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# Deriving Barbed Bisimulations for Bigraphical Reactive Systems

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**Abstract:** We study the definition of a general abstract notion of barbed bisimilarity for reactive systems on bigraphs. More precisely, given a bigraphical reactive system, we define the corresponding barbs from the contextual labels given by the IPO construction, in a general and systematic way. These barbs correspond to observe which names on the interface are actually involved in reactions (and how).

As examples, we apply this construction to the (bigraphical representation of the)  $\pi$ -calculus and of Mobile Ambients, and compare the resulting barbed equivalences with those previously known for these calculi.

**Keywords:** Bigraphs, barbed bisimulations,  $\pi$ -calculus, mobile ambients.

## 1 Introduction

*Bigraphical Reactive Systems* (BRSs) [12] have been proposed as a promising meta-model for concurrent, communicating, mobile systems. They have been successfully used for representing many calculi and models, e.g. CCS,  $\pi$ -calculus, Petri Nets, Mobile Ambients, Fusion calculus among others [8, 11, 2, 5]. BRSs owe much of their expressivity to the fact that their states are *bigraphs*, semi-structured data which can represent at once both the (physical, logical) location and the connections of the components of a system. The dynamics of the system is represented by a set of rewrite rules on this semi-structured data.

An important property of BRSs is that they support the systematic definition of labelled transition systems via so-called *IPO construction* [8], which identifies the labels for a given agent as the *minimal* contexts which trigger a transition. Notably, the strong bisimilarity induced by this LTS is always a congruence, and this has been extended also to weak bisimilarity [6].

In this paper, we continue this line of research by defining a general abstract notion of *barbed bisimilarity* for BRSs. Barbed bisimilarity is interesting for many reasons; e.g., its definition does not rely on a LTS, and usually barbed bisimilarity is coarser than strong bisimilarity.

Intuitively, barbed bisimilarity compares agents by looking only at the channels, or *barbs*, which they expose to the surrounding environment. This turns out to be sufficiently expressive, for calculi whose reactions are essentially triggered by names. However, there is no general and systematic way for identify which names of an agent have to be observed, and which is their role; instead, barbs are usually defined by means of some *ad hoc* intensional definition on the syntax of the calculus; e.g., in the  $\pi$ -calculus, barbs are the subjects of “active” prefixes; in Mobile Ambients, they are the names of top-level ambients.

To overcome this problem, in this paper we propose a general, systematic approach to define barbs for a bigraphic reactive system. We aim to formalize the intuitive idea that “barbs are the

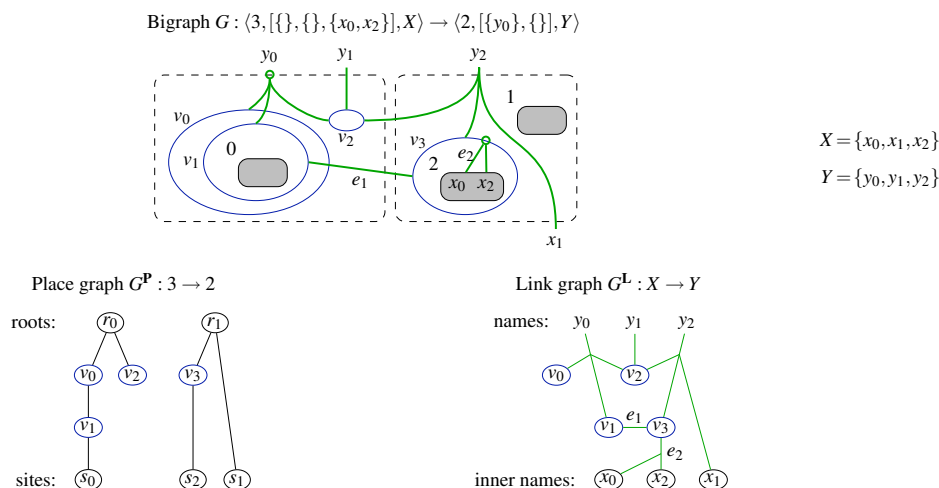


Figure 1: A binding bigraph (picture taken from [1]).

names of an agent, which are involved in reactions”, by using the contextual labels given by the IPO construction: for each agent, we examine only its IPO labels for finding out the names which are effectively used in reactions. This leads to two notions of barb, *IPO barb* and *decorated IPO barb*, which differ on the information about names we extract from IPO labels: in the decorated version, we take advantage of the link and place graphs of the IPO labels to observe also the “role” that each name has in reactions. Notice that we use the IPO labels only for defining barbs, and not the bisimilarity itself which instead relies on the reaction relation only.

In Section 2, after having recalled the basic definitions about binding bigraphs, we introduce the two barbed bisimilarities for binding bigraphs (Definitions 8 and 9). Then, in Sections 3 and 4 we apply this theory to the  $\pi$ -calculus and Mobile Ambients, respectively. For each calculus, we give its encoding as BRS, calculate the IPO labels, and compare the resulting IPO barbed bisimilarity with the standard barbed bisimilarities of these calculi. It turns out that in the case of  $\pi$ -calculus, IPO barbed bisimilarity is finer than standard barbed bisimilarity, but IPO barbed bisimilarity and standard barbed bisimilarity coincide (Proposition 5). On the other hand, in the case of Mobile Ambients, IPO barbed bisimilarities and congruence are finer than their standard counterparts; the reason is that IPO labels allow to observe names also used in capabilities, and not only names of top-level ambients.

## 2 Barbed bisimilarities for Binding Bigraphs

### 2.1 Binding Bigraphs

In this section we recall Milner’s *binding bigraphs*<sup>1</sup>. Intuitively, a binding bigraph represents an open system, so it has an inner and an outer interface to “interact” with subsystems and the surrounding environment, see Figure 1. The *widths* of the interfaces describe the *roots* in the

<sup>1</sup> Binding bigraphs generalize pure bigraphs for dealing with calculi with binders such as  $\pi$ -calculus. The results presented in this papers hold also in the particular case of pure bigraphs, which suffice for e.g. Mobile Ambients.

outer interface (that is, the various locations where the nodes live) and the *sites* in the inner interface (that is, the holes where other bigraphs can be inserted). On the other hand, the *names* in the interfaces describes the free links, that is end points where links from the outside world can be pasted, creating new links among nodes. We refer the reader to [8, 9] for more details.

Let  $\mathcal{K}$  be a *binding signature* of controls, and  $ar: \mathcal{K} \rightarrow \mathbb{N} \times \mathbb{N}$  be the arity function. The arity pair  $(h, k)$  (written  $h \rightarrow k$ ) consists of the *binding arity*  $h$  and the *free arity*  $k$ , indexing respectively the binding and the free ports of a control.

**Definition 1** A *binding interface* is  $\langle m, loc, X \rangle$ , where  $m$  is a finite ordinal (called *width*),  $X$  is a finite set of names, and  $loc: X \rightarrow m \uplus \{\perp\}$  is a *locality map* associating some of the names  $X$  with a site in  $m$ . If  $loc(x) = s$  then  $x$  is *located* at  $s$ , or *local* to  $s$ ; if  $loc(x) = \perp$  then  $x$  is *global*.

