Fully Complete Models for ML Polymorphic Types*

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First we isolate conditions (axioms) for a categorical model of system F to be *fully complete* for *ML polymorphic types*. Then we present an example of such a model, defined by realizability over a *Linear Combinatory Algebra* ([Abr96]), arising from a Geometry of Interaction construction.

In particular, we carry out a *linear* analysis of the standard categorical notion of model for system F. Namely, we consider *adjoint models* of system F, which arise as co-Kleisli indexed categories of *linear indexed categories*, and such that the canonical adjunction between the two categories is monoidal.

Following a pattern common in Full-Abstraction/Full-Completeness proofs of Game Semantics ([AJM96, Abr97]), we identify conditions on the adjoint model which guarantee the validity of a *Decomposition Theorem*. For morphisms which inhabit ML types, this theorem allows to recover the top level structure of the corresponding Böhm trees. The fine analysis of intuitionistic types in terms of linear types plays an essential role here.

In order to build a "concrete" fully complete model, we focus on realizability models, based on Partial Equivalence Relations (PERs) over a *Linear Combinatory Algebra* (LCA). In fact, PER categories over LCAs can be seen to be linear, and to form adjoint models with their co-Klesli categories.

Our concrete fully complete model is obtained via linear realizability on an LCA of *partial involutions*. This LCA arises as a special case of a general categorical construction, i.e. Abramsky's generalized version of Girard's Geometry of Interaction. The proof of full completeness consists in showing that the PER model satisfies the axioms in our axiomatizations. In particular, the most difficult part consists in showing that the model does not contain *infinite* typed Böhm trees. This requires the study of the intermediate model generated by the *Sierpinski* PER, i.e. the model consisting of all (possibly infinite) Böhm trees of the typed lambda-calculus, with constants $\top$, $\bot$. A crucial step in our proof consists in proving that, in the simply typed lambda-calculus with *typical ambiguity* and $\bot$ constants, "totality tests" are lambda-definable by finite typed Böhm trees. A further ingredient is an Approximation Lemma, in the line of [AJM96].

*Paper available at http://www.dimi.uniud.it/˜ lenisa/Papers/Soft-copy-ps/lfcs99.ps.gz

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