Gianluca Gorni

Curriculum Vitae

Summary: 1. Academic career; 2. Teaching; 3. Conferences and seminars; 4. Scientific publications.

1. Academic career

I was born on January 3, 1960, in Roncoferraro (Mantova), Italy.

Between 1978/79 and 81/82 I was a student at the Scuola Normale Superiore and at Pisa University, majoring in Mathematics. I graduated in Mathematics on October 28, 1982 with top grades. The title of my thesis was *"Synthesis of optimal control for deterministic and stochastic control problems with distributed parameters"*, and the supervisor was Prof. Giuseppe Da Prato. I also received the "Diploma di Licenza" from the Scuola Normale Superiore.

I attended the "Perfezionamento" course in Mathematics at the Scuola Normale from January, 1983, to May, 1986, with a year of intermission for the compulsory military draft. As I became "Ricercatore" I did not pursue getting the Perfezionamento any more.

I became "Ricercatore" on May 30, 1986, at the University of Udine, after a competitive examination.

From August 1, 1987 to July 31, 1988, I was "visiting assistant professor" at the Division of Applied Mathematics of **Brown University**, Providence, Rhode Island, USA, with a fellowship granted by the Italian CNR.

From January 16, 1991, to September 15, 1992, I was visiting **Chūō University** of Tōkyō, Japan, with a fellowship from "European Communities Scientific Training Programme in Japan, 1990 Round".

I have been associate professor in Mathematical Analysis still at the University of Udine since November 1, 1992, after a national competitive examination.

Between January 29 and February 17, 1999, I visited the Department of Applied Mathematics of the University of São Paulo, Brasil, guest of Prof. Ângelo Barone-Netto, and sponsored by the Fundação de Amparo à Pesquisa do Estado de São Paulo.

2. Teaching

As **associate professor** in Udine I gave one or two courses a year in Mathematical Analysis, ranging from basic calculus to measure theory and real analysis. Together with Prof. Gaetano ZAMPIERI I was advisor of a Ph.D. thesis.

Since March, 1998, I maintain a web site at the url

`http://www.dimi.uniud.it/~gorni`

where students and any interested person can find lecture notes of mine on diverse topics in Mathematical Analysis, and the texts and detailed solutions to the examination problems given in my courses in the last four years. These are tens of `pdf` files made from TEX and often illustrated with *Mathematica* graphs, totalling over 10 Megabytes of material.

I have given three series of seminars to introduce students to the use of TEX. In September, 1998, I attended a five-day course on *Mathematica* programming given by Prof. Allan HAYES.

3. Conferences and seminars

I attended the following mathematical conferences:

1. Convegno Nazionale su *"Calcolo stocastico e sistemi dinamici stocastici"*, Università di Padova, 12–14 settembre 1983 (with a talk);
2. International Workshop on *"Stochastic space-time models and limit theorems"*, Bremen, West Germany, November 7–10, 1983 (with a talk);
3. Working Conference on *"Stochastic modelling and filtering"*, IASI-CNR, Roma, December 10–14, 1984;
4. Third Bad Honnef Conference on *"Stochastic differential systems"*, Bad Honnef, Bonn, West Germania, June 3–7, 1985;
5. *"Equazioni differenziali stocastiche e applicazioni"*, CIRM, Trento, September 30 to October 5, 1985;
6. Congrès Franco-Québecois: *"Analyse non linéaire appliquée"*, Perpignan, France, June 22–26, 1987;
7. *"Partial differential equations in stochastic control and differential games"*, Brown University, Providence, Rhode Island, USA, August 14, 1987;
8. Fifth Workshop on *"Nonlinear evolution equations and dynamical systems"*, Kolymbari (Chania), Crete, Greece, July 2–16, 1989 (with a talk);
9. Conference on *"Nonlinear partial differential equations"*, Università di Padova, April 18–20, 1990;
10. Convegno su *"Equazioni differenziali ordinarie e applicazioni"*, Università di Firenze, May 17–18, 1990 (with a talk);
11. *"Reunião Sobre Equações Diferenciais"*, homenagem ao Prof. Nelson Onuchic, São Carlos, SP, Brasil, June 12–15, 1990 (*"invited speaker"*);
12. *"Evolution Equations and Nonlinear Problems"*, Kyōto University, Research Institute for Mathematical Sciences, Kyōto, Giappone, October 23–25, 1991 (with a talk);
13. *"Evolution Equations and Nonlinear Problems"*, Kyōto University, Research Institute for Mathematical Sciences, Kyōto, Giappone, July 29–31, 1992;