We shall some times represent the locality map as a vector  $\vec{X} = (X_0, \dots, X_{m-1})$  of disjoint subsets, where  $X_s$  is the set of names local to  $s$ ; thus  $X \setminus \vec{X}$  are the global names. We call an interface *local* (resp. *global*) if all its names are local (resp. global).

**Definition 2** A *binding bigraph*  $G: \langle m, loc, X \rangle \rightarrow \langle n, loc', Y \rangle$  is defined as a (pure) bigraph  $G^u: \langle m, X \rangle \rightarrow \langle n, Y \rangle$  satisfying certain locality conditions (see below).

$G^u$  is defined by composing a *place graph*  $G^P$ , describing the nesting of nodes, and a *link graph*  $G^L$ , describing the (hyper-)links among nodes.

$$\begin{aligned} G^u &= (V, E, ctrl, G^P, G^L): \langle m, X \rangle \rightarrow \langle n, Y \rangle && \text{(pure bigraph)} \\ G^P &= (V, ctrl, prnt): m \rightarrow n && \text{(place graph)} \\ G^L &= (V, E, ctrl, link): X \rightarrow Y && \text{(link graph)} \end{aligned}$$

where  $V, E$  are the sets of nodes and edges respectively,  $ctrl: V \rightarrow \mathcal{K}$  is the *control map*, which assigns a control to each node,  $prnt: m \uplus V \rightarrow V \uplus n$  is the (acyclic) *parent map*,  $P = \sum_{v \in V} \pi_1(ar(ctrl(v)))$  is the set of ports and  $B = \sum_{v \in V} \pi_2(ar(ctrl(v)))$  is the set of bindings (associated to all nodes), and  $link: X \uplus P \rightarrow E \uplus B \uplus Y$  is the *link map*.

The locality conditions are the following:

1. if a link is bound, then its inner names and ports must lie within the node that binds it;
2. if a link is free, with outer name  $x$ , then  $x$  must be located in every region that contains any inner name or port of the link.

**Definition 3** The category of binding bigraphs over a signature  $\mathcal{K}$  ( $\text{BBG}(\mathcal{K})$ ) has local interfaces as objects, and binding bigraphs as morphisms.

Given two local bigraphs  $G: \langle m, loc, X \rangle \rightarrow \langle n, loc', Y \rangle$ ,  $H: \langle n, loc', Y \rangle \rightarrow \langle k, loc'', Z \rangle$ , the composition  $H \circ G: \langle m, loc, X \rangle \rightarrow \langle k, loc'', Z \rangle$  is defined by composing their place and link graphs:

1. the composition of  $G^P: m \rightarrow n$  and  $H^P: n \rightarrow k$  is define as

$$H^P \circ G^P = (V_G \uplus V_H, ctrl_G \uplus ctrl_H, (id_{V_G} \uplus prnt_H) \circ (prnt_G \uplus id_{V_H})): n \rightarrow k;$$

2. the composition of  $G^L: X \rightarrow Y$  and  $H^L: Y \rightarrow Z$  is defined as

$$H^L \circ G^L = (V_G \uplus V_H, E_G \uplus E_H, ctrl_G \uplus ctrl_H, (id_{E_G} \uplus link_H) \circ (link_G \uplus id_{P_H})): X \rightarrow Z.$$

An important operation about (bi)graphs, is the *tensor product*. Intuitively, the tensor product of  $G: \langle m, loc, X \rangle \rightarrow \langle n, loc'', Y \rangle$  and  $H: \langle m', loc', X' \rangle \rightarrow \langle n', loc''', Y' \rangle$ , is a bigraph  $G \otimes H: \langle m + m', loc \uplus loc', X \uplus X' \rangle \rightarrow \langle n + n', loc'' \uplus loc''', Y \uplus Y' \rangle$  is defined when  $X \cap X' = Y \cap Y' = \emptyset$  and it obtained by putting “side by side”  $G$  and  $H$ , without merging any root nor any name in the interfaces. It is easy to check that composition and tensor preserve the locality conditions.

Two useful variant of tensor product can be defined using tensor and composition: the *parallel product*  $\parallel$ , which merges shared global names between two bigraphs, and the *prime product*  $|$ , that moreover merges all roots in a single one. Due to lack of space, we refer the reader to [9].

## 2.2 Bigraphical reactive systems

Bigraphical reactive systems (BRSs) are reactive systems in the sense of [10], built over binding bigraphs [8]. A BRS consists of a set of bigraphical rewrite rules, together with a definition of the contexts (i.e., bigraphs with holes), called *active context*, where rules redexes can be found in order to be rewritten.

**Definition 4** Controls of a signature are either *active*, *passive* or *atomic*. Atomic controls cannot contain any node (and hence must be a leaf of the place and its binding arity is zero). A location (i.e., an element in the domain of *prnt*) is *active* if all its ancestors have active controls (the roots are active); otherwise it is *passive*. A context is *active* if all its holes are active locations.

It is easy to check that active contexts form a compositional reflective sub-category of bigraphs, which we denote by  $\mathcal{A}$ , the category of active contexts.

**Definition 5** A bigraphical reactive system (BRS)  $\mathcal{D}(\mathcal{K}, \mathcal{R})$  is formed by  $\text{BBG}(\mathcal{K})$  equipped with the subcategory  $\mathcal{A}$  of active contexts, and a set  $\mathcal{R}$  of (parametric) *reaction rules*, that is pairs  $l, r: \langle m, loc, X \rangle \rightarrow \langle n, loc', Y \rangle$  (usually written as  $l \rightarrow r$ ).

The *reaction relation*  $\rightarrow$  is the relation between ground bigraphs given by the following rule:

$$\frac{\text{for some } A \in \mathcal{A}, l \rightarrow r \in \mathcal{R}, d \text{ ground : } g = A \circ l \circ d \text{ and } h = A \circ r \circ d}{g \rightarrow h}$$

## 2.3 Minimal labels for BRSs

In order to derive systematically labelled transition systems (LTSs) from reactive systems, Leifer and Milner in [10] proposed to consider as labels the “minimal” contexts which can trigger a reaction. Such minimality can be elegantly defined as a categorial universal property, i.e., introducing the *relative pushouts* (RPOs) and *idem pushouts* (IPOs).

**Definition 6** Given a span  $(f_0, f_1)$  and an its bound  $(g_0, g_1)$  as shown in Figure 2(1)

1. a candidate RPO is a triple  $(h_0, h_1, h)$  such that the diagram in Figure 2(1) commutes.
2. an RPO  $(h_0, h_1, h)$  is a candidate RPO such that given any other candidate RPO  $(k_0, k_1, k)$  there exists a unique mediating arrow  $j$  such that the diagram in Figure 2(2) commutes.
3. an IPO is an RPO having as the third component an identity (as in Figure 2(3)).

