Università degli Studi di Udine

Dottorato di Ricerca in Ingegneria Industriale e dell'Informazione

Ph.D. Thesis

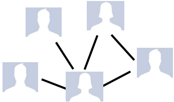
STRUCTURAL ANALYIS AND CONTROL OF DYNAMICAL NETWORKS

Giulia Giordano

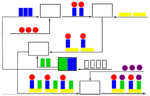
Supervisor: Prof. **Franco Blanchini**

Udine, April 8, 2016

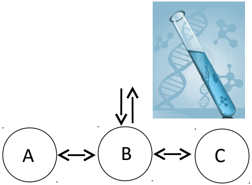
Dynamical networks



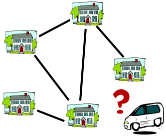
social networks



production and distribution systems

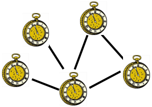


(bio)chemical processes

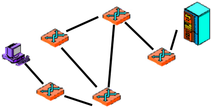


traffic management

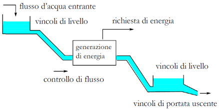
DYNAMICAL NETWORKS



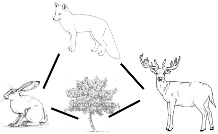
synchronization



telecommunication / data communication networks

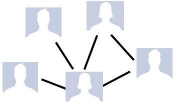


water distribution networks

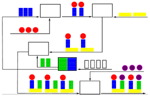


biological systems and ecosystems

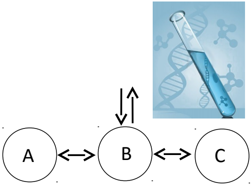
Dynamical networks: graph representation



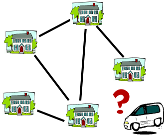
social networks



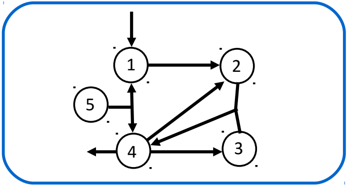
production and
distribution systems

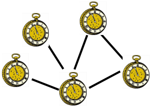


(bio)chemical processes

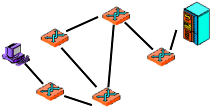


traffic management

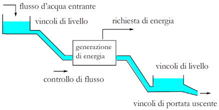




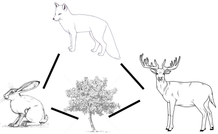
synchronization



telecommunication / data
communication networks

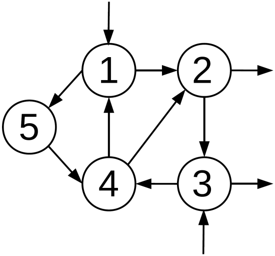


water distribution
networks

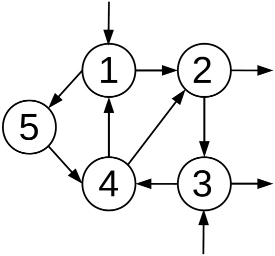


biological systems
and ecosystems

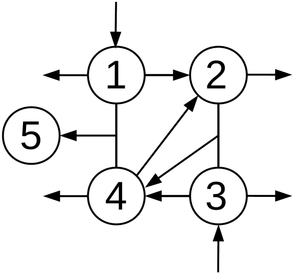
Graph:



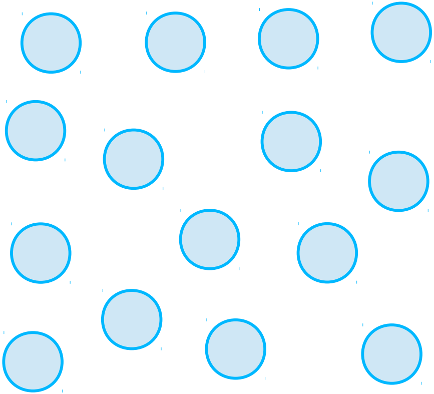
Graph:



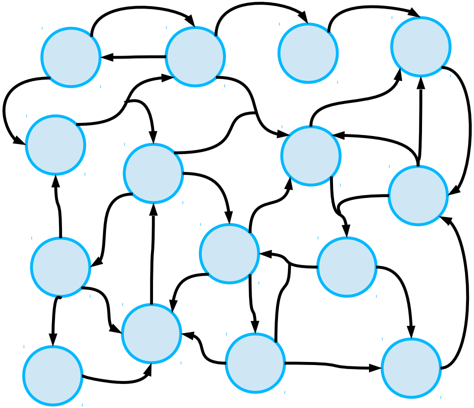
Hypergraph:



Graph representation: nodes

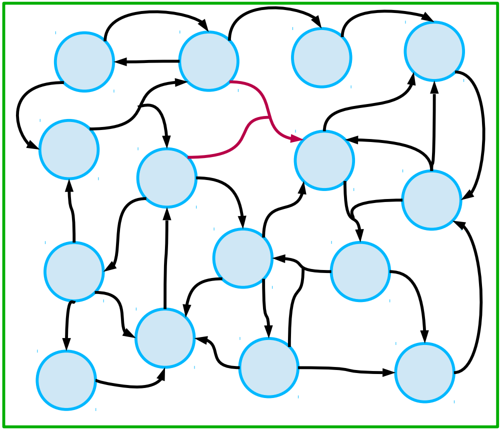


Nodes: Agents



Arcs: Interactions

Local interactions ⇒ Global behaviour



Local Interactions

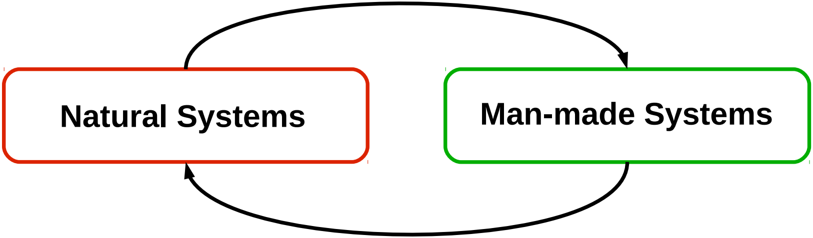
Global Behaviour

A twofold goal

Structural Analysis of Dynamical Networks

Network-Decentralised Control Synthesis

...especially meant for...



Structural analysis

Structural properties

- Parameter-free approach: **system structure**

Structural analysis

Explain behaviours based on the system inherent **structure** (graph)

Structural properties

- Parameter-free approach: system structure

Structural analysis

Explain behaviours based on the system inherent structure (graph)

- **Structurally assess fundamental properties**

Structural properties

Satisfied by all the systems of a family specified by a structure, without numerical bounds.

Structural properties

- Parameter-free approach: **system structure**

Structural analysis

Explain behaviours based on the system inherent **structure** (graph)

- **Structurally assess fundamental properties**

Structural properties

Satisfied by **all** the systems of a **family** specified by a **structure**, without numerical bounds.

- Applications to **biochemical systems**

Structural properties in nature

Biological systems $\rightarrow$ extremely **robust**: fundamental properties **preserved** despite **huge uncertainties and parameter variations**.

The BDC-decomposition

$$\dot{x}(t) = Sg(x(t)) + g_0, \qquad g \text{ monotonic functions}$$

The BDC-decomposition

$$\dot{x}(t) = Sg(x(t)) + g_0, \qquad g \text{ monotonic functions}$$

Local BDC-decomposition

The Jacobian can be decomposed as:

$$J(x) = \frac{\partial Sg(x)}{\partial x} = B\Delta(x)C, \qquad \Delta(x) \succ 0.$$

The BDC-decomposition

$$\dot{x}(t) = Sg(x(t)) + g_0, \qquad g \text{ monotonic functions}$$

Local BDC-decomposition

The Jacobian can be decomposed as:

$$J(x) = \frac{\partial Sg(x)}{\partial x} = B\Delta(x)C, \qquad \Delta(x) \succ 0.$$

