Around Dynamical Systems

University of Udine
Palazzo Toppo-Wassermann
via Gemona 92, Udine
`February 5-6 2018, room T5`

This is a small and very informal workshop on various aspects of dynamical systems, organized by Giovanni Panti and Fabio Zanolin, and supported by the Dipartimento di Scienze Matematiche, Informatiche e Fisiche, Università di Udine. We aim at establishing a friendly atmosphere, with plenty of time for discussion among speakers and partecipants. No registration is required, and everybody is welcome to attend; mail `giovanni.panti@uniud.it` for any information.

Monday February 5

09.15-10.05 Stefano Marmi (SNS, Pisa), *Coupling population and price dynamics: chaotic livestock market cycles*

10.15-11.05 Anna Giordano Bruno (Università di Udine), *Topological and algebraic entropy*

11.15-11.45 Short walk, chat, and coffee break

11.45-12.35 Sergey Kryzhevich (Università di Nova Gorica), *Interval exchange versus chaotic dynamics in a system with hysteresis*

13.00-14.30 Lunch

15.00-15.50 Dimitri Breda (Università di Udine), *Numerical methods for infinite-dimensional dynamical systems from delay equations: a survey on the activities of the CDLab*

16.00-16.50 Oliver Butterley (ICTP, Trieste), *Open sets of exponentially mixing Anosov flows*

17.00-undefined Five o'clock tea, more walk, more chat

19.45-undefined Dinner

Tuesday February 6

09.15-10.05 Gabriele Villari (Università di Firenze), *New results for the existence of limit cycles for the relativistic Liénard equation and the case with curvature*

10.15-11.05 Carlo Carminati (Università di Pisa), *Regularity and bifurcation phenomena in parametric families*

11.15-11.45 Short walk, chat, and coffee break

11.45-12.35 Mauro Artigiani (SNS, Pisa), *Lagrange spectrum and its generalizations*

13.00-14.30 Lunch

15.00-15.50 Paolo Giulietti (SNS, Pisa), *On dynamical zeta functions*

16.00-16.50 Stefano Luzzatto (ICTP, Trieste), *Physical measures for dynamical systems*

17.00-undefined Tea, and parallel brainstorming sessions

(a) *Differential equations and dynamics*, chaired by Duccio Papini and Fabio Zanolin

(b) *Algebra and dynamics*, chaired by Dikran Dikranjan and Giovanni Panti

Abstracts

S. Marmi, *Coupling population and price dynamics: chaotic livestock market cycles.* We propose a new model for the time evolution of livestock commodities which exhibits endogenous deterministic stochastic behaviour. The model is based on the Yoccoz-Birkeland integral equation, a model first developed for studying the time-evolution of single species with high average fertility, short mating season and density dependent reproduction rates. (Joint work with Sylvain Arlot and Duccio Papini.)

A. Giordano Bruno, *Topological and algebraic entropy.* The classical notion of measure entropy for probability measure spaces due to Kolmogorov and Sinai was extended to amenable group actions in [5]. Analogously, the topological entropy for compact spaces introduced in [1] was extended to amenable group actions in [4]. These entropy theories were deeply investigated by many authors as Deninger, Kerr, Li, Lind, Schmidt, Thom, Ward, etc. Recently, the measure and the topological entropy were extended to the case of actions of cancellative amenable semigroups in [2], based on a generalization of Ornstein-Weiss Theorem. Analogously, we generalize the notion of algebraic entropy, gradually crystallized in [1, 6, 3], to actions of cancellative amenable semigroups on abelian groups. We discuss various properties, including an application to Lehmer Problem. In case the abelian groups are torsion, the Yuzvinski's addition formula is available, as well as a connection of the algebraic entropy with the topological entropy by means of Pontryagin duality. (Joint work with Dikran Dikranjan and Antongiulio Fornasiero.)

[1] R. Adler, A. Konheim, M. McAndrew, *Topological entropy*, Trans. AMS 114 (1965) 309–319.

[2] T. Ceccherini-Silberstein, M. Coornaert, F. Krieger, *An analogue of Fekete's lemma for subadditive functions on cancellative amenable semigroups*, J. Anal. Math. 124 (2014) 59–81.

[3] D. Dikranjan, A. Giordano Bruno, *Entropy on abelian groups*, Adv. Math. 298 (2016) 612–653.

[4] J. Moulin Ollagnier, *Ergodic Theory and Statistical Mechanics*, Lecture Notes in Mathematics 1115, Springer-Verlag, 1985.