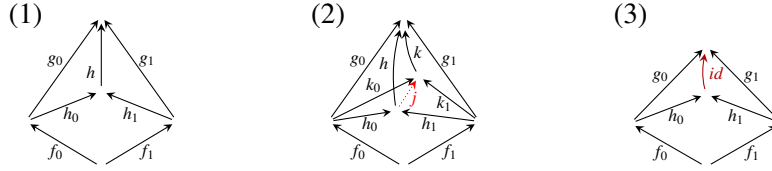


Figure 2: A candidate RPO (1), an RPO (2) and an IPO (3).

**Proposition 1** *The category BBG has RPOs, and hence IPOs<sup>2</sup>.*

Following [10], the labels of an agent are the IPOs with respect to any reaction rule:

**Definition 7** (IPO labels) Let  $\mathcal{D}(\mathcal{K}, \mathcal{R})$  be a BRS, and  $a : \varepsilon \rightarrow \langle m, loc, X \rangle$  an agent. An *IPO label* for  $a$  (in  $\mathcal{D}$ ) is a bigraph  $\ell$  such that there exists a rule  $l \rightarrow r \in \mathcal{R}$ , a parameter  $d$  and an active context  $D$  such that  $(\ell, D, id)$  is an IPO for  $(a, l \circ d)$ .

We will denote by  $\text{IPO}(a)$  the set of all IPO labels for  $a$ .

## 2.4 Defining barbs from IPO labels

In this section we give a way to define systematically barbs over bigraphs using the IPO labels.

In general, a *bigraphic barb* is a predicate over ground bigraphs, observing some names on the interface. More precisely for a bigraph  $a : \varepsilon \rightarrow \langle m, loc, X \rangle$ , a bigraphic barb is a (possibly empty) subset  $X' \subseteq X$  of the outer names of  $a$ ; we denote it as  $a \downarrow X'$ .

The first family of barbs we define are *IPO barbs*. Intuitively, a name of an agent is in the IPO barb if there exists an IPO label that interacts (non trivially) with the agent on such a name.

**Definition 8** (IPO barbs) Let  $\mathcal{D}(\mathcal{K}, \mathcal{R})$  be a BRS, and  $a : \varepsilon \rightarrow \langle m, loc, X \rangle$  a ground bigraph. For each  $\ell \in \text{IPO}(a)$ , the corresponding *IPO barb* (for  $a$ ) is the set  $X' \subseteq X$  defined as  $X' = \{x \in X \mid \text{link}_\ell(x) = \text{link}_\ell(p) \text{ for some } p \in P_\ell \uplus (X \setminus \{x\})\}$ . We write  $a \downarrow_I X'$  for  $X'$  is a IPO barb for  $a$ .

In some cases we need to understand *how* each name is used by the agent. Intuitively, the role of a name can be figured out by looking at the control it is connected to, in the IPO label; e.g., in the case of  $\pi$ -calculus (Section 3), a name acts as an input channel when is connected to a “send” control in the IPO label. We can get this information by looking at the link graph of the label. In general, however, for a reaction to take place not only the names must be connected to the right controls, but these must be in the correct position within the IPO label. Thus, we can say that the role played by the name is given by the control it is connected to, and its position in the label; the latter can be obtained from the place graph of the label. Formally:

**Definition 9** (decorated IPO barb) Let  $\mathcal{D}(\mathcal{K}, \mathcal{R})$  be a BRS, and  $a : \varepsilon \rightarrow \langle m, loc, X \rangle$  a ground bigraph. For each  $\ell \in \text{IPO}(a)$ , the corresponding *decorated IPO barb* (for  $a$ ) is a set of pairs  $X' = \{(x'_0, T_0), \dots, (x'_{n-1}, T_{n-1})\}$  such that  $\{x'_0, \dots, x'_{n-1}\}$  is the IPO barb for  $a$  corresponding to  $\ell$ , and for  $i = 0, \dots, n-1$ ,  $T_i$  is the set satisfying the following properties:

<sup>2</sup> More precisely, RPOs are constructed in the s-category  $'\text{BBG}$  and then mapped down into the category BBG.

1.  $\langle int \rangle \in T_i$  iff there exists  $x \in X \setminus \{x'_i\}$  such that  $link_\ell(x) = link_\ell(x'_i)$  and  $loc(x) = \perp$ ;
2.  $\langle ctrl(w_n), \dots, ctrl(w_1), int \rangle \in T_i$  iff there exists  $x \in X \setminus \{x'_i\}$  such that  $link_\ell(x) = link_\ell(x'_i)$  and  $loc_\ell(x) = j$ , and there exist  $w_1, \dots, w_n \in V_\ell$  and a root  $k$  such that  $prnt_\ell(j) = w_1$ ,  $prnt_\ell(w_z) = w_{z+1}$  ( $z = 1, \dots, n-1$ ),  $prnt_\ell(w_n) = k$ .
3.  $\langle ctrl(w_n), \dots, ctrl(w_1), ctrl(v) \rangle \in T_i$  iff there exists  $v \in V_\ell$  having a port  $p$  such that  $link_\ell(p) = link_\ell(x'_i)$ , and there exist  $w_1, \dots, w_n \in V_\ell$  and a root  $k$  such that  $prnt_\ell(j) = w_1$ ,  $prnt_\ell(w_z) = w_{z+1}$  ( $z = 1, \dots, n-1$ ),  $prnt_\ell(w_n) = k$ .

We write  $a \downarrow_{dl} X'$  to denote that  $X'$  is a decorated IPO label for  $a$ .

Barbed bisimulations about reactive systems with nested and multiple locations, where the role of a control depends also on its position (like e.g. in the case of web services), could benefit from the information about position of controls. An example of a name connected to a nested control is given in the case of Mobile Ambients (label  $in_3$  in Figure 10, Section 4).

In the following, we will omit  $\langle - \rangle$  on unary lists, we will write the pairs  $(x'_i, T_i)$  as  $x'_i(T_i)$ , and we will consider a set like  $\bigcup_{i=0}^{n-1} x_i(T_i)$  also as the set defined as  $\bigcup_{i=0}^{n-1} \bigcup_{t \in T_i} x_i(t)$ .

**Definition 10** Given a BRS  $\mathcal{D}(\mathcal{H}, \mathcal{R})$  and  $\downarrow$  be a predicate over bigraphical agents. A *bigraphical barb bisimulation* is a symmetric relation  $\mathcal{S}$ , such that  $G \mathcal{S} H$  then

- if  $G \downarrow X'$  then  $H \downarrow X'$ ;
- if  $G \rightarrow G'$  then  $H \rightarrow H'$  and  $G' \mathcal{S} H'$ .

The *bigraphical barbed bisimilarity* ( $\sim$ ) is the biggest bigraphical barbed bisimulation.

The *bigraphical barbed congruence* ( $\sim^C$ ) is the biggest congruence contained in bigraphical barbed bisimilarity. It can be defined by closing  $\sim$  under all contexts, i.e.,  $G \sim^C H$  if for all contexts  $C$ ,  $C \circ G \sim C \circ H$ .

