Department of Mathematics, Computer Science and Physics, University of Udine

The Safety Fragment of Temporal Logics on Infinite Sequences

Lesson 12

Luca Geatti
luca.geatti@uniud.it
Angelo Montanari
angelo.montanari@uniud.it

May 13th, 2024

Outline

Outline

IC3

Incremental Construction of Inductive Clauses of Indubitable Correctness

1969. Hoare's logic:

$$\phi P \psi$$

Charles Antony Richard Hoare (1969). "An axiomatic basis for computer programming". **In:** ***Communications of the ACM*** **12.10, pp. 576–580**

Program verification - Timeline

1969. Hoare's logic:

$$\phi P \psi$$

Charles Antony Richard Hoare (1969). "An axiomatic basis for computer programming". In: *Communications of the ACM* 12.10, pp. 576–580

1977. Temporal verification of reactive systems:

$$\phi P \psi \text{ + temporal rules}$$

Zohar Manna and Amir Pnueli (1995). ***Temporal verification of reactive systems - safety*.** Springer. ISBN: 978-0-387-94459-3

Program verification - Timeline

1969. Hoare's logic:

$$\phi P \psi$$

Charles Antony Richard Hoare (1969). "An axiomatic basis for computer programming". In: *Communications of the ACM* 12.10, pp. 576–580

1977. Temporal verification of reactive systems:

$$\phi P \psi + \text{temporal rules}$$

Zohar Manna and Amir Pnueli (1995). *Temporal verification of reactive systems - safety*. Springer. ISBN: 978-0-387-94459-3

1986. Model checking (fully automatic):

$$\mathcal{M}, \sigma \models \phi \qquad \forall \text{ paths } \sigma$$

where $\mathcal{M}$ is a representation of machine and ϕ is a temporal formula.
Edmund M. Clarke, E Allen Emerson, and A Prasad Sistla (1986). "Automatic verification of finite-state concurrent systems using temporal logic specifications". In: *ACM Transactions on Programming Languages and Systems (TOPLAS)* 8.2, pp. 244–263

- (Symbolic) Finite-state transition system $\mathcal{M} = (\overline{x}, I, T)$
  - $\overline{x}$ is a set of state variables;
  - $I(\overline{x})$ is the formula for initial states;
  - $T(\overline{x}, \overline{x}')$ is the formula for the transition relation;

(Symbolic) Transition System

- (Symbolic) Finite-state transition system $\mathcal{M} = (\overline{x}, I, T)$
  - $\overline{x}$ is a set of state variables;
  - $I(\overline{x})$ is the formula for initial states;
  - $T(\overline{x}, \overline{x}')$ is the formula for the transition relation;
- a state s of the system is a cube over $\overline{x}$ (*i.e.*, a conjunction of literals), *e.g.*:

$$\begin{aligned} s &= x_1 \wedge \neg x_2 \wedge x_3 \wedge \neg x_4 \wedge \neg x_5 \\ &= \langle 1, 0, 1, 0, 0 \rangle \end{aligned}$$

(Symbolic) Transition System

- (Symbolic) Finite-state transition system $\mathcal{M} = (\overline{x}, I, T)$
  - $\overline{x}$ is a set of state variables;
  - $I(\overline{x})$ is the formula for initial states;
  - $T(\overline{x}, \overline{x}')$ is the formula for the transition relation;
- a state s of the system is a cube over $\overline{x}$ (*i.e.*, a conjunction of literals), *e.g.*:

$$\begin{aligned} s &= x_1 \wedge \neg x_2 \wedge x_3 \wedge \neg x_4 \wedge \neg x_5 \\ &= \langle 1, 0, 1, 0, 0 \rangle \end{aligned}$$

- a trace of the system is a sequence $s_0, s_1, \ldots$ such that $s_0 \models I$ and $s_i, s'_{i+1} \models T \ \forall i \geq 0$.

(Symbolic) Transition System

- a Boolean formula F over $\overline{x}$ denotes the set of states $[\![F]\!] = \{s \in \{0,1\}^n \mid s \models F\}$:

$$s \models F \Leftrightarrow s \in [\![F]\!]$$

  s is called an F-state;

(Symbolic) Transition System

- a Boolean formula F over $\overline{x}$ denotes the set of states $[\![F]\!] = \{s \in \{0,1\}^n \mid s \models F\}$:

$$s \models F \Leftrightarrow s \in [\![F]\!]$$

  s is called an F-state;
- if $F \Rightarrow G$, then:

$$[\![F]\!] \subseteq [\![G]\!]$$

(Symbolic) Transition System

- a Boolean formula F over $\overline{x}$ denotes the set of states $[\![F]\!] = \{s \in \{0,1\}^n \mid s \models F\}$:

$$s \models F \Leftrightarrow s \in [\![F]\!]$$

  s is called an *F-state*;
- if $F \Rightarrow G$, then:

$$[\![F]\!] \subseteq [\![G]\!]$$

- a clause c is a disjunction of literals. A subclause $d \subseteq c$ is a clause whose literals are a subset of c's literals. It holds that:

$$d \Rightarrow c$$

We are not interested here in full LTL, but only on invariant properties:

$$\mathsf{G}(\varphi) \quad \text{in LTL}$$

Invariance checking

We are not interested here in full LTL, but only on invariant properties:

$$\mathsf{G}(\varphi) \quad \text{in LTL}$$

- Invariant property $P(\overline{x})$: boolean formula that asserts that only P-states are reachable.

We are not interested here in full LTL, but only on invariant properties:

$$\mathsf{G}(\varphi) \quad \text{in LTL}$$

- Invariant property $P(\overline{x})$: boolean formula that asserts that only P-states are reachable.
- P is $\mathcal{M}$-invariant if $P(\overline{x})$ holds for system $\mathcal{M}$. If this is not the case, there exists a counterexample trace $s_0, s_1, \ldots, s_k$ such that $s_k \not\models P$.

$\Rightarrow$ reachability problem

The problem may seem very simple to solve efficiently, but . . .

The problem may seem very simple to solve efficiently, but . . .

1. the system $\mathcal{M}$ is usually too large to keep it in memory: the symbolic representation is not a choice but a necessity;

$$\Rightarrow \text{no standard explicit algorithms for reachability}$$

The problem may seem very simple to solve efficiently, but . . .

1. the system $\mathcal{M}$ is usually too large to keep it in memory: the symbolic representation is not a choice but a necessity;

   $\Rightarrow$ no standard explicit algorithms for reachability

2. BDD-based techniques: state-space explosion problem;
   Kenneth L McMillan (1993). "Symbolic model checking". In: *Symbolic Model Checking*. Springer, pp. 25–60

Invariance checking

The problem may seem very simple to solve efficiently, but ...