14. Workshop on *"Recent results on the global asymptotic stability Jacobian conjecture"*, Università di Trento, September 14–17, 1993 (with a talk);
15. International meeting on *"Ordinary differential equations and their applications"*, in occasione del settantesimo compleanno di Roberto Conti e Gaetano Villari, Università di Firenze, September 20–23, 1993 (with a talk).
16. *"The second world congress of nonlinear analysts"*, University of Athens, Greece, July 10–17, 1996 (with a talk).
17. *"Conference honoring the mathematical work of Gary H. Meisters"*, University of Nebraska at Lincoln, USA, May 9–10, 1997 (with a talk).
18. *"Miniworkshop on global invertibility of polynomial functions"* within *"Turin fortnight on nonlinear analysis"*, Università di Torino, Dipartimento di Matematica, September 1–2, 1997 (with a talk).
19. *"Poly'99, polynomial automorphisms and related topics"*, Jagiellonian University, Cracow (Poland), July 7–10, 1999 (with a talk).
20. *"Equadiff 99"*, Freie Universität Berlin, Fachbereich Mathematik und Informatik, Berlin (Germany), August 1–7, 1999 (with a talk).
21. *"Dynamical Systems"* within *"Second Turin fortnight on nonlinear analysis"*, University of Torino, Dipartimento di Matematica, September 23–24, 1999 (with a talk).

I gave seminars on my research at Brown University of Providence, RI, USA (May 3, 1988), University of Padova (April 18, 1989 and March 26, 1990), University of São Paulo, Brasil (June 11, 1990), Scuola Normale Superiore of Pisa (July 17, 1990), Chūō University of Tōkyō (November 7, 1991), Jagiellonian University of Cracow, Poland (June 5, 1996), University of Torino (December 11, 1998).

4. Scientific Publications

Summary:

4.1 Analisi convessa applicata al controllo stocastico (4 titles).
4.2 Convex analysis (2 titles).
4.3 Scattering Hamiltonian system with a cone potential (4 titles).
4.4 Markus-Yamabe conjecture (2 titles).
4.5 Invertibility in the large for local homeo- and diffeomorphisms (3 titles).
4.6 Extremal points of real analytic functions (2 titles).
4.7 Jacobian conjecture (6 titles).
4.8 Non-holonomic mechanics (1 titolo).
4.9 Lyapunov stability (1 titolo).
4.10 Miscellaneous papers (4 titles).

Le pubblicazioni sono raggruppate qui per argomenti ma la numerazione è nell'ordine in cui sono state scritte in origine.

4.1 Convex analysis applied to stochastic control

[1] Gianluca Gorni, *Synthesis of stochastic optimal control for a convex optimization problem in Hilbert spaces. Rendiconti dell'Accademia Nazionale dei Lincei,* Classe di Scienze fisiche, matematiche e naturali, Serie VIII, vol. LXXIV, fasc. 3, (marzo 1983), 143–148. MR `85f:93079`, ZB `575.93070`.

[2] Gianluca Gorni, *Optimality principle and synthesis for a stochastic control problem in Hilbert spaces. Stochastics* **12** (1984), 215–227. MR `86c:49031`, ZB `544.93078`.

[3] Gianluca Gorni, *The dynamic programming equation for stochastic optimal control in Hilbert spaces: a variational approach. Stochastics* **15** (1985), 69–111. MR `87b:93070`, ZB `571.93068`.

[4] Gianluca Gorni, *A variational method for a stochastic control problem in Hilbert spaces. Bollettino dell'Unione Matematica Italiana,* Analisi Funzionale e Applicazioni, Serie VI, vol. IV–C, n. 1, (1986), 323–327. MR `86j:49043`, ZB `576.93068`.