Notice that the closure under all contexts is necessary, because in general bigraphical barbed bisimilarities are not congruences (as usual with barbed bisimilarities).

### 3 Application: The $\pi$ -calculus

In this section we apply the theory of bigraphic barbed bisimilarity introduced in the previous section to the synchronous  $\pi$ -calculus. As a first step, we provide an encoding of the synchronous  $\pi$ -calculus as a BRS. This is almost the same as in [8]. We refer the reader to usual literature for the technical details about  $\pi$ -calculus e.g. [14].

#### 3.1 $\pi$ -calculus

Recall that the syntax of the (recursion-free)  $\pi$ -calculus is defined as follows:

$$P, Q ::= \mathbf{0} \mid z(x).P \mid \bar{z}x.P \mid P|Q \mid \nu x.P.$$

Processes are taken up to the following structural equivalence ( $\equiv$ ).

$$\begin{array}{lll}
 P \mid \mathbf{0} \equiv P & P \mid Q \equiv Q \mid P & (P \mid Q) \mid R \equiv P \mid (Q \mid R) \\
 \nu x. \mathbf{0} \equiv \mathbf{0} & \nu x. \nu y. P \equiv \nu y. \nu x. P & \nu x. (P \mid Q) \equiv P \mid \nu x. Q \text{ if } x \notin \text{fn}(P) \\
 \nu x. \bar{x}y. P \equiv \bar{x}y. (\nu x. P) \text{ if } x \notin \{z, y\} & & \nu x. z(y). P \equiv z(y). (\nu x. P) \text{ if } x \notin \{z, y\}
 \end{array}$$

The reduction semantics is defined by the following set of rules, closed under  $\equiv$ :

$$\text{Com} \frac{}{\bar{x}x.P \mid z(y).Q \rightarrow P \mid Q\{x/y\}} \quad \frac{P \rightarrow Q}{P \mid R \rightarrow Q \mid R} \quad \frac{P \rightarrow Q}{\nu x. P \rightarrow \nu x. Q}$$

A barb is a predicate over processes, it is usually written as  $P \downarrow a$ , read  $P$  commits to  $a$ , where  $P$  is a process and  $a$  a name.

**Definition 11** Let  $\downarrow$  be a barb predicate. A *barbed bisimulation* is a symmetric relation  $\mathcal{S}$ , such that if  $P \mathcal{S} Q$ , then

- if  $P \downarrow a$ , then  $Q \downarrow a$ ;
- if  $P \rightarrow P'$ , then  $Q \rightarrow Q'$  and  $P' \mathcal{S} Q'$ .

The *barbed bisimilarity* ( $\sim$ ) is the biggest barbed bisimulation.

To obtain the biggest congruence, called *barbed congruence* ( $\simeq^c$ ), contained into the barbed bisimilarity, we close  $\sim$  under all contexts, i.e.,  $P \simeq^c Q$  if for all  $C[-]$   $C[P] \sim C[Q]$ .

In literature, two notions of barbs for the  $\pi$ -calculus have been introduced:

**Definition 12** Let  $P$  be a  $\pi$ -process,

- (unsorted barbs)  $P$  commits to  $a$  (written  $P \downarrow_{\pi} a$ ) iff  $P \equiv a(b).Q \mid R$  or  $P \equiv \bar{a}b.Q \mid R$ ;
- (sorted barbs)  $P$  commits to  $a$  (written  $P \downarrow_{\pi'} a$ ) iff  $P \equiv a(b).Q \mid R$  and  $P$  commits to  $\bar{a}$  (written  $P \downarrow_{\pi'} \bar{a}$ ) iff  $P \equiv \bar{a}b.Q \mid R$ .

We will denote with  $\sim_{\pi}$  and  $\simeq_{\pi}^c$  the barbed bisimilarity and barbed congruence for  $\downarrow_{\pi}$ , respectively; and we will denote with  $\sim_{\pi'}$  and  $\simeq_{\pi'}^c$  the barbed bisimilarity and barbed congruence for  $\downarrow_{\pi'}$ , respectively. The following result is well-known [14]:

**Proposition 2**  $\simeq_{\pi}^c = \simeq_{\pi'}^c \subset \sim_{\pi'} \subset \sim_{\pi}$ .

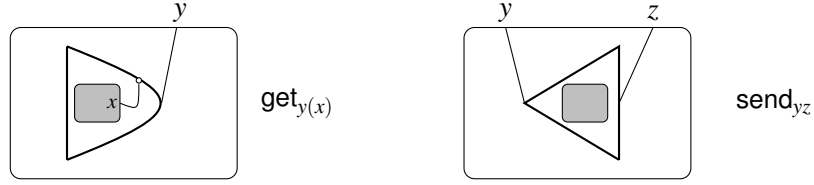
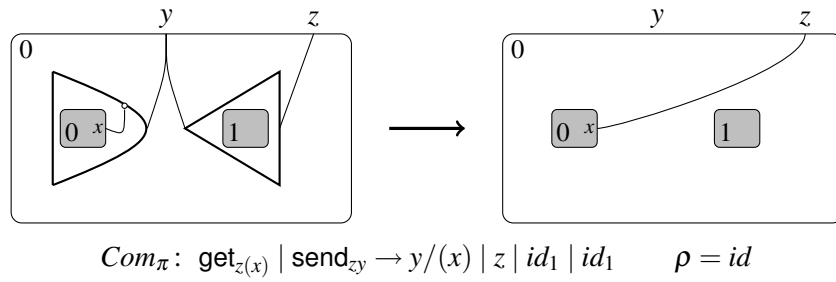
### 3.2 $\pi$ -calculus as BRS

The signature for representing the  $\pi$ -calculus processes in binding bigraphs is

$$\mathcal{H}_{\pi} \triangleq \{\text{get}: 1 \rightarrow 1, \text{send}: 0 \rightarrow 2\}$$

where get and send are both passive. Notice that get has a binding port, representing the bound local variable. An example of the controls is given in Figure 3.




 Figure 3: The controls of the signature for the  $\pi$ -calculus.

 Figure 4: Reaction rules  $\mathcal{R}_{\pi}$  for the  $\pi$ -calculus.

A  $\pi$ -process  $P$  is translated into a bigraph of  $\text{BBG}(\mathcal{K}_{\pi})$  as follows:

$$\begin{aligned} \llbracket \mathbf{0} \rrbracket_X &= 1 \mid X & \llbracket P \mid Q \rrbracket_X &= \llbracket P \rrbracket_X \mid \llbracket Q \rrbracket_X & \llbracket \nu x.P \rrbracket_X &= /x \circ \llbracket P \rrbracket_{X \uplus \{x\}} \\ \llbracket z(x).P \rrbracket_X &= \text{get}_{z(x)} \circ \llbracket P \rrbracket_{X \uplus \{x\}} \text{ if } z \in X & \llbracket \bar{z}x.P \rrbracket_X &= \text{send}_{zx} \circ \llbracket P \rrbracket_X \text{ if } z, x \in X \end{aligned}$$

The outer upward names represent free names; restricted names are encoded as edges; finally, input bindings are represented as binding ports localized in the get control.