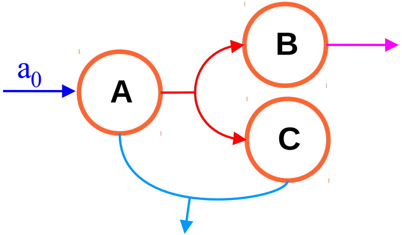
Global BDC-decomposition

Given the equilibrium $\bar{x}$, $z \doteq x - \bar{x}$. The system can be rewritten as:

$$\dot{z}(t) = [BD(z)C]\ z(t), \qquad D(z) \succ 0.$$

BDC-decomposition: example

$$\begin{bmatrix} \dot{a} \\ \dot{b} \\ \dot{c} \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 \\ 1 & 0 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} g_a(a) \\ g_{ac}(a,c) \\ g_b(b) \end{bmatrix} + \begin{bmatrix} a_0 \\ 0 \\ 0 \end{bmatrix}$$



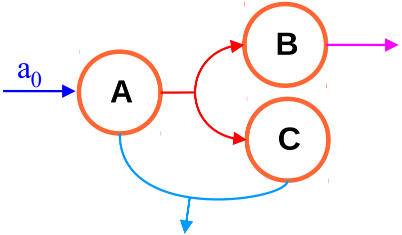
$$\dot{z} = BDCz$$

$$D = \text{diag}\left\{ \frac{\partial g_a}{\partial a}, \frac{\partial g_{ac}}{\partial a}, \frac{\partial g_{ac}}{\partial c}, \frac{\partial g_b}{\partial b} \right\} \succ 0$$

$$B = \begin{bmatrix} -1 & -1 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & -1 & -1 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

BDC-decomposition: example

$$\begin{bmatrix} \dot{a} \\ \dot{b} \\ \dot{c} \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 \\ 1 & 0 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} g_a(a) \\ g_{ac}(a,c) \\ g_b(b) \end{bmatrix} + \begin{bmatrix} a_0 \\ 0 \\ 0 \end{bmatrix}$$



$$\dot{z} = BDCz$$

$$D = \mathrm{diag}\left\{ \frac{\partial g_a}{\partial a}, \frac{\partial g_{ac}}{\partial a}, \frac{\partial g_{ac}}{\partial c}, \frac{\partial g_b}{\partial b} \right\} \succ 0$$

$$B = \begin{bmatrix} -1 & -1 & -1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & -1 & -1 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Structure: **parameter free**, no numerical bounds.

Structurally assess stability: polyhedral Lyapunov functions

Absorb the nonlinear system in a Linear Differential Inclusion

$D(z(t)) \to D(t)$

$$\dot{z}(t) = [BD(t)C]\ z(t), \qquad D(t) \succ 0.$$

F. Blanchini and G. Giordano, "Piecewise-linear Lyapunov Functions for Structural Stability of Biochemical Networks", *Automatica*, 2014

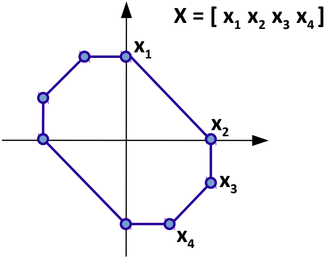
Structurally assess stability: polyhedral Lyapunov functions

Absorb the nonlinear system in a Linear Differential Inclusion

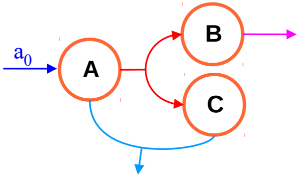
$D(z(t)) \rightarrow D(t)$

$$\dot{z}(t) = [BD(t)C]\ z(t), \qquad D(t) \succ 0.$$

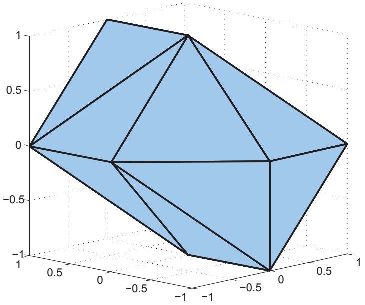
Iterative algorithm to compute
the **unit ball of the polyhedral Lyapunov function** (if any).



