A Dynamical Approach to Phase Transitions

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The last years have seen an increasing interest in establishing both good mathematical models for evolutive phase transitions and consequently in proving rigorous results of existence, (non)uniqueness, stability of solutions. In this talk I shall focus on models in one space dimension, where both previous items have been received satisfactory answers.

Initial-value problems for the most naive models of phase transitions have been recognized since long, [7], to be not well posed: uniqueness fails. The reason has been understood both from a physical and a mathematical point of view; Abeyaratne, Knowles and Truskinovsky, [1, 8] suggested to include the entropy dissipation across the interface as a criterion to single out solutions.

These papers originated the so called *kinetic* approach to phase transitions. In a general setting a phase transitions is then modelled as a weak solutions to a system of conservation laws,

$$\partial_t u + \partial_x \left[f(u)\right] = 0\,.$$

where the convective term f is defined in two *disjoints* open sets Ω_1 and Ω_2 of $\mathbb{R}^n$, the phase states. A phase boundary is then a discontinuity separating values of the solutions in Ω_1 from those in Ω_2. Conditions on kinetic functions in order to obtain globally defined solutions with bounded variation have been given for many continuum and fluid models in [2, 6, 4, 3, 5]. Applications to elastodynamics shall be given here.

References

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