**Proposition 3** (Syntax adequacy) *Let  $P, Q$  be two  $\pi$ -processes and let  $X$  be a name set; then  $P \equiv Q$  iff  $\llbracket P \rrbracket_X = \llbracket Q \rrbracket_X$ .*

The set of reaction rules ( $\mathcal{R}_{\pi}$ ) is shown in Figure 4. Notice that communication simply connects the links “stopped” by get to the resources provided by send. We denote this BRS as  $\mathcal{D}_{\pi} \triangleq \mathcal{D}(\mathcal{K}_{\pi}, \mathcal{R}_{\pi})$ , where the parametric rules can be instantiated with discrete bigraphs.

**Proposition 4** (Semantics adequacy) *Let  $P, Q$  be two  $\pi$ -processes and let  $X$  be a name set; then  $P \rightarrow Q$  iff  $\llbracket P \rrbracket_X \rightarrow \llbracket Q \rrbracket_X$ .*

### 3.3 Bigraphical barbed bisimilarity for $\pi$ -calculus

The IPO labels for the BRS  $\mathcal{D}_{\pi}$  are in Figure 5, and in Figure 6 we show the corresponding barbs (both bigraphic and decorated). Recalling that bigraphic barbs identify names on the agent interface which are “effectively involved” in reactions, an intuitive explanation of these barbs is the following. The barb  $a \downarrow_I \{y\}$  on the left-hand side, arises from the fact that the agent uses

	Agent ( $a : \varepsilon \rightarrow X$ )	Label	conditions
com1	$/Z \circ (\text{send}_{yx} \circ d \mid q)$	$\text{get}_{y(z)} \circ c$	$y \in X$ and $c$ discrete with names not in $X \cup Z$
com2	$/Z \circ (\text{get}_{y(x)} \circ d \mid q)$	$\text{send}_{yz} \circ c$	$y \in X$ and $c$ discrete with names not in $X \cup Z$
com3	$/Z \circ ((\text{send}_{y_0x} \circ d) \mid (\text{get}_{y_1(z)} \circ c) \mid q)$	$id \mid w/y_0 \mid w/y_1$	$y_0, y_1 \in X$
com4	$/Z \circ ((\text{send}_{yx} \circ d) \mid (\text{get}_{y(z)} \circ c) \mid q)$	$id$	-

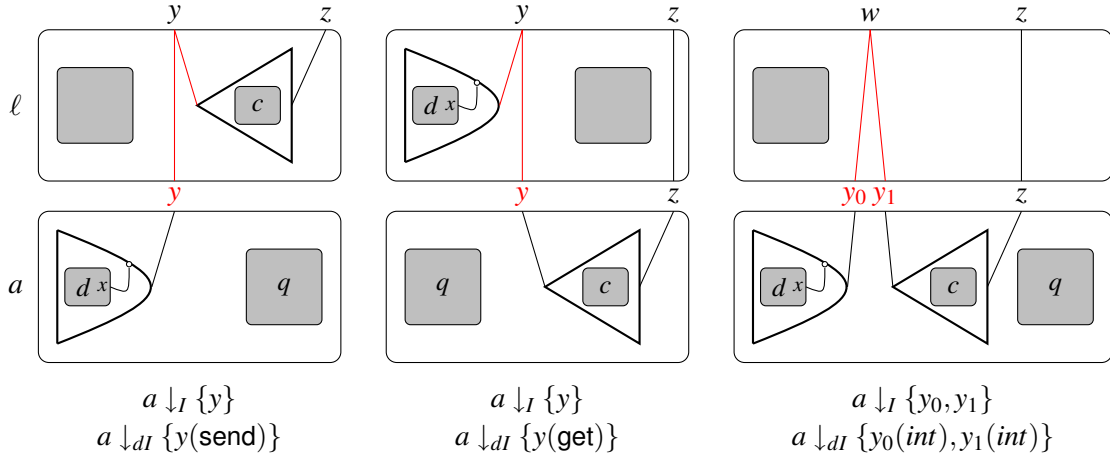
 Figure 5: The set of labels for the  $\pi$ -calculus derived using the IPO construction.


Figure 6: The barbs derived from the labels of Figure 5.

$y$  as input channel, and the IPO label completes the transition by providing an output along the same name. The same barb arises also in the second case; in order to distinguish the two labels, we have to consider also how the name  $y$  is used. In fact, the decorated barbs differ in these two cases, because they carry also the linking information about  $y$  in the IPO label. Finally, the third barb arises because the IPO label triggers a transition by unifying the channel names of the two controls; hence, in this case the two names are involved in an unification. Notice that the name  $z$  in the agent interface does not take part to the reactions, and hence it does not appear in the barbs.

We will denote with  $\sim_{I\pi}$  (resp.  $\sim_{dI\pi}$ ) the IPO barbed bisimilarity (resp. decorated IPO barbed bisimilarity), and with  $\simeq_{I\pi}^c$  and  $\simeq_{dI\pi}^c$  the corresponding congruences. Now we can state and prove some results on the derived barbed bisimilarities and congruences.

**Proposition 5**  $\simeq_{I\pi}^c = \simeq_{dI\pi}^c = \simeq_{\pi}^c = \simeq_{\pi'}^c \subset \sim_{\pi'} = \sim_{dI\pi} \subset \sim_{I\pi} \subset \sim_{\pi}$ .

*Proof.* The equality  $\simeq_{I\pi}^c = \simeq_{dI\pi}^c = \simeq_{\pi}^c = \simeq_{\pi'}^c$  is quite simple to prove. Just notice that all the extra information you can see in  $\sim_{\pi'}$  (discriminates by  $a$  and  $\bar{a}$ ),  $\sim_{I\pi}$  (can check if equating two names a process performs a communication), or  $\sim_{dI\pi}$  (can check communication enabling, as  $\sim_{I\pi}$ , and besides it can see the nodes linked to the names inside the labels) w.r.t.  $\sim_{\pi}$  are captured by the closure under all contexts.

The inclusion  $\dots \simeq_{\pi'}^c \subset \simeq_{\pi'} \dots$  is trivially a consequence of Proposition 2.

For  $\simeq_{\pi'} = \simeq_{dI\pi}$ , notice that every time a bigraph  $G \downarrow_{dI} \{x(int), y(int)\}$ , then either  $G \downarrow_{dI} \{x(send)\}$  and  $G \downarrow_{dI} y(get)$  or  $G \downarrow_{dI} \{x(get)\}$  and  $G \downarrow_{dI} y(send)$ . For the barbs  $x(send)$  and  $x(get)$  there is a one to one correspondence with the barbs  $x$  and  $\bar{x}$ , respectively.

$\simeq_{dI\pi} \subset \simeq_{I\pi}$  because the former can observe the controls of nodes linked to names. The inclusion is strict, in fact  $\llbracket a(x) \mid \bar{b}z \rrbracket \simeq_{I\pi} \llbracket b(x) \mid \bar{a}z \rrbracket$ , but they are not decorated IPO barbed bisimilar.