We study mainly the optimal control of a linear system perturbed by a white noise:

$$\begin{cases} dy_t = (Ay_t + u_t)\,dt + dw_t\,, & t \in [t_0, T]\,, \\ y(t_0) = X\,, \end{cases}$$

$$J(t_0, x, u) := E\Big(\int_{t_0}^{T} \big(V(y_s) + F(u_s)\big)ds + \varphi(y_T)\Big),$$

$$W(t_0, x) := \inf_u J(t_0, x, u) \qquad \text{for } x \in \mathbb{H}\,,$$

where w_t is a Brownian motion on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, with values in a Hilbert space $\mathbb{H}$; y_t and u_t are square-integrable stochastic processes, adapted by the filtration generated by w_t; A is the infinitesimal generator of a strongly continuous semigroup on $\mathbb{H}$. The functions V, F, φ are always convex, with more assumptions added in each paper.

The first results in [1] and [2] are for the case when V, F, φ are lower semicontinuous and with a certain growth at infinity. In [3] we add the assumption that V, φ and the conjugate F^* are of class C^2 with bounded second derivatives. In [4] we extend the results of [1] and [2] in the case when to the evolution equation we add a diffusion term depending linearly on the state: $(Cy_t + D)\,dw_t$ instead of Cdw_t.

4.2 Convex analysis.

[9] Gianluca Gorni, *Conjugation and second order properties of convex functions. Journal of Mathematical Analysis and Applications* **158** (1991), 293–315. MR `92g:49030`, ZB `739.26009`.

[30] Gianluca Gorni, *Locally Convex Homogenous Multifunctions and their Conjugates.* Università di Udine, Dipartimento di Matematica e Informatica, Rapporto di Ricerca UDMI/8/2000/RR, 1–11.

The study of second order properties of the value function of stochastic problems described above led me to an interest in the generalized second-order derivatives of convex functions. Different definitions for such derivatives have been proposed in the literature. In the approach developed in [9] we don't use more mathematical structure than what is necessary to define the concept of convex function: we can formulate it in abstract linear space without topology. A draw-back of such an approach is that the correspondence with Fréchet derivatives is good only in the nondegenerate case. An advantage is that this "algebraic" theory harmonizes nicely with such basically algebraic concept as conjugation and first-order subdifferential.

In [30] we introduce a concept of *"locally convex multifunction"* with the purpose to extend the concept of conjugation of (strictly) convex functions to functions that are locally strictly convex but are defined on a nonconvex set and have noninjective gradients. For simplicity's sake the theory is limited to the case of homogeneous functions in dimension 2. Several examples are worked out.

4.3 Scattering Hamiltonian system with a cone potential

[8] Gianluca Gorni, Gaetano Zampieri, *Complete integrability for Hamiltonian systems with a cone potential. Journal of Differential Equations* **85,** No. 2, (1990), 302–337. MR `91d:58070`, ZB `713.34003`.

[10] Gianluca Gorni, Gaetano Zampieri, *Reducing scattering problems under cone potential to normal form by global canonical transformations. Journal of Differential Equations* **88,** No. 1, (1990), 71–86. MR `91i:58127`, ZB `724.34093`.

[11] Gianluca Gorni, Gaetano Zampieri, *A class of integrable Hamiltonian systems including scattering of particles on the line with repulsive interactions. Differential and Integral Equations* **4,** No. 2 (1991), 305–329. MR `92k:58111`, ZB `722.34046`.

[12] Gianluca Gorni, Gaetano Zampieri, *Liouville-Arnold integrability for scattering under cone potentials. Nonlinear Evolution Equations and Dynamical Systems,* S. Carillo and O. Ragnisco Eds., Research Reports in Physics, Springer Verlag, (1990), 173–180. MR `91g:70014`, ZB `696.58003`.

Consider a particle q that moves in the field generated by a potential $\mathcal{V}$:

$$\dot q = p\,, \qquad \dot p = -\nabla\mathcal{V}(q)\,, \qquad p,q \in \mathbb{R}^n.$$

Suppose that $\mathcal{V} \in C^2(\mathbb{R}^n)$ and that

i) $\mathcal{V}$ be bounded from below;
ii) $\mathcal{V}$ is a *cone potential*, which means that the force $-\nabla\mathcal{V}(q)$ lies always in a closed convex cone which is "proper", in the sense that it never contains a nonzero vector together with its opposite.