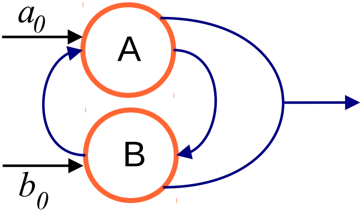
F. Blanchini and G. Giordano, "Piecewise-linear Lyapunov Functions for Structural Stability of Biochemical Networks", *Automatica*, 2014

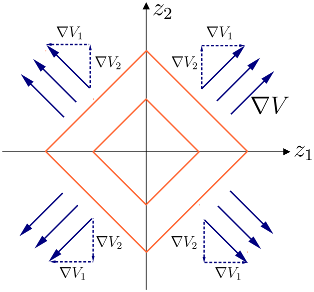


The procedure converges $\Longrightarrow$ structurally stable



Structurally assess stability: polyhedral... why?

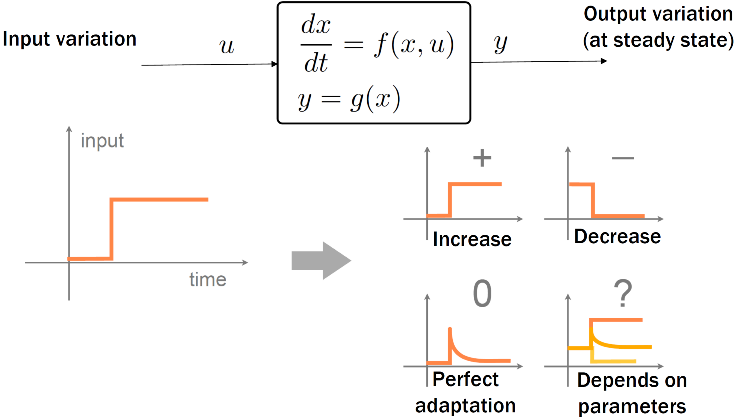




Claim

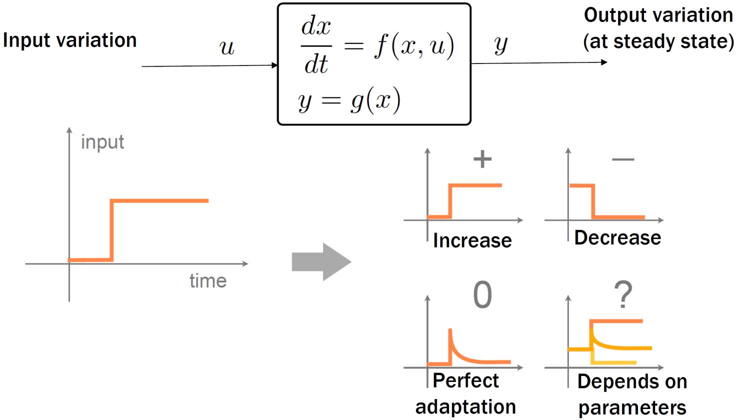
The only **structural** *Lyapunov function is* **polyhedral***!*

Structural steady-state analysis: the influence matrix



G. Giordano, C. Cuba Samaniego, E. Franco, F. Blanchini, "Computing the Structural Influence Matrix for Biological Systems", *J. Math. Biol.*, 2015

Structural steady-state analysis: the influence matrix



Structural influence of variable j on variable i

Assuming stability, $\Sigma_{ij} \in \{+, -, 0, ?\}$: sign of the steady-state variation of $x_i(\infty)$ due to a step input acting on x_j.

G. Giordano, C. Cuba Samaniego, E. Franco, F. Blanchini, "Computing the Structural Influence Matrix for Biological Systems", *J. Math. Biol.*, 2015

The structural influence matrix

For systems admitting a *BDC* decomposition

$$\Sigma_{ij} = H_i(-BDC)^{-1}E_j,$$

H output matrix, E input matrix $\rightarrow$ efficient **vertex algorithm**

G. Giordano, C. Cuba Samaniego, E. Franco, F. Blanchini, "Computing the Structural Influence Matrix for Biological Systems", *J. Math. Biol.*, 2015

