On the joint spectral radius

University of Udine
Dept. of Mathematics, Informatics and Physics
via delle Scienze 206, Udine

`Wednesday November 20 2019, aula multimediale`

`Gli Estesi Mercoledì del DMIF`

The joint spectral radius of finite matrix sets was introduced by Rota and Strang in 1960. It remained unnoticed for more than 30 years, and then abruptly entered the forefront of research, attracting too many people to be mentioned here. Part of the research of the "Sistemi Dinamici e Applicazioni" project, founded by the DMIF, focused on the jsr, and in this final workshop we present results and look at directions for future research.

We aim at establishing a friendly atmosphere, with plenty of time for discussion among participants. No registration is required, and everybody is welcome to attend; mail `giovanni.panti@uniud.it` for any information.

11:30-12:20	Giovanni Panti (Università di Udine), *Pythagorean triples, billiards, and mean free paths*
12:20-14:30	Discussion, (informal) lunch, coffee, chat.
14:30-15:20	Marino Zennaro (Università di Trieste), *On the most stable switching laws of linear switched systems*
15:20-15:50	Discussion and short break
15:50-16:40	Franco Blanchini (Università di Udine), *Linear differential and difference inclusions: stability control and convergence*
16:40-17:10	Discussion and short break
17:10-18:00	Ian Morris (University of Surrey), *Fast approximation of the p-norm joint spectral radius*
18:00-undefined	Discussion
19:30-undefined	Dinner

Abstracts

G. Panti, *Pythagorean triples, billiards, and mean free paths.* We show that primitive pythagorean triples can be enumerated by playing billiard on a m-gonal billiard table in the Poincaré disk. The resulting first-return billiard map B is a $(m-1)$-to-1 orientation-reversing covering map of the circle, a property shared by the group character $T(z) = z^{-(m-1)}$. We prove that there exists a homeomorphism Φ, unique up to postcomposition with elements in a dihedral group, that conjugates B with T. This homeomorphism —whose prototype is the Minkowski question mark function— establishes a bijection between the set of points of degree ≤ 2 over $\mathbb{Q}(i)$ and the torsion subgroup of the circle. We prove that Φ is singular and Hölder continuous with exponent $\log(m-1)$ divided by the maximal mean free path in the billiard table (or, equivalently, twice the logarithm of the joint spectral radius of the set of matrices determining reflections in the billiard walls).

M. Zennaro, *On the most stable switching laws of linear switched systems.* We deal with discrete-time linear switched systems of the form

$$x(n+1) = A_{\sigma(n)}x(n), \quad \sigma : \mathbb{N} \to \{1, 2, \ldots, m\}.$$

where $x(0) \in \mathbb{R}^k$, the matrix $A_{\sigma(n) \in \mathbb{R}^{k \times k}}$ belongs to a finite family $\mathcal{F} = \{A_i\}_{1 \leq i \leq m}$ and σ denotes the *switching law.*

It is known that the most stable switching laws are associated to the so-called spectrum-minimizing products of the family $\mathcal{F}$. Moreover, for a normalized family $\mathcal{F}$ of matrices (i.e., with lower spectral radius $\check{\rho}(F) = 1$) that share an invariant cone K, all the most stable trajectories starting from an initial value $x(0)$ in the interior of K lie on the boundary of an antiball of a so-called invariant Barabanov antinorm. Under suitable conditions, a canonical constructive procedure for Barabanov antinorms of polytope type has been proposed by Guglielmi & Z. (2015).

For families $\mathcal{F}$ sharing an invariant cone K, in this talk we show how to provide lower bounds to $\check{\rho}(F)$ by a suitable adaptation of the Gelfand limit in the framework of antinorms (Guglielmi & Z. (2019)), which could be of some practical interest when the above mentioned constructive procedure fails. Then we consider families of matrices $\mathcal{F}$ that share an invariant multicone K_{mul} (Brundu & Z. (2018, 2019)) and show how to generalize some of the known results on antinorms to this more general setting (Guglielmi & Z. (in progress)). These generalizations are of interest because common invariant multicones may well exist when common invariant cones do not

F. Blanchini, *Linear differential and difference inclusions: stability control and convergence.* In this talk we consider the problems of stability analysis and stabilization of both linear difference and linear differential inclusions.

In the first part a review of the history of the problem is proposed. In particular the equivalence will be described among the robust stability analysis, the spectral radius determination and the existence of a classes of Lyapunov functions. Duality properties and computational issues will be presented. It will be shown how these results have been extended to the robust stabilization problem.

We finally present some recent results about:

(a) structural stability analysis of biochemical networks

(b) the problem of convergence in differential-difference inclusions.

I. Morris, *Fast approximation of the p-norm joint spectral radius*. The joint spectral radius of a finite collection of matrices $A_1, \ldots, A_N$ describes the growth rate of the largest product of a given length which can be formed using those matrices. The p-norm joint spectral radius considers instead the growth rate of the ℓ^p-average of the norms of products of a given length, and has been studied on the basis of its applications to wavelet regularity and fractal geometry. I will describe an algorithm for approximating the p-norm joint spectral radius of positive matrices which is closely related to the work of Pollicott on approximating Lyapunov exponents.