It is well-known and easy to prove that all trajectories have a finite *asymptotic velocity* $p_\infty := \lim_{t\to+\infty} p(t)$. The existence of asymptotic velocities is typical of scattering problems, and in fact the notion of cone potential was born as a generalization of the scattering of particles in one dimension with mutual repulsion.

The n components of p_∞ are obviously constants of motion. Some heuristics make these components as candidates to be a set of n first integrals independent and in involution, as called for in the theory of *integrable Hamiltonian systems* (our system has $\mathcal{H} := |p|^2/2 + \mathcal{V}(q)$ as hamiltonian function). This conjecture, proposed by E. Gutkin in 1984, is false, however, in the general hypotheses given so far. In particular, the asymptotic velocities need not be continuous, let alone differentiable, as functions of the initial data.

In the three papers [8], [10] and [11], joint works with Gaetano Zampieri (University of Padova, later Messina, now Torino), we develop a theory of C^k integrability ($2 \le k \le +\infty$) for Hamiltonian systems with cone potentials under suitable assumptions on $\mathcal{V}$.

In particular in [10] we prove that if we restrict to potentials $\mathcal{V}$ with a fast enough decay at infinity, the motion becomes asymptotic to a uniform straight motion: $q(t) = a_\infty + p_\infty t + o(1)$ as $t \to +\infty$, and moreover the *asymptotic data* (p_∞, a_∞), as functions of the initial data, define a *global canonical diffeomorphism* $(p,q) \mapsto (P,Q)$ that brings the original hamiltonian system into *normal form*

$$\dot P = 0\,, \qquad \dot Q = P\,.$$

The article [12] is a summary of the ideas and of the results of the theory.

4.4 Markus-Yamabe conjecture

[13] Gaetano Zampieri, Gianluca Gorni, *On the Jacobian conjecture for global asymptotic stability. Journal of Dynamics and Differential Equations* **4** (1992), 43–55. MR `93a:34063`, ZB `739.34047`.

[16] Gianluca Gorni, Gaetano Zampieri, *Global sinks for planar vector fields.* Proceedings of the Kyōto University RIMS symposium on *Evolution Equations and Nonlinear Problems.* RIMS Kokyuroku **785** (1992), 134–138.
MR 93m:35002, ZB 836.35057.

Markus-Yamabe conjecture, harking back at least to an article by Krasovskii of 1959, and then proved to be true in 1994, states that if $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a C^1 function vanishing at the origin and if the Jacobian matrix $f'(x)$ has eigenvalues with strictly negative linear parts for all $x \in \mathbb{R}^2$, then all trajectories of the differential equation

$$\dot{x}(t) = f\big(x(t)\big)$$

converge to the origin as $t \to +\infty$ (global asymptotic stability).

Olech in 1963 had proved that global asymptotic stability for our system is equivalent to injectivity of f.

In the paper [13], joint work with Gaetano Zampieri, we introduce a new method for the study of the injectivity of f. We translate the problem into a solvability of a boundary value problem for an ordinary differential equation depending on a parameter. The simplest additional assumption on f where the approach is successful is when $f \in C^2$ and

$$\text{the quantity} \quad \frac{\|f'(x)\|}{\sqrt{\det f'(x)}} \quad \text{is bounded as } x \in \mathbb{R}^2 .$$

The note [16] summarizes the results of [13] and applies them to a new example that clarifies how the assumptions in [13] do not imply surjectivity of f.

4.5 Invertibility in the large for local homeo- and diffeomorphisms

[14] Gianluca Gorni, *A criterion of invertibility in the large for local diffeomorphisms between Banach spaces. Nonlinear Analysis TMA* **21** (1993), 43–47 MR 94h:58022, ZB 940.06053.

[17] Gaetano Zampieri, Gianluca Gorni, *Local homeo- and diffeomorphisms: invertibility and convex image. Bulletin of the Australian Mathematical Society* **49** (1994), 377–398. MR 95b:58018, ZB 814.58010.

[18] Gianluca Gorni, Gaetano Zampieri, *Injectivity onto a star-shaped set for local homeomorphisms in n-space. Annales Polonici Mathematici* **59** (1994), 171–196. MR 95c:58017, ZB 814.58009.