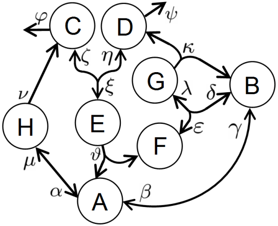
The structural influence matrix

For systems admitting a *BDC* decomposition

$$\Sigma_{ij} = H_i(-BDC)^{-1}E_j,$$

H output matrix, E input matrix $\rightarrow$ efficient **vertex algorithm**

Network from Shinar&Feinberg (2010)



$$\begin{bmatrix} + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + \\ + & + & + & 0 & + & 0 & + & + \\ 0 & 0 & 0 & + & + & + & + & 0 \\ + & + & + & + & + & + & + & + \\ ? & ? & ? & ? & ? & ? & ? & ? \\ + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + \end{bmatrix}$$

G. Giordano, C. Cuba Samaniego, E. Franco, F. Blanchini, "Computing the Structural Influence Matrix for Biological Systems", *J. Math. Biol.*, 2015

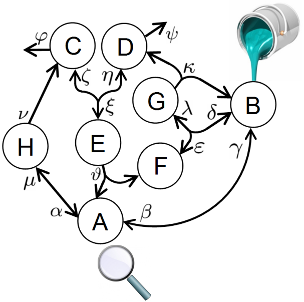
The structural influence matrix

For systems admitting a *BDC* decomposition

$$\Sigma_{ij} = H_i(-BDC)^{-1}E_j,$$

H output matrix, E input matrix $\rightarrow$ efficient vertex algorithm

Network from Shinar&Feinberg (2010)



$$\begin{bmatrix} + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + \\ + & + & + & 0 & + & 0 & + & + \\ 0 & 0 & 0 & + & + & + & + & 0 \\ + & + & + & + & + & + & + & + \\ ? & ? & ? & ? & ? & ? & ? & ? \\ + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + \end{bmatrix}$$

G. Giordano, C. Cuba Samaniego, E. Franco, F. Blanchini, "Computing the Structural Influence Matrix for Biological Systems", *J. Math. Biol.*, 2015

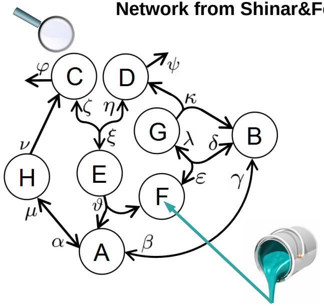
The structural influence matrix

For systems admitting a *BDC* decomposition

$$\Sigma_{ij} = H_i(-BDC)^{-1}E_j,$$

H output matrix, E input matrix $\rightarrow$ efficient **vertex algorithm**

Network from Shinar&Feinberg (2010)



$$\begin{bmatrix} + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + \\ + & + & + & 0 & + & \boxed{0} & + & + \\ 0 & 0 & 0 & + & + & + & + & 0 \\ + & + & + & + & + & + & + & + \\ ? & ? & ? & ? & ? & ? & ? & ? \\ + & + & + & + & + & + & + & + \\ + & + & + & + & + & + & + & + \end{bmatrix}$$

G. Giordano, C. Cuba Samaniego, E. Franco, F. Blanchini, "Computing the Structural Influence Matrix for Biological Systems", *J. Math. Biol.*, 2015

Oscillatory/multistationary systems: structural classification

Sign-definite systems, **globally bounded**

	Candidate oscillator	Candidate multistationary
Weak	A negative cycle exists	A positive cycle exists
Strong	All cycles are negative	All cycles are positive

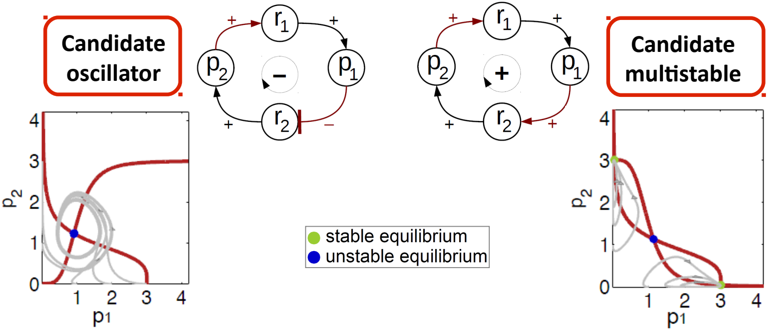
F. Blanchini, E. Franco and G. Giordano, "A Structural Classification of Candidate Oscillators and Multistationary Biochemical Systems", *Bull. Math. Biol.*, 2014