Finally,  $\simeq_{I\pi} \subset \simeq_{\pi}$  because there is a correspondence between the barb  $\{x\}$  used by  $\simeq_{I\pi}$  and the barb  $x$  in  $\simeq_{\pi}$ . The strictness of the inclusion is given by the fact that  $a(x) \mid \bar{b}z \simeq_{\pi} \bar{a}x \mid \bar{b}z$ , but  $\llbracket a(x) \mid \bar{b}z \rrbracket \not\simeq_{I\pi} \llbracket \bar{a}x \mid \bar{b}z \rrbracket$ .  $\square$

IPO barbed bisimilarity is finer than (unsorted) bisimilarity due to the third barb in Figure 6, allowing to observe when the agent can perform an input and an output on two different names, at once. On the other hand, the decorated barb arising from this case does not add any discriminating power with respect to sorted barbed bisimilarity  $\simeq_{\pi'}$ , because the sorted barbs allow to observe when the agent exposes both an input and an output name. Finally, when we consider the contextual congruences of these bisimilarities, we get the same equivalence because any extra observations yielded by the (decorated) IPO barbs are recovered by suitable contexts.

## 4 Application: Mobile Ambients

In this section we analyze another application of the theory of barbed bigraphical bisimilarity, i.e., to the fragment of recursion-free Mobile Ambients. The encoding resembles mostly the one presented in [7]. See [4] for a detailed introduction to Mobile Ambients.

### 4.1 Mobile Ambients

Recall that the syntax of (our fragment of) Mobile Ambients is defined as follows:

$$P, Q ::= \mathbf{0} \mid n[P] \mid P|Q \mid \nu n.P \mid in\ n.P \mid out\ n.P \mid open\ n.P.$$

Processes are taken up to the following structural equivalence ( $\equiv$ ):

$$\begin{array}{ll} P \mid \mathbf{0} \equiv P & P \mid Q \equiv Q \mid P \\ (P \mid Q) \mid R \equiv P \mid (Q \mid R) & \nu n.\mathbf{0} \equiv \mathbf{0} \\ \nu n.\nu m.P \equiv \nu m.\nu n.P & \nu n.(P \mid Q) \equiv P \mid \nu n.Q \text{ if } n \notin fn(p) \\ \nu m.n[P] \equiv n[\nu m.P] \text{ if } m \neq n & \nu n.in\ m.P \equiv in\ m.\nu x.P \text{ if } n \neq m \\ \nu n.out\ m.P \equiv out\ m.\nu n.P \text{ if } n \neq m & \nu n.open\ m.P \equiv open\ m.\nu x.P \text{ if } n \neq m \end{array}$$

The reduction semantics is defined by the following set of rules, closed under  $\equiv$ :

$$\begin{array}{l} In \frac{}{n[in\ m.P \mid Q] \mid m[R] \rightarrow m[n[P \mid Q] \mid R]} \quad Out \frac{}{m[n[out\ m.P \mid Q] \mid R] \rightarrow n[P \mid Q] \mid m[R]} \\ Open \frac{}{open\ n.P \mid n[Q] \rightarrow P \mid Q} \quad \frac{P \rightarrow Q}{P \mid R \rightarrow Q \mid R} \quad \frac{P \rightarrow Q}{\nu x.P \rightarrow \nu x.Q} \quad \frac{P \rightarrow Q}{n[P] \rightarrow n[Q]} \end{array}$$

As for  $\pi$ -calculus, we give a notion of barb for Mobile Ambients. It is defined as follows.

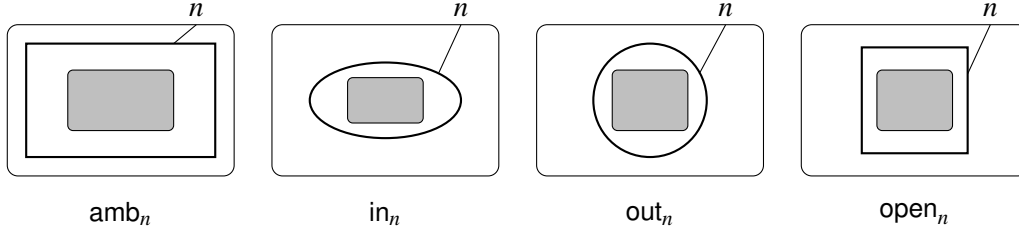


Figure 7: The controls of the signature for the Mobile Ambients.

**Definition 13** Let  $P$  be an ambient process.  $P$  commits to  $n$  (written  $P \downarrow_A n$ ) iff  $P \equiv n[Q] \mid R$ ;

We denote with  $\sim_A$  and  $\simeq_A^c$  the barbed bisimilarity and barbed congruence for  $\downarrow_A$ , respectively.

## 4.2 Mobile Ambients as BRSs

The signature for representing the Mobile Ambients in binding bigraphs is

$$\mathcal{K}_A \triangleq \{\text{amb}: 0 \rightarrow 1, \text{in}: 0 \rightarrow 1, \text{out}: 0 \rightarrow 1, \text{open}: 0 \rightarrow 1\}$$

where  $\text{amb}$  is active and  $\text{in}$ ,  $\text{out}$ ,  $\text{open}$  are passive. An example of the controls is given in Figure 7.

An ambient process  $P$  is translated into a bigraph of  $\text{BBG}(\mathcal{K}_A)$  as follows:

$$\begin{aligned} \llbracket \mathbf{0} \rrbracket_X &= 1 \mid X & \llbracket P \mid Q \rrbracket_X &= \llbracket P \rrbracket_X \mid \llbracket Q \rrbracket_X \\ \llbracket \nu x.P \rrbracket_X &= /x \circ \llbracket P \rrbracket_{X \uplus \{x\}} & \llbracket n[P] \rrbracket_X &= \text{amb}_n \circ \llbracket P \rrbracket_X \text{ if } n \in X \\ \llbracket \text{in } n.P \rrbracket_X &= \text{in}_n \circ \llbracket P \rrbracket_X \text{ if } n \in X & \llbracket \text{out } n.P \rrbracket_X &= \text{out}_n \circ \llbracket P \rrbracket_X \text{ if } n \in X \\ \llbracket \text{open } n.P \rrbracket_X &= \text{open}_n \circ \llbracket P \rrbracket_X \text{ if } n \in X \end{aligned}$$

The outer upward names represent free names; restricted names are encoded as edges.