1. the system $\mathcal{M}$ is usually too large to keep it in memory: the symbolic representation is not a choice but a necessity;

   $\Rightarrow$ no standard explicit algorithms for reachability

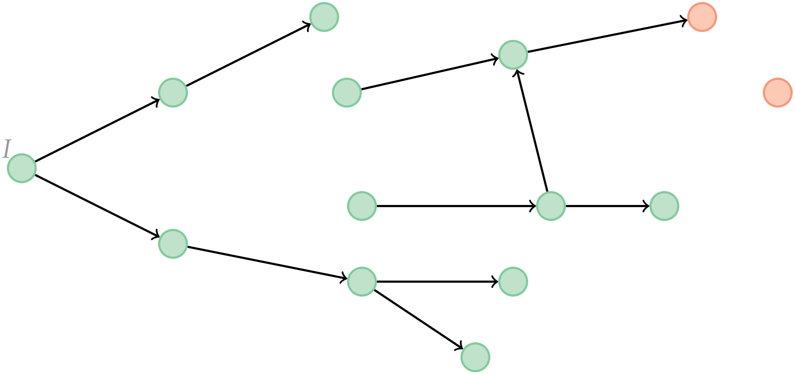
2. BDD-based techniques: state-space explosion problem;
   Kenneth L McMillan (1993). "Symbolic model checking". In: *Symbolic Model Checking*. Springer, pp. 25–60

3. Bounded Model Checking (BMC): unrolling of the transition relation;
   Armin Biere et al. (1999). "Symbolic model checking without BDDs". In: *International Conference on Tools and Algorithms for the Construction and Analysis of Systems (TACAS)*. Springer, pp. 193–207

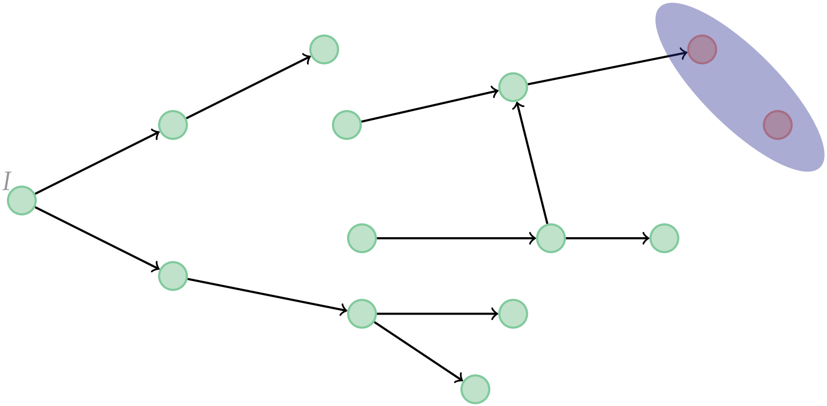
Symbolic Algorithms for Reachability

BDD-based backward algorithms

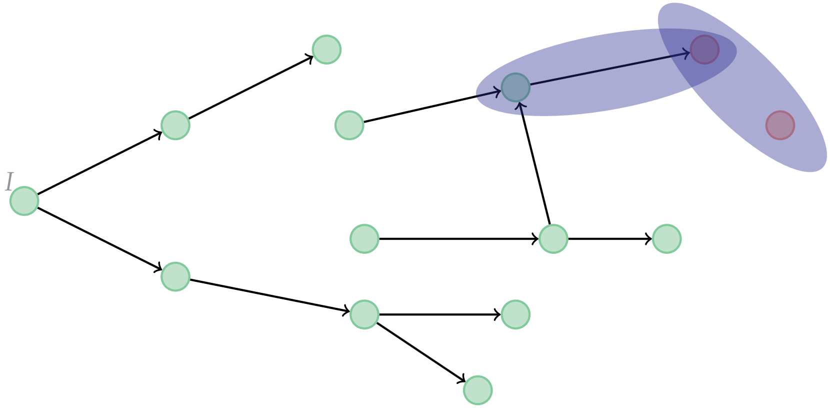
Start with $\neg P$ and proceed backward until fixpoint F. If the BDD for F contains an I-state, then a $\neg P$-state is reachable: counterexample trace.



Start with $\neg P$ and proceed backward until fixpoint F. If the BDD for F contains an I-state, then a $\neg P$-state is reachable: counterexample trace.

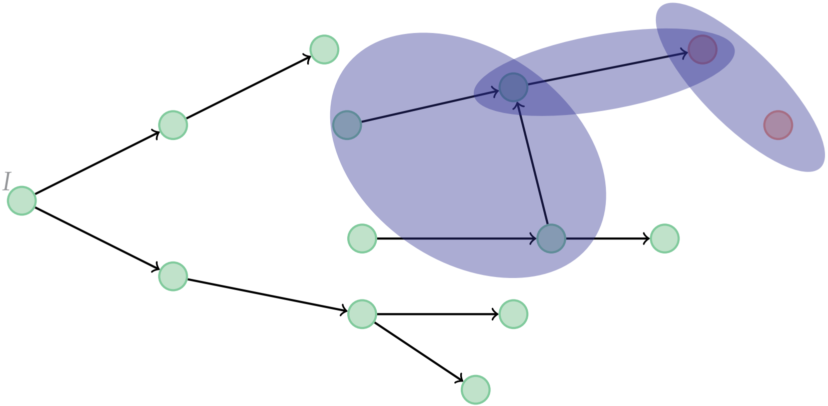


Start with $\neg P$ and proceed backward until fixpoint F. If the BDD for F contains an I-state, then a $\neg P$-state is reachable: counterexample trace.

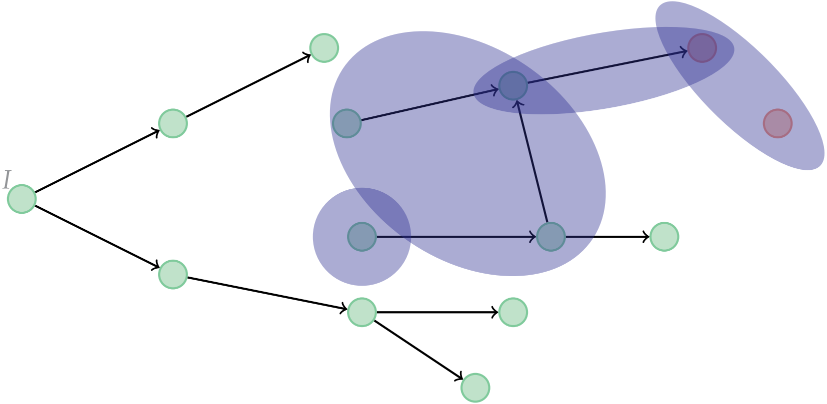


BDD-based backward algorithms

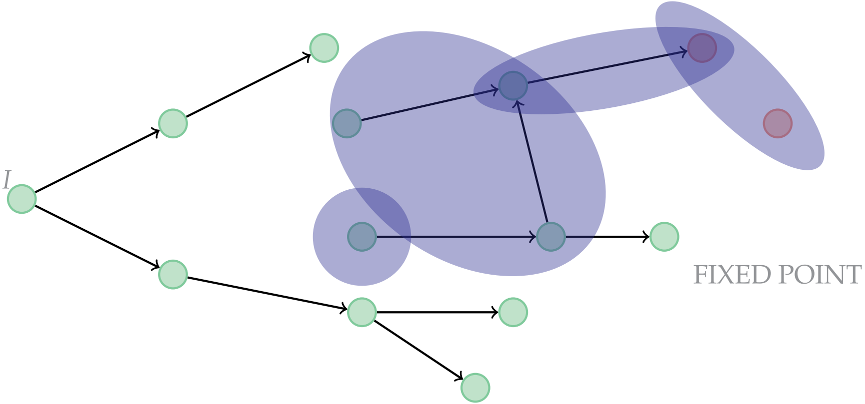
Start with $\neg P$ and proceed backward until fixpoint F. If the BDD for F contains an I-state, then a $\neg P$-state is reachable: counterexample trace.



Start with $\neg P$ and proceed backward until fixpoint F. If the BDD for F contains an I-state, then a $\neg P$-state is reachable: counterexample trace.



Start with $\neg P$ and proceed backward until fixpoint F. If the BDD for F contains an I-state, then a $\neg P$-state is reachable: counterexample trace.