Sign-definite systems, globally bounded

	Candidate oscillator	Candidate multistationary
Weak	A negative cycle exists	A positive cycle exists
Strong	All cycles are negative	All cycles are positive

Candidate oscillator

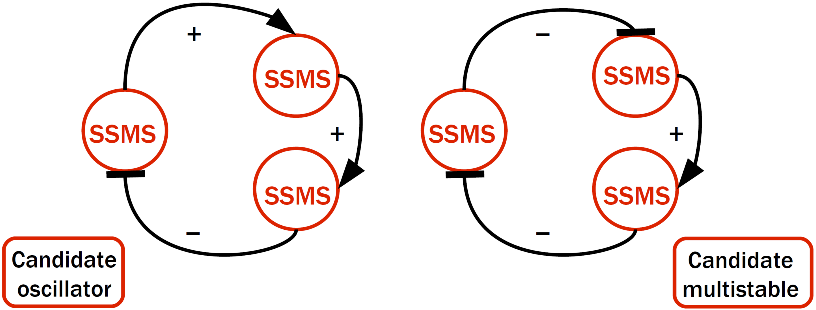
Candidate multistable



F. Blanchini, E. Franco and G. Giordano, "A Structural Classification of Candidate Oscillators and Multistationary Biochemical Systems", *Bull. Math. Biol.*, 2014

Oscillatory/multistationary aggregate monotone systems

Analogous results for the **sign-definite interconnection** of **Structurally Stable Monotone Subsystems**



F. Blanchini, E. Franco and G. Giordano, "Structural Conditions for Oscillations and Multistationarity in Aggregate Monotone Systems", *IEEE CDC*, 2015

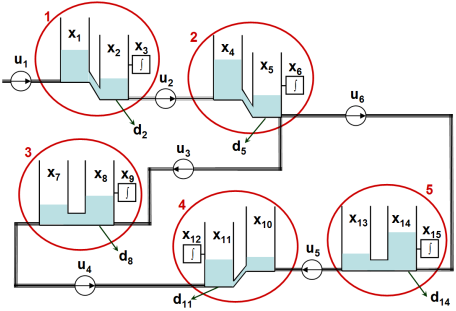
Network-decentralised control

Decoupled subsystems interact due to control agents

System consisting of N subsystems, connected by control agents

$$\dot{x}(t) = Ax(t) + Bu(t) + d(t)$$

$A = \text{diag}\{A_1, A_2, \dots A_N\}$ **block-diagonal**; B **block-structured**

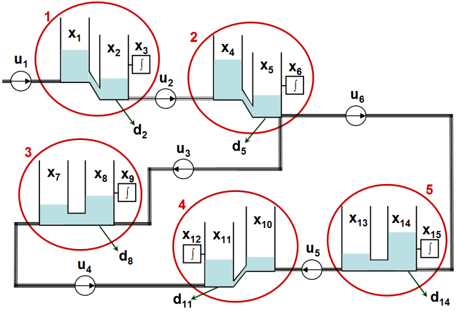


Decoupled subsystems interact due to control agents

System consisting of N subsystems, connected by control agents

$$\dot{x}(t) = Ax(t) + Bu(t) + d(t)$$

$A = \text{diag}\{A_1, A_2, \dots A_N\}$ **block-diagonal**; B **block-structured**



- A centralised control can be too expensive or infeasible

Network-decentralised control: the concept

Local control agents govern **global** behaviour

Network-decentralised state-feedback control

Agents control a **subset of subsystems** with a strategy based on information about **those subsystems only**. **Linear case**: the feedback matrix has a **structure** ($B^\top$) given by the graph.