[19] Giuseppe De Marco, Gianluca Gorni, Gaetano Zampieri, *Global inversion of functions: an introduction. Nonlinear Differential Equations and Applications* **1**, (1994), 229–248. MR 95h:58014, ZB 820.58008.

The injectivity problem rising from Markus-Yamabe conjecture stimulated the interest in finding sufficient conditions for a local homeo- or diffeomorphism to be globally injective.

In [14] the following result is proved:

Let X, Y be Banach spaces, Ω an open neighbourhood of the origin in X, $: \Omega \to Y$ a C^1 function such that $f(0) = 0$ and $f'(x)$ is invertible for all $x \in \Omega$. Let $r > 0$ be such that the ball $B := \{x \in X \ : \ |x|_X \leq r\}$ be contained in Ω, and suppose that

1. *the directional derivatives of the norm of X in every point x of the boundary of B in the direction $-f'(x)^{-1}f(x)$ is strictly negative, and moreover*
2. *$|f'(x)f(x)|_X$ sia limitato su B.*

Then f is injective on B and $f(B)$ is star-shaped with respect to the origin of Y.

Every ball B with small enough radius satisfies the assumptions of the theorem. The proof of the result is elementary and is based on the concept of lifting through f of the half-lines emanating from the origin of Y.

The paper [17], joint work with Gaetano Zampieri, proves first that a necessary and sufficient condition for a local homeomorphism from an open connected set in $\mathbb{R}^n$ to $\mathbb{R}^n$ to be at the same time injective and with **convex range** is that there exists a scalar function defined on the domain of f, proper (i.e., the inverse images of the compact sets of $\mathbb{R}$ are compact) and bounded from below, whose composition with any local inverse of f is locally convex. The general result is then particularized to local diffeomorphisms, giving conditions in terms of first and second derivatives. Last, it is shown that any local diffeomorphism with locally Lipschitz first derivatives maps any euclidean ball with small enough radius into a convex set, a property which is false for diffeomorphism with merely continuous first derivatives, as a counterexample shows.

The paper [18] is largely parallel to [17] and arises from the same collaboration. We characterize the local homeomorphisms f in finite dimension which are at the same time invertible and whose range is **star-shaped** with respect to some point p in terms of the existence on the domain of f of a scalar function which is continuous, bounded from below and proper, such that its composition with any local inverse of f is increasing along all half-lines departing from p. As a corollary we give a sufficient criterion of injectivity on a ball for local diffeomorphisms, and the fact that any local C^1 diffeomorphism in finite dimension maps any euclidean ball with a given center and small enough radius onto a set that is star-shaped with respect to the image of the center of the ball. An examples shows that the property is false for bilipschitz homeomorphisms.

The two papers [17] and [18] are clearly related to each other, but none of them is a corollary of the other, and require distinct proofs.

The work [19], in collaboration with Giuseppe De Marco (University of Padova), and Gaetano Zampieri, is divided into two parts. One gives a proof of a classical theorem of Hadamard and Caccioppoli: *Let X, Y be pathwise connected Hausdorff spaces, with Y simply connected, and $f : X \to Y$ a local homeomorphism; then f is bijective if and only if f is proper.* The proof uses some concepts borrowed from the theory of dynamical system. The second part reworks some results of [18] in infinite dimension.

4.6 Extremal points of real analytic functions

[20] Ângelo Barone-Netto, Gianluca Gorni, Gaetano Zampieri, *Local Extrema of Analytic Functions*. *Nonlinear Differential Equations and Applications* **3**, (1996), 287–303. MR 97j:26017, ZB 876.26018.

[21] Ângelo Barone-Netto, Gianluca Gorni, Gaetano Zampieri, *Sur les points critiques des fonctions analytiques réelles*. *Comptes Rendus de l'Académie des Sciences.* **321**, Série I (1995), 821–824. MR 96h:32007, ZB 924.32008.

Let f be a real analytic function defined in a neighbourhood of $x_0 \in \mathbb{R}^n$, with values in $\mathbb{R}$. We say that **the extremality type of f if k-decidable** if we can decide whether x_0 is a weak or strict minimum or maximum, or none of the above, based only on the partial derivatives of f at x_0 up to order k. We will say that we have finite decidability if there is k-decidability for some $k \in \mathbb{N}$.