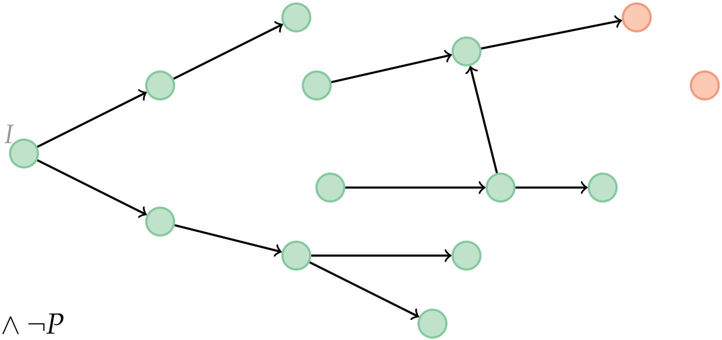
BDDs are much more than a representation of a boolean formula.

Compressed truth tables: BDDs represent all the models of a boolean formula.

$\Rightarrow$ often too much large

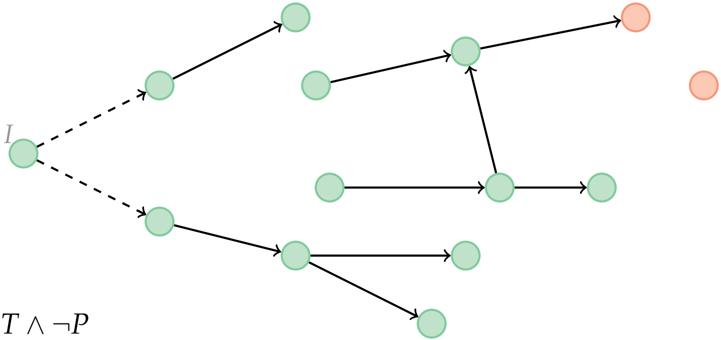
At iteration k, check if $I \wedge \bigwedge_{i=0}^{k-1} T^i \wedge \neg P^k$ is SAT. If so, stop with a counterexample of length k.

At iteration k, check if $I \wedge \bigwedge_{i=0}^{k-1} T^i \wedge \neg P^k$ is SAT. If so, stop with a counterexample of length k.



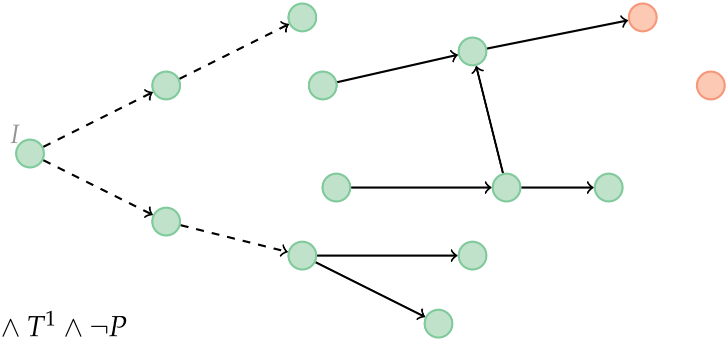
$I \wedge \neg P$

At iteration k, check if $I \wedge \bigwedge_{i=0}^{k-1} T^i \wedge \neg P^k$ is SAT. If so, stop with a counterexample of length k.



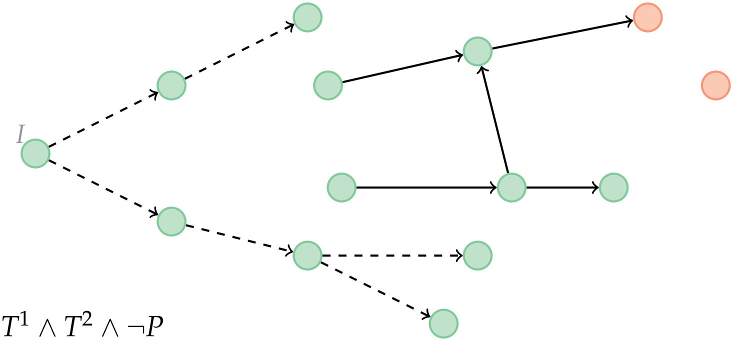
$I \wedge T \wedge \neg P$

At iteration k, check if $I \wedge \bigwedge_{i=0}^{k-1} T^i \wedge \neg P^k$ is SAT. If so, stop with a counterexample of length k.



$I \wedge T \wedge T^1 \wedge \neg P$

At iteration k, check if $I \wedge \bigwedge_{i=0}^{k-1} T^i \wedge \neg P^k$ is SAT. If so, stop with a counterexample of length k.



$I \wedge T \wedge T^1 \wedge T^2 \wedge \neg P$

- it leverages the big progress made in SAT solvers in the last decades;

- it leverages the big progress made in SAT solvers in the last decades;
- BMC looks for counterexamples of length k and increases k only if the formula of the current iteration is UNSAT;

- it leverages the big progress made in SAT solvers in the last decades;
- BMC looks for counterexamples of length k and increases k only if the formula of the current iteration is UNSAT;
- drawbacks:
  - in general it is not complete: we have to compute a big QBF to know the diameter of the graph;

- it leverages the big progress made in SAT solvers in the last decades;
- BMC looks for counterexamples of length k and increases k only if the formula of the current iteration is UNSAT;
- drawbacks:
  - in general it is not complete: we have to compute a big QBF to know the diameter of the graph;
  - it requires the unrolling of the transition relation:

$$I \wedge \left(T \wedge T^1 \wedge \cdots \wedge T^{k-1}\right) \wedge \neg P^k$$

  Both T and k can be very large: the formula can become too large for the SAT solver.

First Attempts to Incremental Inductive Verification

In order to prove that $P(x)$ is $\mathcal{M}$-invariant, one possibility is to check if P is inductive. With two SAT-solver calls, we check the validity of:

$$\begin{aligned} \text{(initiation)} \quad & I \Rightarrow P \\ \text{(consecution)} \quad & P \wedge T \Rightarrow P' \end{aligned}$$

In order to prove that $P(x)$ is $\mathcal{M}$-invariant, one possibility is to check if P is inductive. With two SAT-solver calls, we check the validity of:

$$\begin{aligned} \text{(initiation)} \quad & I \Rightarrow P \\ \text{(consecution)} \quad & P \land T \Rightarrow P' \end{aligned}$$

It is a sufficient condition to prove invariance for P. It is not also a necessary condition.

In order to prove that $P(x)$ is $\mathcal{M}$-invariant, one possibility is to check if P is inductive. With two SAT-solver calls, we check the validity of:

$$\begin{aligned} \text{(initiation)} \quad & I \Rightarrow P \\ \text{(consecution)} \quad & P \wedge T \Rightarrow P' \end{aligned}$$

It is a sufficient condition to prove invariance for P. It is not also a necessary condition.

Why?

If consecution fails, then:

If consecution fails, then:

- **Monolithic approach**: look for a stronger assertion F such that $F \wedge P$ is inductive. $F \wedge P$ is called an inductive strenghtening.

Incremental vs Monolithic

If consecution fails, then:

- **Monolithic approach**: look for a stronger assertion F such that $F \wedge P$ is inductive. $F \wedge P$ is called an inductive strenghtening.
- **Incremental proof**: look for a sequence of lemmata $\phi_1, \phi_2, \ldots, \phi_k = P$ such that ϕ_i is inductive relative to $\phi_1 \wedge \cdots \wedge \phi_{i-1}$, for all $1 < i \leq k$, *i.e.*,
  - $I \Rightarrow \phi_i$
  - $\phi_1 \wedge \cdots \wedge \phi_{i-1} \wedge \phi_i \wedge T \Rightarrow \phi_i'$

Incremental vs Monolithic

If consecution fails, then:

- **Monolithic approach**: look for a stronger assertion F such that $F \wedge P$ is inductive. $F \wedge P$ is called an inductive strenghtening.
- **Incremental proof**: look for a sequence of lemmata $\phi_1, \phi_2, \ldots, \phi_k = P$ such that ϕ_i is inductive relative to $\phi_1 \wedge \cdots \wedge \phi_{i-1}$, for all $1 < i \leq k$, *i.e.*,
  - $I \Rightarrow \phi_i$
  - $\phi_1 \wedge \cdots \wedge \phi_{i-1} \wedge \phi_i \wedge T \Rightarrow \phi_i'$

It follows that $P \wedge \bigwedge_{i=1}^{k-1} \phi_i$ is an inductive strengthening.