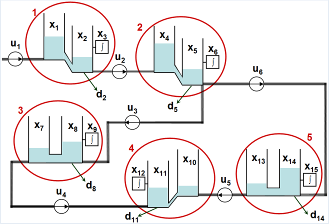
F. Blanchini, E. Franco, G. Giordano, "Structured-LMI Conditions for Stabilizing Network-Decentralized Control", *IEEE CDC*, 2013

Local control agents govern global behaviour

Network-decentralised state-feedback control

Agents control a **subset of subsystems** with a strategy based on information about **those subsystems only**. **Linear case**: the feedback matrix has a structure ($B^\top$) given by the graph.

$$K = \begin{bmatrix} K_{11} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ K_{21} & K_{22} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & K_{32} & K_{33} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & K_{43} & K_{44} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & K_{54} & K_{55} \\ \mathbf{0} & K_{62} & \mathbf{0} & \mathbf{0} & K_{65} \end{bmatrix}$$



Restricted structure: **0** blocks must be zero!

F. Blanchini, E. Franco, G. Giordano, "Structured-LMI Conditions for Stabilizing Network-Decentralized Control", *IEEE CDC*, 2013

Stabilisability: a general theorem

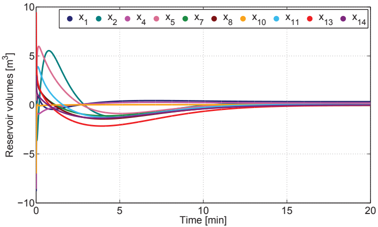
Theorem (Network-decentralised stabilisability)

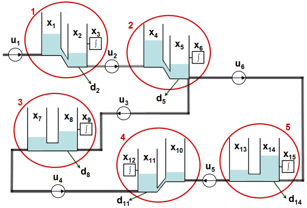
Assume that there are **no common eigenvalues with nonnegative real part** *between two distinct blocks A_i and A_k. Then the system can be* **stabilised** *by a* **network-decentralised** *control* **if and only if** *it can be* **stabilised**.

F. Blanchini, E. Franco, G. Giordano, "Network-Decentralized Control Strategies for Stabilization", *IEEE Trans. Autom. Control*, 2015

Theorem (Network-decentralised stabilisability)

Assume that there are **no common eigenvalues with nonnegative real part** *between two distinct blocks A_i and A_k. Then the system can be* **stabilised** *by a* **network-decentralised** *control* **if and only if** *it can be* **stabilised**.



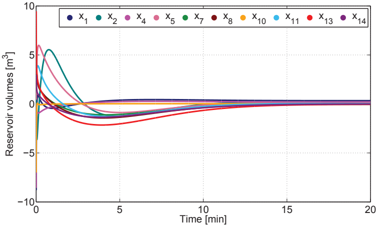


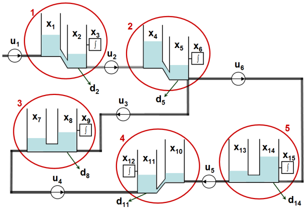
F. Blanchini, E. Franco, G. Giordano, "Network-Decentralized Control Strategies for Stabilization", *IEEE Trans. Autom. Control*, 2015

Stabilisability: a general theorem

Theorem (Network-decentralised stabilisability)

Assume that there are **no common eigenvalues with nonnegative real part** *between two distinct blocks A_i and A_k. Then the system can be* **stabilised** *by a* **network-decentralised** *control* **if and only if** *it can be* **stabilised**.





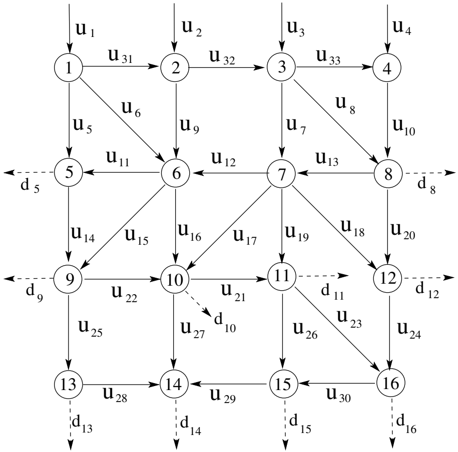
The result holds also for extended buffer systems
(A_i **asymptotically stable**/**marginally stable** with $\lambda = 0$).