It is obvious that there are functions that at some points are not finitely decidable. For example the constants can change extremality type with perturbation of arbitrary order. More generally, any function that in x_0 has a weak extremum that is not strict are also not finitely decidable. Severi in 1930 had proved that in dimension 2 there are no other counterexamples.

In the papers [20] and [21], joint works with Prof. Ângelo Barone-Netto of the University of São Paulo, Brazil, and Gaetano Zampieri, it is proved that if S_r is the euclidean sphere of radius r centered in x_0 and f is real analytic in a neighbourhood of x_0, then we have asymptotic expansions of the form

$$\inf_{S_r} f = f(x_0) + a(r^\alpha + o(r^\alpha)), \qquad \sup_{S_r} f = f(x_0) + b(r^\beta + o(r^\beta)) \qquad \text{for } r \to 0^+ ,$$

where α, β are rational numbers and $a \leq b$ are reals. The proof uses results in the theory of semianalytic sets.

A corollary is that in fact in any dimension the only analytic functions that in x_0 are not finitely decidable are the ones for which x_0 is a weak non-strict extremum.

4.7 Jacobian conjecture

[22] Gianluca Gorni, Gaetano Zampieri, *On the existence of global analytic conjugations for polynomial mappings of Yagzhev type*. *Journal of Mathematical Analysis and Applications* **201**, (1996), 880–896. MR 97g:32024, ZB 868.32028.

[23] Gianluca Gorni, Gaetano Zampieri, *Yagzhev polynomial mappings: on the structure of the Taylor expansion of their local inverse*. *Annales Polonici Mathematici* **64** (1996), 285–290. MR 97g:14012, ZB 868.12001.

[24] Gianluca Gorni, Gaetano Zampieri, *On cubic-linear polynomial mappings*. *Indagationes Mathematicae, N. S.* **8** *(4)* (1997), 471–492.
MR 99e:14016, ZB 904.58005.

[25] Gianluca Gorni, *A nonlinearizable cubic-linear mapping*. Proceedings of *A conference on Polynomial Maps and the Jacobian Conjecture, in honour of the mathematical work of Gary Meisters*. University of Nebraska-Lincoln, Lincoln, Nebraska, USA,

May 9–10, 1997. Engelbert Hubbers, editor, April 1998. Pages 63–67. Electronic document available at the Internet site
`http://www.math.unl.edu/Dept/Conferences/Meisters97`.

[26] Gianluca Gorni, Halszka Tutaj-Gasińska, Gaetano Zampieri, *Drużkowski matrix search and D-nilpotent automorphisms. Indagationes Mathematicae, N. S.* **10** *(9)* (1999), 235–245.

[28] Halszka Tutaj-Gasińska, Gianluca Gorni, *On the Span Invariant for Cubic Similarity*. Università di Udine, Dipartimento di Matematica e Informatica, Rapporto di Ricerca UDMI/6/2000/RR, 1–7.

A famous conjecture dating back to O. H. Keller in 1939 can be stated very easily: *if $f: \mathbb{C}^n \to \mathbb{C}^n$ is a function with polynomial components and if the Jacobian determinant of f is a nonvanishing constant throughout $\mathbb{C}^n$, then f is a global diffeomorphism of $\mathbb{C}^n$.* The problem is still open in any dimension ≥ 2, and it was also included by Smale into his list of mathematical problems for the twenty-first century.

The papers [22], [23], [24] and [25], joint works with Gaetano Zampieri, are about **conjugability** of polynomial mappings $f: \mathbb{C}^n \to \mathbb{C}^n$, that is, on the existence of values of the parameter $\lambda \in \mathbb{C} \setminus \{0\}$ and of **(analytic or polynomial) homeomorphisms** $k_\lambda: \mathbb{C}^n \to \mathbb{C}^n$ for which the following diagram commutes:

$$\begin{array}{ccc} \mathbb{C}^n & \xleftarrow{k_\lambda} & \mathbb{C}^n \\ \Big\downarrow \lambda f & & \Big\downarrow \lambda f'(0) \\ \mathbb{C}^n & \xleftarrow{k_\lambda} & \mathbb{C}^n \end{array}$$