Abstraction

Note that:

- both methods do *not* compute a formula R for the exact set of reachable states in $\mathcal{M}$;

Note that:

- both methods do *not* compute a formula R for the exact set of reachable states in $\mathcal{M}$;
- rather, they find a formula $F \land P$ that represents a larger set of states *all satisfying* $F \land P$:
  - $\Rightarrow$ this F is a much smaller formula than R.

Naïve algorithm for finding an inductive strengthening:

1. $IS := P$

Monolithic Approach - Naïve algorithm

Naïve algorithm for finding an inductive strengthening:

1. $IS := P$
2. if IS is inductive, then we have found an inductive strengthening; stop.

Monolithic Approach - Naïve algorithm

Naïve algorithm for finding an inductive strengthening:

1. $IS := P$
2. if IS is inductive, then we have found an inductive strengthening; stop.
3. else find a CTI err (counterexample to inductiveness):

$$err \models IS \wedge T \wedge \neg IS'$$

Monolithic Approach - Naïve algorithm

Naïve algorithm for finding an inductive strengthening:

1. $IS := P$
2. if IS is inductive, then we have found an inductive strengthening; stop.
3. else find a CTI err (counterexample to inductiveness):

$$err \models IS \land T \land \neg IS'$$

   1. if $err \land I$ is SAT, then stop: P is NOT invariant;

Monolithic Approach - Naïve algorithm

Naïve algorithm for finding an inductive strengthening:

1. $IS := P$
2. if IS is inductive, then we have found an inductive strengthening; stop.
3. else find a CTI err (counterexample to inductiveness):

$$err \models IS \land T \land \neg IS'$$

   1. if $err \land I$ is SAT, then stop: P is NOT invariant;
   2. else set $IS := IS \land \neg err$ and go to Item 2.

Naïve algorithm for finding an inductive strengthening:

1. $IS := P$
2. if IS is inductive, then we have found an inductive strengthening; stop.
3. else find a CTI err (counterexample to inductiveness):

$$err \models IS \wedge T \wedge \neg IS'$$

    1. if $err \wedge I$ is SAT, then stop: P is NOT invariant;
    2. else set $IS := IS \wedge \neg err$ and go to Item 2.

At the end, the inductive strengthening (if any) will be:

$$P \wedge \bigwedge_{err \in CTI} \neg err$$

Consider the program $\mathcal{M}_1$:

```
x,y := 1,1
while *:
  x,y := x+1,y+x
```

Consider the program $\mathcal{M}_1$:

```
x,y := 1,1
while *:
  x,y := x+1,y+x
```

We want to prove that $y \geq 1$ is $\mathcal{M}_1$-invariant:

Consider the program $\mathcal{M}_1$:

```
x,y := 1,1
while *:
  x,y := x+1,y+x
```

We want to prove that $y \geq 1$ is $\mathcal{M}_1$-invariant:

- $\underbrace{x = 1 \wedge y = 1}_{I} \Rightarrow \underbrace{y \geq 1}_{P}$

Consider the program $\mathcal{M}_1$:

```
x,y := 1,1
while *:
  x,y := x+1,y+x

```

We want to prove that $y \geq 1$ is $\mathcal{M}_1$-invariant:

- $\underbrace{x = 1 \land y = 1}_{I} \Rightarrow \underbrace{y \geq 1}_{P}$
- $\underbrace{y \geq 1}_{P} \underbrace{\land x' = x + 1 \land y' = y + x}_{T} \not\Rightarrow \underbrace{y' \geq 1}_{P'}$

Consider the program $\mathcal{M}_1$:

```
x,y := 1,1
while *:
  x,y := x+1,y+x

```

We want to prove that $y \geq 1$ is $\mathcal{M}_1$-invariant:

- $\underbrace{x = 1 \wedge y = 1}_{I} \Rightarrow \underbrace{y \geq 1}_{P}$
- $\underbrace{y \geq 1}_{P} \wedge \underbrace{x' = x + 1 \wedge y' = y + x}_{T} \not\Rightarrow \underbrace{y' \geq 1}_{P'}$

Why?

Incremental Proof - Example

We establish the first inductive incremental lemma $\phi_1 \coloneqq x \geq 0$:

- $x = 1 \wedge y = 1 \Rightarrow x \geq 0$
- $x \geq 0 \wedge x' = x + 1 \wedge y' = y + x \Rightarrow x' \geq 0$

Incremental Proof - Example

We establish the first inductive incremental lemma $\phi_1 := x \geq 0$:

- $x = 1 \wedge y = 1 \Rightarrow x \geq 0$
- $x \geq 0 \wedge x' = x + 1 \wedge y' = y + x \Rightarrow x' \geq 0$

Now, $\phi_2 := y \geq 1$ is inductive relative to ϕ_1:

- $x = 1 \wedge y = 1 \Rightarrow y \geq 1$
- $\underbrace{x \geq 0}_{\phi_1} \wedge y \geq 1 \wedge x' = x + 1 \wedge y' = y + x \Rightarrow y' \geq 1$

Incremental Proof - Example

We establish the first inductive incremental lemma $\phi_1 \coloneqq x \geq 0$:

- $x = 1 \wedge y = 1 \Rightarrow x \geq 0$
- $x \geq 0 \wedge x' = x + 1 \wedge y' = y + x \Rightarrow x' \geq 0$

Now, $\phi_2 \coloneqq y \geq 1$ is inductive relative to ϕ_1:

- $x = 1 \wedge y = 1 \Rightarrow y \geq 1$
- $\underbrace{x \geq 0}_{\phi_1} \wedge y \geq 1 \wedge x' = x + 1 \wedge y' = y + x \Rightarrow y' \geq 1$

We have found the inductive strengthening $\phi_1 \wedge \phi_2$, by means of an incremental proof.

Limitation of incremental proofs

Consider the program $\mathcal{M}_2$:

```
x,y := 1,1
while *:
  x,y := x+y,y+x
```

We want to prove $\phi_2 := y \geq 1$.

Consider the program $\mathcal{M}_2$:

```
x,y := 1,1
while *:
  x,y := x+y,y+x

```

We want to prove $\phi_2 := y \geq 1$.
Like before, ϕ_2 is not inductive on its own.