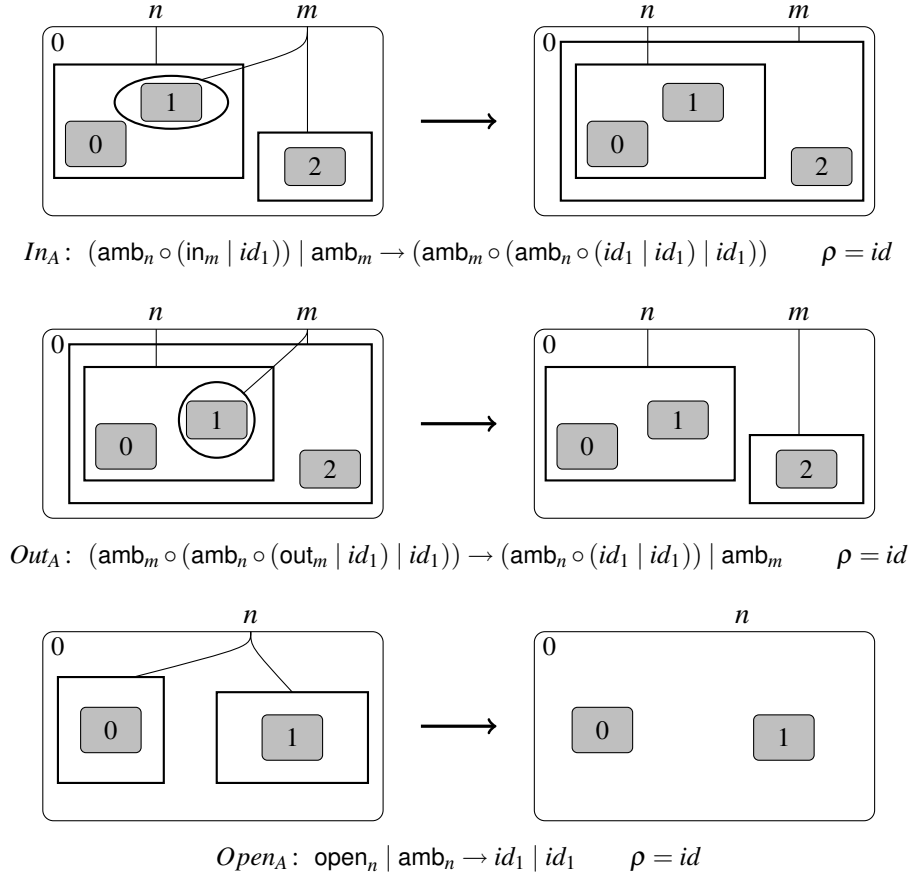
**Proposition 6** (Syntax adequacy) *Let  $P, Q$  be two ambient processes and let  $X$  be a name set; then  $P \equiv Q$  iff  $\llbracket P \rrbracket_X = \llbracket Q \rrbracket_X$ .*

The set of rules  $\mathcal{R}_A$  is shown in Figure 8. The first rule moves an ambient inside another one following the  $\text{in}$  capability; the second one moves an ambient outside another one by means of the capability  $\text{out}$ ; finally the third rule open the ambient linked to the  $\text{open}$  capability. We denote this BRS as  $\mathcal{D}_A(\mathcal{K}_A, \mathcal{R}_A)$ , whose parametric rules can be instantiated with discrete bigraphs.

**Proposition 7** (Semantics adequacy) *Let  $P, Q$  be two ambient processes and let  $X$  be a name set; then  $P \rightarrow Q$  iff  $\llbracket P \rrbracket_X \rightarrow \llbracket Q \rrbracket_X$ .*

## 4.3 Bigraphical barbed bisimilarity for Mobile Ambients

Analogously to the  $\pi$ -calculus case, the set of labels derived by applying the IPO construction are shown in Figure 9; then, the derived barbs are shown in Figure 10.


 Figure 8: Reaction rules  $\mathcal{R}_A$  for the Mobile Ambients.

Barbs in3 and open1 let us to observe that  $n$  is the name of a top-level ambient (which can be entered, or opened, by a parallel agent). On the other hand, barbs in4, out3 and open3 arise from the IPO labels which trigger a reaction by unifying two names; notice that these names can be used at any position in the agent, not only at top level.

Barbs in1 and in2, instead, arise when  $n$  is used by a “in” capability in the agent; in the first case, this capability is contained by an ambient at top-level, whilst in the second case the capability is directly exposed at top-level. Interestingly, this topological difference is not observable in either kinds of IPO barbs. A similar situation holds for out1 and out2. Finally, open2 arises when  $n$  is a name used by an “open” capability at the top-level in the agent.

We will denote with  $\sim_{IA}$  (resp.  $\sim_{dIA}$ ) the IPO barbed bisimilarity (resp. decorated IPO barbed bisimilarity), and with  $\simeq_{IA}^c$  and  $\simeq_{dIA}^c$  the corresponding congruences. We can prove the following

**Proposition 8** 1.  $\sim_{dIA} \subset \sim_{IA} \subset \sim_A$ ;

2.  $\simeq_{dIA}^c = \simeq_{IA}^c \subset \simeq_A^c$ ;

3.  $\sim_{dIA} \not\subset \simeq_A^c$  and  $\simeq_A^c \not\subset \sim_{IA}$ ;

	Agent ( $a : \varepsilon \rightarrow X$ )	Label	Conditions
in1	$/Z \circ (\text{amb}_m \circ (\text{in}_n \circ p \mid q) \mid d)$	$(\text{amb}_n \circ c) \mid id$	$n \in X$ and $c$ discrete with names not in $X \cup Z$
in2	$/Z \circ (\text{in}_n \circ p \mid q)$	$\text{amb}_n \circ c \mid \text{amb}_m \circ id$	$n \in X$ , $c$ discrete with names not in $X \cup Z$ and $m \notin X \cup Z$
in3	$/Z \circ (\text{amb}_n \circ d \mid c)$	$\text{amb}_m \circ (\text{in}_n \circ p \mid q) \mid id$	$n \in X$ , $p, q$ discretized with names not in $X \cup Z$ and $m \notin X \cup Z$
in4	$\mathcal{T} \circ (\text{amb}_m \circ (\text{in}_{n_0} \circ p \mid q) \mid \text{amb}_{n_1} \circ d)$	$id \mid n/n_0 \mid n/n_1$	$n_0, n_1 \in X$ and $\mathcal{T}$ active
in5	$\mathcal{T} \circ (\text{amb}_m \circ (\text{in}_n \circ p \mid q) \mid \text{amb}_n \circ d)$	$id$	$\mathcal{T}$ active
out1	$/Z \circ (\text{amb}_m \circ (\text{out}_n \circ p \mid q) \mid d)$	$\text{amb}_n \circ (c \mid id)$	$n \in X$ and $c$ discrete with names not in $X \cup Z$
out2	$/Z \circ (\text{out}_n \circ p \mid q)$	$\text{amb}_n \circ \text{amb}_m \circ (c \mid id)$	$n \in X$ , $c$ discrete with names not in $X \cup Z$ and $m \notin X \cup Z$
out3	$\mathcal{T} \circ \text{amb}_{n_0} \circ (\text{amb}_m \circ (\text{out}_{n_1} \circ p \mid q) \mid d)$	$id \mid n/n_0 \mid n/n_1$	$n_0, n_1 \in X$ and $\mathcal{T}$ active
out4	$\mathcal{T} \circ \text{amb}_n \circ (\text{amb}_m \circ (\text{out}_n \circ p \mid q) \mid d)$	$id$	$\mathcal{T}$ active
open1	$/Z \circ (\text{amb}_n \circ p \mid q)$	$\text{open}_n \circ c \mid id$	$n \in X$ and $c$ discrete with names not in $X \cup Z$
open2	$/Z \circ (\text{open}_n \circ p \mid q)$	$\text{amb}_n \circ c \mid id$	$n \in X$ and $c$ discrete with names not in $X \cup Z$
open3	$\mathcal{T} \circ (\text{open}_{n_0} \circ p \mid \text{amb}_{n_1} \circ q \mid d)$	$id \mid n/n_0 \mid n/n_1$	$n_0, n_1 \in X$ and $\mathcal{T}$ active
open4	$\mathcal{T} \circ (\text{open}_n \circ p \mid \text{amb}_n \circ q \mid d)$	$id$	$\mathcal{T}$ active

Figure 9: The set of labels for Mobile Ambients derived using IPO construction.

