mathcal{H}$. If $B_\mathcal{H} = 0$, then we fall in the case of *EC*, whose complexity has been shown to be $O(n^3)$. When $B_\mathcal{H} > 0$, the evaluation of each of the $O(n)$ meta-predicate *init* or *term* of Definition 2.2 results in the evaluation of at worst $C_\mathcal{H}$ preconditions and then the evaluation of each of these conditions results in $O(n^2)$ satisfiability tests with a $B_\mathcal{H} - 1$ nesting level. The relationship between the complexities $Comp(B_\mathcal{H})$ and $Comp(B_\mathcal{H} - 1)$ is expressed by the following recurrent expression:

$$Comp(B_\mathcal{H}) = O(n) \cdot O(n^2) \cdot Comp(B_\mathcal{H} - 1).$$

By induction hypothesis, $Comp(B_\mathcal{H}) = O(n^3) \cdot O(n^{3\cdot B_\mathcal{H}})$, and hence $Comp(B_\mathcal{H}) = O(n^{3\cdot(B_\mathcal{H}+1)})$. ■

Let us prove now that if we extend *PEC* with modal operators and boolean connectives, the resulting calculus *PGMEC* is NP-hard. To this end, we consider the simple linguistic fragment of *PGMEC* we called *PMEC*. To determine the complexity of the satisfiability test in *PMEC*, we can exploit Dean and Boddy's results reported in [7]. They consider the problem of computing which facts must be or may possibly be true over certain time intervals with respect to the set of *completions* of the current partial order in a framework including preconditions but devoid of propositional connectives. They showed that this computation is NP-hard in the general case. *PMEC* tests the satisfiability of a *PMEC*-formula with respect to the set of *extensions* of the current partial order. In Section 2, we showed that the approach that considers all the extensions is more general than the one that restricts itself to completions. It is easy to show that there exists a polynomial reduction of Dean and Boddy's problem to the satisfiability problem in *PMEC*. Hence, the satisfiability problem in *PMEC* is at least as complex as a problem that Dean and Boddy proved to be NP-hard. From this result, it immediately follows that the satisfiability test in *PGMEC* is NP-hard.

**Corollary 4.2** (*Cost of model checking in PGMEC*)

*Given a PGMEC-instance* $(\mathcal{H}, w, \varphi)$*, the satisfiability test* $w \models \varphi$ *is NP-hard.* □

## 5 Conclusions

In this paper we studied the expressiveness and complexity of extensions of *EC* with preconditions and modal operators. We also extensively discussed the application of the resulting modal event calculi with preconditions to two real-world examples.

Elsewhere [2, 4, 13], we systematically investigated modal extensions of *EC* without preconditions. In particular, we considered the modal event calculi *MEC* and *GMEC* that are obtained from *PMEC* and *PGMEC*, respectively, by substituting *EC* for *PEC*, that is, by making the effects of event occurrences context-independent. We proved that model checking in *MEC* has cost $O(n^3)$, while it is NP-hard for *GMEC*. Moreover, the attempt of characterizing *GMEC* within the rich taxonomy of modal logics reveals *Sobocinski logic*, also known as *system K1.1* [15], as its closest relative. This characterization allowed us to establish a number of interesting logical properties of *GMEC* that have been used to improve the efficiency of its implementations. In [8] we have shown that *PGMEC* inherits most of these logical properties.

We are currently investigating the interplay between preconditions, modal operators, and boolean connectives. We already know that the addition of preconditions to *MEC* makes the resulting calculus, *PMEC*, NP-hard. In this respect, we are looking for both polynomial approximations to MVI computation and expressive sublanguages that admit exact polynomial procedures for this task. Instead, the problem of characterizing the relationships between boolean connectives and preconditions in a modal framework is still open. More precisely, we do not know if the addition of boolean connectives to *PMEC* or of preconditions to *GMEC* makes the resulting calculus, *PGMEC*, (strictly) more expressive than *PMEC* and *GMEC*, respectively. The complexity results we obtained are compatible with all the alternatives.

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