Consider the program $\mathcal{M}_2$:

```
x,y := 1,1
while *:
  x,y := x+y,y+x

```

We want to prove $\phi_2 := y \geq 1$.
Like before, ϕ_2 is not inductive on its own.
$\cdots$ but now neither is $\phi := x \geq 0$:

- $x = 1 \wedge y = 1 \Rightarrow x \geq 0$
- $x \geq 0 \wedge x' = x + y \wedge y' = y + x \not\Rightarrow x' \geq 0$

Limitation of incremental proofs

Consider the program $\mathcal{M}_2$:

```
x,y := 1,1
while *:
  x,y := x+y,y+x

```

We want to prove $\phi_2 := y \geq 1$.
Like before, ϕ_2 is not inductive on its own.
$\cdots$ but now neither is $\phi := x \geq 0$:

- $x = 1 \land y = 1 \Rightarrow x \geq 0$
- $x \geq 0 \land x' = x + y \land y' = y + x \not\Rightarrow x' \geq 0$

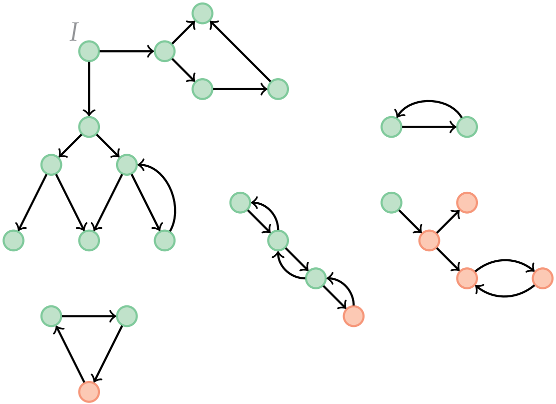
Monolithic approach = worst case of incremental proofs.

- FSIS: Finite-State Inductive Strengthening. It follows the incremental methodology.
  Aaron R Bradley and Zohar Manna (2007). "Checking safety by inductive generalization of counterexamples to induction". **In: *Formal Methods in Computer Aided Design (FMCAD'07)*. IEEE, pp. 173–180**

- FSIS: Finite-State Inductive Strengthening. It follows the incremental methodology.
  Aaron R Bradley and Zohar Manna (2007). "Checking safety by inductive generalization of counterexamples to induction". In: *Formal Methods in Computer Aided Design (FMCAD'07)*. IEEE, pp. 173–180
- *"this algorithm is a result of asking the question: if the incremental method is often better for humans, might it be better for algorithms as well?"*
  Aaron R Bradley (2012). "Understanding ic3". In: *International Conference on Theory and Applications of Satisfiability Testing*. Springer, pp. 1–14

FSIS - Algorithm

- FSIS: Finite-State Inductive Strengthening. It follows the incremental methodology.
  Aaron R Bradley and Zohar Manna (2007). "Checking safety by inductive generalization of counterexamples to induction". **In: *Formal Methods in Computer Aided Design (FMCAD'07)*. IEEE, pp. 173–180**
- *"this algorithm is a result of asking the question: if the incremental method is often better for humans, might it be better for algorithms as well?"*
  Aaron R Bradley (2012). "Understanding ic3". **In: *International Conference on Theory and Applications of Satisfiability Testing*. Springer, pp. 1–14**
- the core of the algorithm is the generalization an error state.

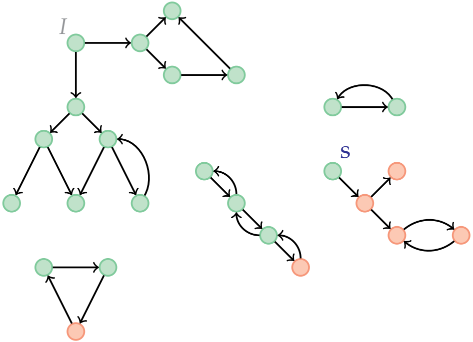

I

Check if P is inductive (relative to nobody). Check the validity of:

✓ $I \Rightarrow P$

✗ $P \wedge T \Rightarrow P'$

State s is a CTI.



FSIS - Example - 1^{st} iteration

- s is a cube returned by the SAT-solver; $\neg s$ is a clause encoding all states different from s;

FSIS - Example - 1^{st} iteration

- s is a cube returned by the SAT-solver; $\neg s$ is a clause encoding all states different from s;
- generalization of error state s: find a clause ϕ_1 such that
  - $\phi_1 \subseteq \neg s$; *(it excludes s)*
  - ϕ_1 is inductive (relative to nobody); *(it includes at least all the reachable states)*
  - ϕ_1 is minimal. *(it excludes the maximal number of non-reachable states)*
    - recall the nice property of clauses: if $c \subseteq d$ then $[\![c]\!] \subseteq [\![d]\!]$

FSIS - Example - 1^{st} iteration

- s is a cube returned by the SAT-solver; $\neg s$ is a clause encoding all states different from s;
- generalization of error state s: find a clause ϕ_1 such that
  - $\phi_1 \subseteq \neg s$; *(it excludes s)*
  - ϕ_1 is inductive (relative to nobody); *(it includes at least all the reachable states)*
  - ϕ_1 is minimal. *(it excludes the maximal number of non-reachable states)*
    - recall the nice property of clauses: if $c \subseteq d$ then $[\![c]\!] \subseteq [\![d]\!]$
- if ϕ_1 does exist, it becomes the first incremental lemma.

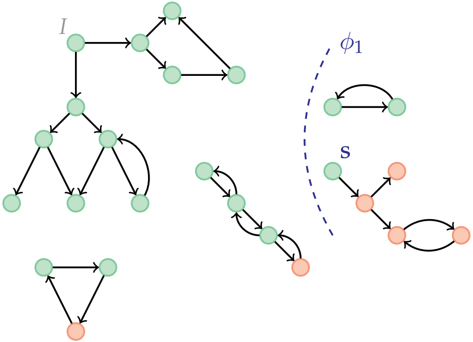
FSIS - Example - 1^{st} iteration

ϕ_1 can be thought as a "boolean" cutting plane.

Which states are excluded by ϕ_1?

(i) those who can reach s in one step

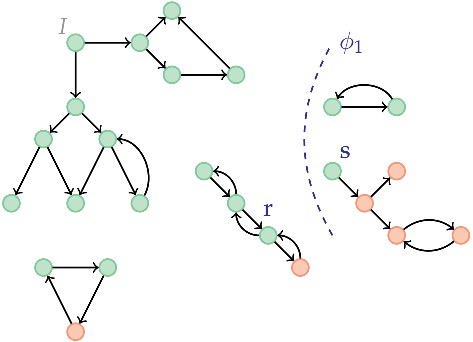
(ii) states “similar" to s (they share with s the dropped literals).



Check if P is inductive relative to ϕ_1:

- ✓ $I \Rightarrow P$
- ✗ $\phi_1 \wedge P \wedge T \Rightarrow P'$

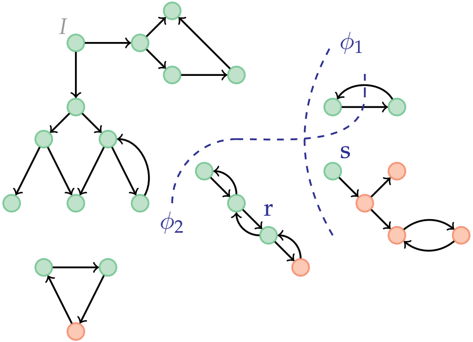
State r is a CTI.



FSIS - Example - 2^{nd} iteration

- generalization of error state r:
  - $\phi_2 \subseteq \neg r$;
  - ϕ_2 is inductive relative to ϕ_1;
  - ϕ_2 is minimal;
- ϕ_2 is the second incremental lemma.

- generalization of error state r:
  - $\phi_2 \subseteq \neg r$;
  - ϕ_2 is inductive relative to ϕ_1;
  - ϕ_2 is minimal;
- ϕ_2 is the second incremental lemma.



Why “inductive relative to"?

