BOLZA PROBLEMS WITH DISCONTINUOUS LAGRANGIANS: REGULARITY OF QUASI–MINIMIZERS AND RELAXATION

ANDREA DAVINI

ABSTRACT. We consider the classical Bolza Problem of the Calculus of Variations associated to an autonomous Lagrangian $L(x, q)$, convex and superlinear in q, but only Borel–measurable in x. Under these assumptions, the associated integral functional is not lower semicontinuous with respect to the suitable topology which assures the existence of minimizers. It is therefore natural to consider its relaxed formulation. However, the possibility to find an integral representation for the associated functional is strictly related to the regularity of quasi–minimizers of the initial problem. We propose a new approach, based on suitable reparametrization arguments, to derive the desired *a priori* estimates on the Lipschitz constants of quasi–minimizers. Our result can be compared with the ones obtained by M. Amar, G. Bellettini and S. Venturini (*Proc. Roy. Soc. Edinburgh,* **128A**, pp. 193–217) for abstract functionals satisfying polynomial growth conditions of order $p > 1$.