[5] D. Ornstein, B. Weiss, *Entropy and isomorphism theorems for actions of amenable groups*, J. Analyse Math. 48 (1987) 1–141.

[6] J. Peters, *Entropy on discrete abelian groups*, Adv. Math. 33 (1979) 1–13.

S. Kryzhevich, *Interval exchange versus chaotic dynamics in a system with hysteresis.* This is a preliminary result of my joint research with Maxim Arnold, Viktor Avrutin, Nikita Begun and Dmitrii Rachinskii. We are studying a model of behaviour of a stock exchange player. This model is given as a discontinuous map on the plane. The discontinuity is provided by the so-called stop operator which represents a hysteresis element of the system. My coauthors have studied the "hyperbolic" case of the system. It was shown that the system can be reduced to a 1D map. It was demonstrated that in that case either a finite number of stable fixed points or a chaotic invariant set. However, the limit case that is also very interesting for applications, remained unstudied. I demonstrate that in this case the one-dimensional map is "interval exchange with overlaps". Main properties of such maps are studied. It is demonstrated that "interval exchange maps with overlaps" do always have non-atomic invariant measures. They are metrically equivalent to usual interval exchange maps, particularly, they can have a finite number of ergodic non-atomic invariant measures. So, the map cannot be chaotic in any sense. Techniques from Ergodic Theory and Discontinuous 1D dynamics are involved. Some open problems will be formulated.

D. Breda, *Numerical methods for infinite-dimensional dynamical systems from delay equations: a survey on the activities of the CDLab.* This seminar aims at summarizing the research of the "Computational Dynamics Laboratory", recently founded at the Department of Mathematics, Computer Science and Physics of the University of Udine. The activities mostly concern numerical methods for the stability analysis of infinite-dimensional dynamical systems mainly generated by retarded functional equations. Applications of interest range from population dynamics to control theory.

Oliver Butterley, *Open sets of exponentially mixing Anosov flows.* If a flow is sufficiently close to a volume-preserving Anosov flow with 1D stable bundle and unstable bundle at least 2D then the flow mixes exponentially whenever the stable and unstable bundles are not jointly integrable (similarly if the requirements on stable and unstable bundle are reversed). This implies the existence of non-empty open sets of exponentially mixing Anosov flows. We will discuss why such questions are of importance, what we already knew about this topic and what remains unknown. (Joint work with Khadim War.)

G. Villari, *New results for the existence of limit cycles for the relativistic Liénard equation and the case with curvature.* The aim of this talk is to discuss the phase-portrait of the revisited Liénard equation in the relativistic case and in the case of curvature. Such problems have been investigated in presence of a periodic forcing term, but up to now very little is present

in the literature in the autonomous case. Some recent results concerning the existence/not existence of limit cycles will be presented, as well as some open questions and a conjecture for the Van der Pol case with the curvature. (Joint work with Jean Mawhin and Fabio Zanolin.)

C. Carminati, *Regularity and bifurcation phenomena in parametric families.* We study the mechanism which leads to stability and bifurcation phenomena in parametric families. In order to understand better this phenomenon we shall focus on the case of a simple family of piecewise affine maps, where the occurrence of bifurcations explains some features of the behavior of the entropy. In the case examined we detected also a phenomenon analogous to the tuning for the logistic family. (Joint work with Henk Bruin, Stefano Marmi and Alessandro Profeti.)

M. Artigiani, *Lagrange spectrum and its generalizations.* The Lagrange spectrum is a classical object in Diophantine approximation on the real line. It can be also seen as the spectrum of asymptotic penetration of hyperbolic geodesics into the cusp of the modular surface. This interpretation yielded many generalizations of the spectrum to non-compact, finite volume, negatively curved surfaces and higher dimensional manifolds. In this talk we will survey some of the results on the classical spectrum and discuss some generalizations to hyperbolic and translation surfaces. In particular, we will focus on the presence of an infinite interval in the spectrum, known as Hall ray. Parts of this talk are from joint works with L. Marchese and C. Ulcigrai.

P. Giulietti, *On dynamical zeta functions.* I will present a general overview of the interplay between zeta functions, dynamical determinants and trace formulas. I will present a flexible functional analytic strategy to face counting problems in dynamical systems, which relies on the properties of the transfer operator.

S. Luzzatto, *Physical measures for dynamical systems.* We present the Palis and Viana conjectures on the existence of physical measures, review some of the literature, and describe some recent joint work with Climenhaga and Pesin on the existence of Sinai-Ruelle-Bowen physical measures for nonuniformly hyperbolic surface diffeomorphisms.