- it would have been correct to generate ϕ_2 inductive (relative to its own), but it’s more than what we need;
  - at the end we will consider the AND of all the lemmata;

FSIS - Example - 2^{nd} iteration

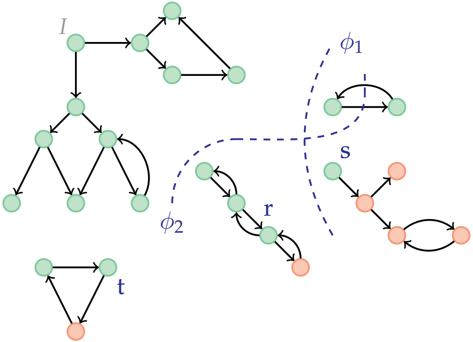
Why “inductive relative to"?

- it would have been correct to generate ϕ_2 inductive (relative to its own), but it’s more than what we need;
  - at the end we will consider the AND of all the lemmata;
- in general, it is faster to generate “inductive relative to" clauses.
  - intuitively, we are considering many fewer states of the system.

Check if P is inductive relative to $\phi_1 \land \phi_2$:

- ✓ $I \Rightarrow P$
- ✗ $\phi_1 \land \phi_2 \land P \land T \not\Rightarrow P'$

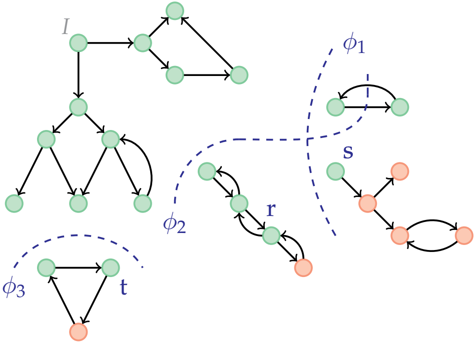
State t is a CTI.



FSIS - Example - 3^{rd} iteration

- generalization of error state t:
  - $\phi_3 \subseteq \neg t$;
  - ϕ_3 is inductive relative to $\phi_1 \wedge \phi_2$;
  - ϕ_3 is minimal;
- ϕ_3 is the second incremental lemma.

- generalization of error state t:
  - $\phi_3 \subseteq \neg t$;
  - ϕ_3 is inductive relative to $\phi_1 \wedge \phi_2$;
  - ϕ_3 is minimal;
- ϕ_3 is the second incremental lemma.



FSIS - Example - 4^{th} iteration

Check if P is inductive relative to $\phi_1 \wedge \phi_2 \wedge \phi_3$:

✓ $I \Rightarrow P$

✗ $\phi_1 \wedge \phi_2 \wedge \phi_3 \wedge P \wedge T \Rightarrow P'$

FSIS - Example - 4^{th} iteration

Check if P is inductive relative to $\phi_1 \wedge \phi_2 \wedge \phi_3$:

✓ $I \Rightarrow P$

✗ $\phi_1 \wedge \phi_2 \wedge \phi_3 \wedge P \wedge T \Rightarrow P'$

- $\phi_1 \wedge \phi_2 \wedge \phi_3 \wedge P$ is an inductive strengthening.
- P is $\mathcal{M}$-invariant.

Worst case

- suppose that an error state s does *not* have a minimal inductive generalization;

Worst case

- suppose that an error state s does *not* have a minimal inductive generalization;
- worst case: we proceed with the monolithic technique;

$$P := P \land \neg s$$

Worst case

- suppose that an error state s does *not* have a minimal inductive generalization;
- worst case: we proceed with the monolithic technique;

$$P := P \wedge \neg s$$

- eventually,
  - either $I \wedge \neg P$ is SAT: P is not invariant;
  - or we find an inductive strengthening $P \wedge \bigwedge_{i=0}^{n} \phi_i$;

Complexity:

- it is on the convergence of the procedure, not on the calls to the SAT-solver as before;
- each SAT-solver call is relatively small compared to those made by BMC.

Complexity:

- it is on the convergence of the procedure, not on the calls to the SAT-solver as before;
- each SAT-solver call is relatively small compared to those made by BMC.

Parallelization:

- straightforward; *"by simply using a randomized decision procedure to obtain the CTIs, each process is likely to analyze a different part of the state-space."*
  Aaron R Bradley and Zohar Manna (2007). "Checking safety by inductive generalization of counterexamples to induction". In: *Formal Methods in Computer Aided Design (FMCAD'07)*. IEEE, pp. 173–180

IC3

Incremental Construction of Inductive Clauses of Indubitable Correctness

- FSIS sometimes enters a long search for the next relatively inductive clauses;
- IC3 de-emphasizes global information in favor of stepwise information: we will generate clauses that ensure that an error is unreachable up to some number of steps.

Sequence of frames $F_0(= I), F_1, F_2, \ldots, F_k$:

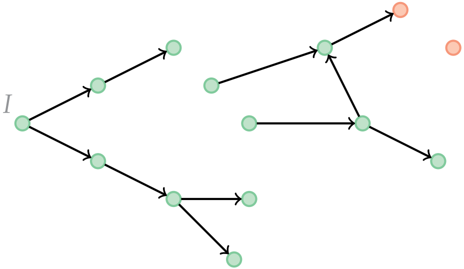
- each F_i is a set of clauses, *i.e.*, a CNF formula;
- each F_i is an over-approximation of the set of states reachable in at most k steps;
- the algorithm stops when $F_i \equiv F_{i+1}$. We will maintain the invariance that $clauses(F_{i+1}) \subseteq clauses(F_i)$: the equivalence check is simply a syntactic test: $F_i = F_{i+1}$.
- F_0 is a special frame always equal to I.

IC3 - 1^{st} iteration

Check if there are counterexamples of length 0 or 1 with these two SAT-queries:

✗ $I \wedge \neg P$

✗ $F_0 (= I) \wedge T \wedge \neg P'$



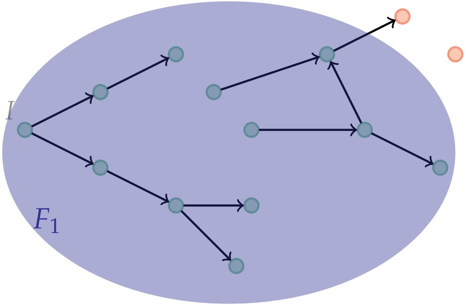
IC3 - 1^{st} iteration

Check if there are counterexamples of length 0 or 1 with these two SAT-queries:

✗ $I \wedge \neg P$

✗ $F_0 (= I) \wedge T \wedge \neg P'$

Since $F_0 \wedge T \Rightarrow P'$, we set $F_1 := P$. (over-approximation)



IC3 - 2^{nd} iteration

At iteration k, check if $F_k \wedge T \wedge \neg P'$; in this case ($k = 1$):

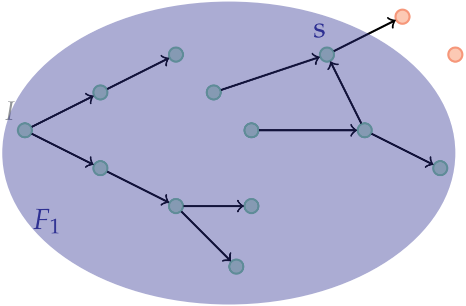
$$✓ \quad F_1 \wedge T \wedge \neg P'$$

i.e., there exists an F_k-state that leads in one step to an error state?

At iteration k, check if $F_k \wedge T \wedge \neg P'$; in this case ($k = 1$):

✓ $F_1 \wedge T \wedge \neg P'$

i.e., there exists an F_k-state that leads in one step to an error state?



IC3 - 2^{nd} iteration (blocking phase)

At iteration $k (= 1)$, we check whether $\neg s$ is inductive relative to $F_{k-1} = (F_0 = I)$: ✓
$\Rightarrow$ error state s is not reachable in at least $k = 1$ step.