F. Blanchini, E. Franco, G. Giordano, "Network-Decentralized Control Strategies for Stabilization", *IEEE Trans. Autom. Control*, 2015

Network-decentralised robustness and optimality

Saturated network-decentralised control

In the presence of control constraints (limited flow)
$\rightarrow$ asymptotically optimal in norm

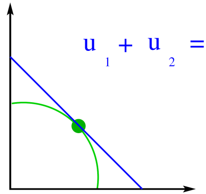


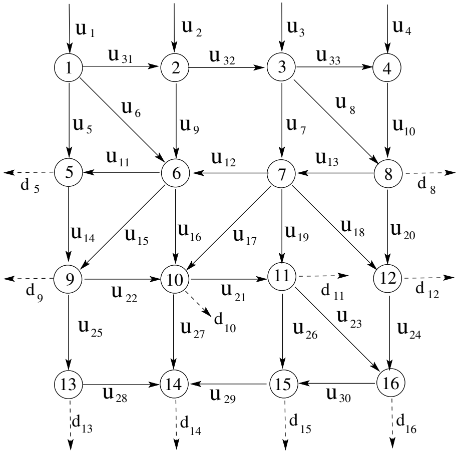
F. Blanchini, E. Franco, G. Giordano, V. Mardanlou, P. L. Montessoro, "Compartmental Flow Control: Decentralisation, Robustness and Optimality", *Automatica*, 2016

Saturated network-decentralised control

In the presence of control constraints (**limited** flow)
$\rightarrow$ **asymptotically optimal** in norm

Minimum norm means fairness, **equi-distribution**

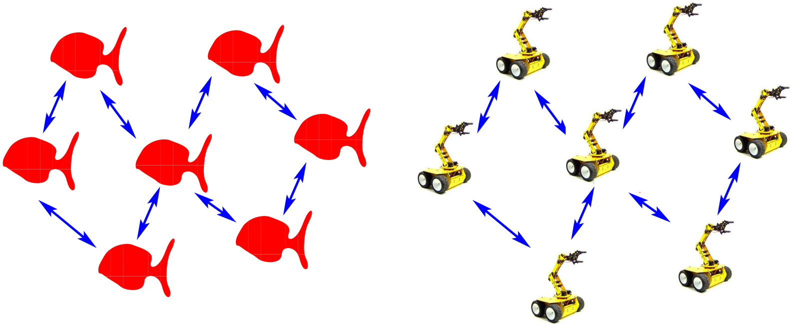




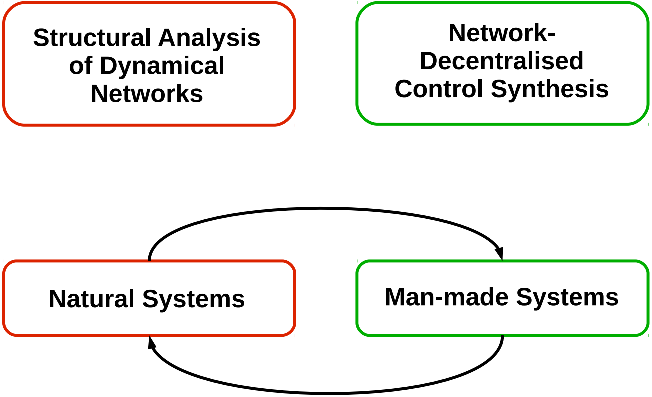
F. Blanchini, E. Franco, G. Giordano, V. Mardanlou, P. L. Montessoro, "Compartmental Flow Control: Decentralisation, Robustness and Optimality", *Automatica*, 2016

Learn from network-decentralised control in nature

Collective animal behaviour $\Longleftrightarrow$ Swarm robotics



Thank you!


Structural Analysis of Dynamical Networks
Network-Decentralised Control Synthesis
Natural Systems
Man-made Systems