*Proof.* 1. The first inclusion is justified by the fact that decorated IPO barbs are the same as the IPO barbs, but the former barbs have some extra information, so they are able to distinguish more than the latter ones. It is strict, consider  $\text{in } n$  and  $n[]$  as shown in Figure 6,  $\llbracket \text{in } n \rrbracket \downarrow_I \{n\}$  and  $\llbracket n[] \rrbracket \downarrow_I \{n\}$ , but  $\llbracket \text{in } n \rrbracket \downarrow_{dI} \{n(\text{amb})\}$  and  $\llbracket n[] \rrbracket \downarrow_{dI} \{n(\langle \text{amb}, \text{in} \rangle)\}$ .

The second inclusion holds because the set of barbs of  $BA$  is a subset of the set of IPO barbs. To prove the strictness consider  $\text{open } n$  and  $\text{open } m$ ,  $\text{open } n \sim_A \text{open } m$  because they do nothing, but  $\llbracket \text{open } n \rrbracket \not\sim_{IA} \llbracket \text{open } m \rrbracket$  because  $\llbracket \text{open } n \rrbracket \downarrow_I \{n\}$  but  $\llbracket \text{open } m \rrbracket \downarrow_I \{m\}$ .

2. The equality is justified by the fact that the closure by all contexts catch all the extra information added by the decoration of IPO barbs. The inclusion is a direct consequence of point 1. It is strict, consider  $\text{vn}.n[\text{open } m \mid k[]]$  and  $\text{vn}.n[]$ , they are barbed congruent, but not IPO barbed bisimilar, indeed  $\llbracket \text{vn}.n[\text{open } m \mid k[]] \rrbracket \downarrow_I \{k, m\}$  and  $\llbracket \text{vn}.n[] \rrbracket \not\downarrow_I \{k, m\}$ .

3.  $\sim_{dIA} \not\subseteq \simeq_A^c$  is trivial, consider  $m[n[]]$  and  $m[k[]]$ : their encodings under  $\llbracket - \rrbracket$  are decorated IPO bisimilar, but they are not barbed congruent.

For  $\simeq_A^c \not\subseteq \sim_{IA}$  consider again  $\text{vn}.n[\text{open } m \mid k[]]$  and  $\text{vn}.n[]$ : they are barbed congruent, but not IPO barbed bisimilar, indeed  $\llbracket \text{vn}.n[\text{open } m \mid k[]] \rrbracket \downarrow_I \{k, m\}$  and  $\llbracket \text{vn}.n[] \rrbracket \not\downarrow_I \{k, m\}$ .  $\square$

Intuitively, IPO barbed bisimilarity is finer than barbed bisimilarity because IPO barbs can

Rule	IPO barb	Decorated IPO barb
in1	$\{n\}$	$\{n(\text{amb})\}$
in2	$\{n\}$	$\{n(\text{amb})\}$
in3	$\{n\}$	$\{n(\langle \text{amb}, \text{in} \rangle)\}$
in4	$\{n_0, n_1\}$	$\{n_0(\text{int}), n_1(\text{int})\}$
out1	$\{n\}$	$\{n(\text{amb})\}$
out2	$\{n\}$	$\{n(\text{amb})\}$
out3	$\{n_0, n_1\}$	$\{n_0(\text{int}), n_1(\text{int})\}$
open1	$\{n\}$	$\{n(\text{open})\}$
open2	$\{n\}$	$\{n(\text{amb})\}$
open3	$\{n_0, n_1\}$	$\{n_0(\text{int}), n_1(\text{int})\}$

Figure 10: The barbs derived from the labels in Figure 9.

observe names of ambients together with names in capabilities, which may be not at the top level of the agent; these barbs arise from the fact that an unification of these names will trigger a reaction; see e.g. the counterexample given in the proof of Proposition 8.2. Notice that the contextual closures of these bisimilarities still do not coincide, because there is no ambient context which can observe inside a restricted ambient, like in the example above.

## 5 Conclusions

In this paper we have presented a general notion of *barb* and *barbed bisimilarity* for bigraphical reactive systems. Intuitively, barbs are the names in the interfaces of an agent which are *effectively* involved in reactions. We have shown how these names can be found by looking inside the contextual labels of the agent given by the IPO construction. Also, a finer notion of barb (called “decorated”) can be defined by observing also *how* the names are used by the agent. The role played by a name can be figured out by looking at the controls it is connected to, in the IPO label.

As example applications of this new notion of bisimilarity, we have considered the IPO barbed bisimilarities for the (BRS encodings of)  $\pi$ -calculus and Mobile Ambients. For  $\pi$ -calculus, IPO barbed bisimilarity is finer than standard barbed bisimilarity, but decorated IPO barbed bisimilarity coincides with sorted bisimilarity, and IPO barbed congruence coincides with standard barbed congruence. For Mobile Ambients, IPO barbed bisimilarities and congruence are finer than their standard counterparts. This is because IPO labels allow to observe also names used in inner capabilities and ambients, and not only names of top-level ambients.

The problem of providing an abstract definition of barbs has been faced also in [13]. There, a general definition of barb is given around a notion of *closure* of an agent, that is, a minimal set of processes which offer an immediate interaction with the agent. Although this notion of closure seems related to the IPO construction used in the present work, their development is carried out in the quite different setting of *biorthogonality*. A future work will be to compare the IPO bigraphical barbs and those coming from the biorthogonality framework.

Another interesting application of the IPO construction to barbed bisimilarities has been in-

investigated in [3], where the notion of *barbed semisaturated bisimulation* is introduced. Although it turns out to be finer than usual barbed congruence, this bisimulation is a congruence without the need of contextual closures, and moreover it can be characterized using the minimal labels defined by the IPO construction. However, [3] does not specify any particular notion of barb, and hence it can be seen as an complementary research with respect to our work.

Finally, we plan to apply this theory to other calculi, such as Fusion calculus and those for security aspects (such as spi-calculus). To this end, the bigraphical barbed bisimilarity can be extended easily to *directed bigraphs* [5], which allow to represent also calculi with fusions.

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