IC3 - 2^{nd} iteration (blocking phase)

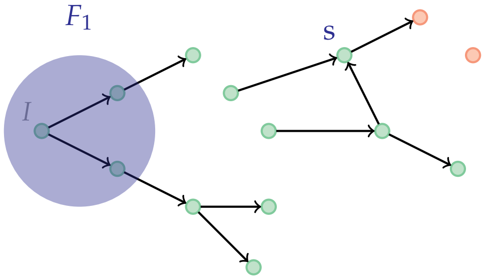
At iteration $k(= 1)$, we check whether $\neg s$ is inductive relative to $F_{k-1} = (F_0 = I)$: ✓
$\Rightarrow$ error state s is not reachable in at least $k = 1$ step.
We find a minimal $\phi_1 \subseteq \neg s$ such that ϕ_1 is inductive relative to $F_0(= I)$.

$\Rightarrow \phi_1$ excludes the error state s (and similar states) but contains at least all the states reachable in at most $k = 1$ steps.

At iteration $k(=1)$, we check whether $\neg s$ is inductive relative to $F_{k-1} = (F_0 = I)$: ✓
$\Rightarrow$ error state s is not reachable in at least $k = 1$ step.
We find a minimal $\phi_1 \subseteq \neg s$ such that ϕ_1 is inductive relative to $F_0(= I)$.

$\Rightarrow$ ϕ_1 excludes the error state s (and similar states) but contains at least all the states reachable in at most $k = 1$ steps.

We add ϕ_1 to all the previous frames. In this case $F_1 := F_1 \wedge \phi_1$.



We have found a CTI s such that $s \models F_k \wedge T \wedge \neg P'$.

$\Rightarrow$ we want to generalize the error s or to prove that it's reachable from an initial state

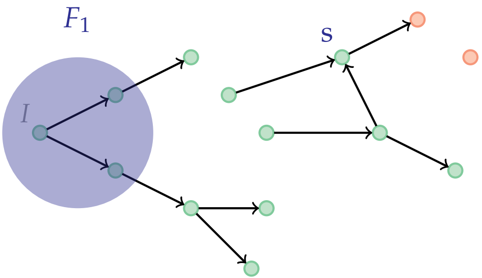
We have found a CTI s such that $s \models F_k \wedge T \wedge \neg P'$.

$\Rightarrow$ we want to generalize the error s or to prove that it's reachable from an initial state

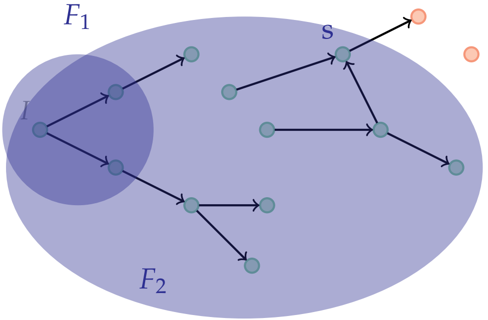
if $\neg s$ is inductive relative to F_{k-1}, then generate a minimal subclause $c \subseteq \neg s$ inductive relative to F_{k-1}, *i.e.*, c holds for at least all states reachable in i steps.

$\Rightarrow$ add c to frames $F_0 \ldots F_{k+1}$, *i.e.*, refine the over-approximations.

We create a new frame only when $F_k \wedge T \Rightarrow P'$ is valid.



We create a new frame only when $F_k \wedge T \Rightarrow P'$ is valid.
In this case, $F_1 \wedge T \Rightarrow P'$ is valid. We create a new frame $F_2 := P$.



Propagation phase: After creating a new frame $F_{k+1} := P$, we perform the propagation phase: we push forward the clause discovered in frame F_i for some i.

Propagation phase: After creating a new frame $F_{k+1} := P$, we perform the propagation phase: we push forward the clause discovered in frame F_i for some i. For all $0 \leq i \leq k$ and $c \in F_i$, check if

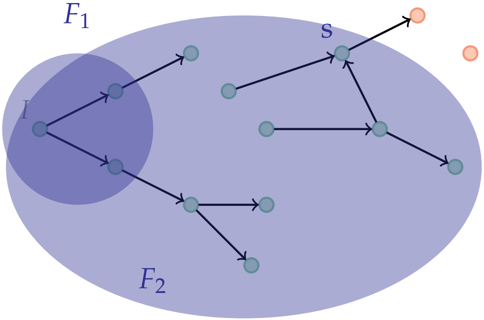
$$F_i \wedge T \Rightarrow c'$$

If $c \notin clauses(F_{i+1})$, then set $F_{i+1} := F_{i+1} \cup \{c\}$

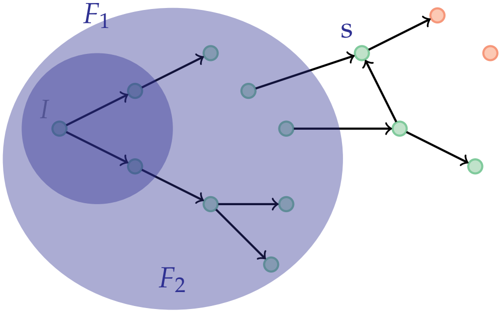
$\Rightarrow$ it propagates forward the errors
$\Rightarrow$ it helps the discovery of mutually inductive clauses

IC3 - 3^{rd} iteration

Check if $F_2 \wedge T \wedge \neg P'$ (✓): counterexample s.
Check if $\neg s$ is inductive relative to F_1: ✓ $\Rightarrow$ error state s is not reachable for at least $k = 2$ steps.

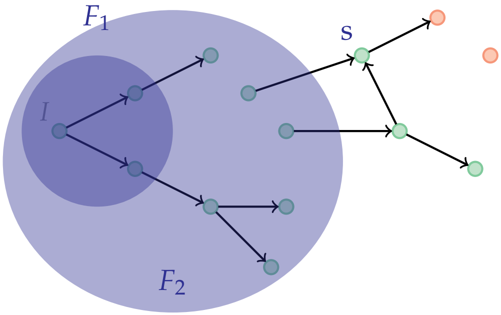


Check if $F_2 \wedge T \wedge \neg P'$ (✓): counterexample s.
Check if $\neg s$ is inductive relative to F_1: ✓ $\Rightarrow$ error state s is not reachable for at least $k = 2$ steps.
Blocking phase: find minimal subclause $\phi_2 \subseteq \neg s$ inductive relative to F_1. Add ϕ_2 to frames F_0 and F_1.

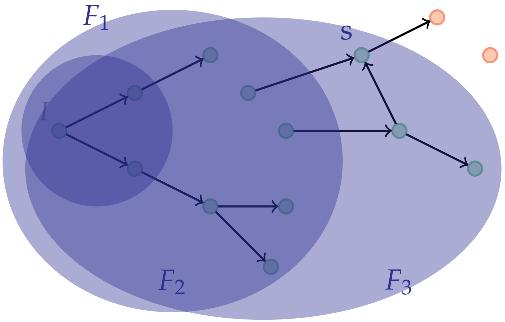


IC3 - 4^{rd} iteration

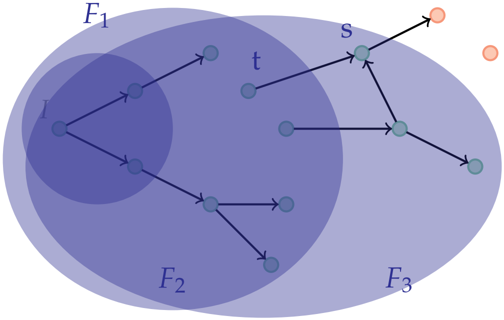
Since $F_2 \wedge T \Rightarrow P'$ is valid, we create a new frame $F_3 := P$.



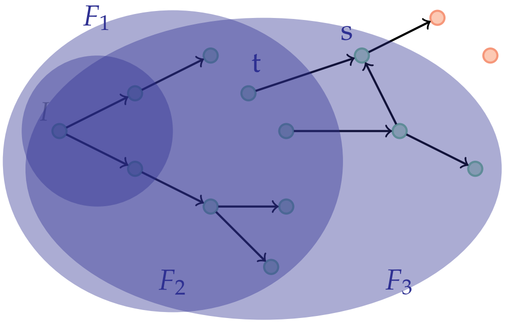
Since $F_2 \wedge T \Rightarrow P'$ is valid, we create a new frame $F_3 := P$.



Again $F_3 \wedge T \wedge \neg P'$ (✓): counterexample s.
But now $\neg s$ is not inductive relative to F_2



Again $F_3 \wedge T \wedge \neg P'$ (✓): counterexample s.
But now $\neg s$ is not inductive relative to F_2: error state s *could* be reachable in $k = 3$ steps . . .



Instead of generating a clause that excludes s (it is possible), we call the algorithm recursively on the predecessor t of s

... remember that t could still be reachable as far as we know ...

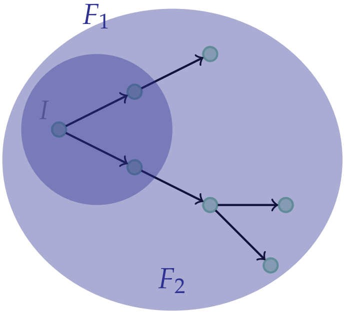
IC3 - 4^{rd} iteration

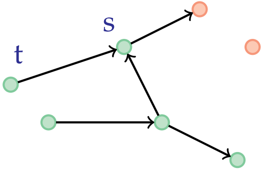
Instead of generating a clause that excludes s (it is possible), we call the algorithm recursively on the predecessor t of s

... remember that t could still be reachable as far as we know ...

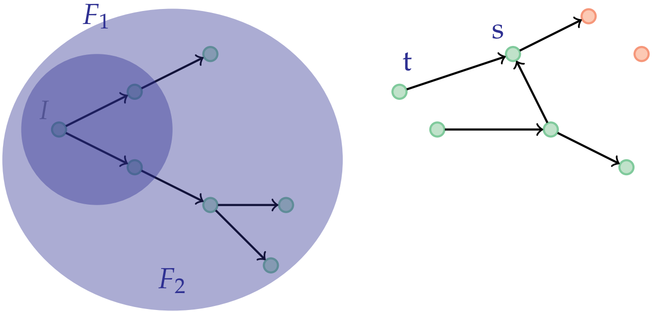
"t is the new s" ;-)

We want to remove error state t from F_2. $\neg t$ is inductive relative to F_1: find min subclause $\phi_4 \subseteq \neg t$ and add it to F_1 and F_2.



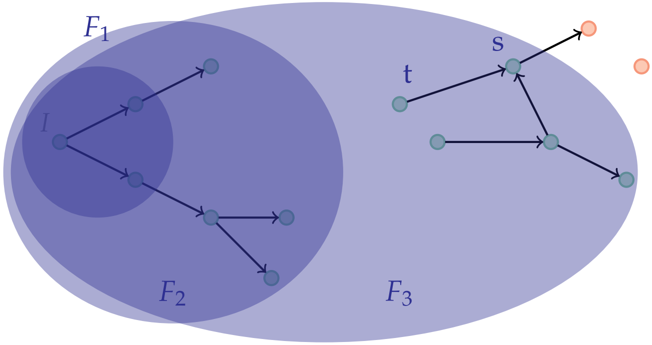


We want to remove error state t from F_2. $\neg t$ is inductive relative to F_1: find min subclause $\phi_4 \subseteq \neg t$ and add it to F_1 and F_2.

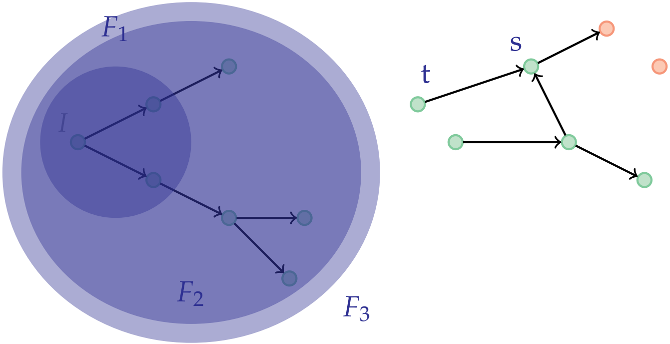


If in this process we go back with recursion until an initial state, then we would have found a counterexample.

Now error state s in frame F_3 can be generalized: find min clause $\phi_5 \subseteq \neg s$ inductive relative to F_2.



Now error state s in frame F_3 can be generalized: find min clause $\phi_5 \subseteq \neg s$ inductive relative to F_2.



$F_2 = F_3$: IC3 terminates with True.

- FAIR: IC3 for ω-regular properties (*e.g.*, LTL).
  Aaron R Bradley, Fabio Somenzi, et al. (2011). "An incremental approach to model checking progress properties". In: *2011 Formal Methods in Computer-Aided Design (FMCAD)*. IEEE, pp. 144–153
- IICTL: IC3 for CTL properties.
  Zyad Hassan, Aaron R Bradley, and Fabio Somenzi (2012). "Incremental, inductive CTL model checking". In: *International Conference on Computer Aided Verification*. Springer, pp. 532–547
- Infinite-state: software model checking via IC3.
  Alessandro Cimatti and Alberto Griggio (2012). "Software model checking via IC3". In: *International Conference on Computer Aided Verification*. Springer, pp. 277–293

REFERENCES

Armin Biere et al. (1999). "Symbolic model checking without BDDs". In: *International Conference on Tools and Algorithms for the Construction and Analysis of Systems (TACAS)*. Springer, pp. 193–207.

Aaron R Bradley (2012). "Understanding ic3". In: *International Conference on Theory and Applications of Satisfiability Testing*. Springer, pp. 1–14.

Aaron R Bradley and Zohar Manna (2007). "Checking safety by inductive generalization of counterexamples to induction". In: *Formal Methods in Computer Aided Design (FMCAD'07)*. IEEE, pp. 173–180.

Aaron R Bradley, Fabio Somenzi, et al. (2011). "An incremental approach to model checking progress properties". In: *2011 Formal Methods in Computer-Aided Design (FMCAD)*. IEEE, pp. 144–153.

Alessandro Cimatti and Alberto Griggio (2012). "Software model checking via IC3". In: *International Conference on Computer Aided Verification*. Springer, pp. 277–293.

Edmund M. Clarke, E Allen Emerson, and A Prasad Sistla (1986). "Automatic verification of finite-state concurrent systems using temporal logic specifications". In: *ACM Transactions on Programming Languages and Systems (TOPLAS)* 8.2, pp. 244–263.

Zyad Hassan, Aaron R Bradley, and Fabio Somenzi (2012). "Incremental, inductive CTL model checking". In: *International Conference on Computer Aided Verification*. Springer, pp. 532–547.

Charles Antony Richard Hoare (1969). "An axiomatic basis for computer programming". In: *Communications of the ACM* 12.10, pp. 576–580.

Zohar Manna and Amir Pnueli (1995). ***Temporal verification of reactive systems - safety*.** Springer. ISBN: 978-0-387-94459-3.

Kenneth L McMillan (1993). "Symbolic model checking". In: *Symbolic Model Checking*. Springer, pp. 25–60.