The problem of conjugability is connected to the Jacobian conjecture, because any conjugable mappings with nonvanishing Jacobian determinant are also invertible. As a program to attack the Jacobian conjecture the conjugability approach eventually failed, because of counterexamples found by Arno Van Den Essen, but this whole line of research helped clarifying a number of issues in both the Jacobian conjecture and the Markus-Yamabe conjecture in dimension ≥ 3. Prof. Arno van den Essen (Nijmegen, Holland) gives a lively account of the history of the various conjugability conjectures, and of Zampieri's and my role in it, in the following review available on-line:

`http://www.math.unl.edu/Dept/Conferences/Meisters97/papers/essen.pdf`

The paper [26], joint work with Halszka Tutaj-Gasińska (Jagiellonian University, Crakow, Poland) and Gaetano Zampieri, is about "cubic-linear" mappings, that is, the class of polynomial mappings of $\mathbb{C}^n$ to itself of the form $f(x) = x + (Ax)^{*3}$, where A is a constant $n \times n$ matrix and the exponent $*3$ the componentwise cubic power. A special subclass of the cubic linear mappings is characterized. Cubic-linear mappings are familiar to people working in the Jacobian conjecture, because it is well-known that the whole conjecture would follow if it were proved for cubic-linear mappings.

Given a mapping $\varphi: \mathbb{R}^n \to \mathbb{R}^n$, let us denote by $\operatorname{span}\varphi$ the smallest linear subspace of $\mathbb{R}^n$ that contains the range of φ. In the paper [28], in collaboration with Halszka Tutaj-Gasińska, we study the linear space $\operatorname{span}(x \mapsto (Ax)^{*k})$, where A is an arbitrary $n \times n$

real matrix and $k \in \mathbb{N}$. This space is of interest in the Jacobian conjecture, in that for $k = 3$ its dimension is an invariant for cubic-linear similarity. In the paper we conjecture that

$$\operatorname{span}\bigl(x \mapsto (Ax)^{*k}\bigr) = \operatorname{span}\bigl(x \mapsto (A^T A)^{*k} x\bigr)\,,$$

where A^T is the transpose of A. The conjecture arose from thousands of automated trials on a computer with A random integer matrices of various dimensions and ranks. We also provide a proof of the conjecture for $n = 2$ and $n = 3$ for all $k \in \mathbb{N}$.

4.8 Non-holonomic mechanics

[27] Gianluca Gorni, Gaetano Zampieri, *Time reversibility and energy conservation with nonlinear nonholonomic constraints. Reports on Mathematical Physics* **45** *(2)* (2000), 217–227.

A **mechanical system** with Lagrangian $L(t, q, \dot q)$ is called **non-holonomic** if it has constraints not only in q but also in $\dot q$, of the form $b(t, q, \dot q) = 0$, with b a scalar or vector-valued function that is possibly nonlinear in $\dot q$.

In the paper [27], join work with Gaetano Zampieri, we start by giving very general assumptions on L and b such that every trajectory of the system is time-reversible. Roughly speaking, the assumptions are that both the Lagrangian function $L(t, q, \dot q)$ and the set of admissible states $\{(t, q, \dot q) \mid b(t, q, \dot q) = 0\}$ be symmetric with respect to the exchange of $(t, q, \dot q)$ with $(-t, q, -\dot q)$, plus some more technical conditions (e.g., Jacobian of maximum rank) on the constraint function b.

We then prove that a sufficient condition for energy conservation for an autonomous non-holonomic Lagrangian system is that the set of the admissible states be a cone with respect to $\dot q$, that is, $b(q, \dot q) = 0$ implies $b(q, \lambda \dot q) = 0$ for all $\lambda \geq 0$.

For example, there is reversibility and energy conservation for all system with a "natural" Lagrangian and constraint functions that are homogeneous in the velocity.

4.9 Lyapunov stability

[29] Ângelo Barone-Netto, Mauro de Oliveira Cesar, Gianluca Gorni, *Explicit Criteria for the Stability.* Università di Udine, Dipartimento di Matematica e Informatica, Rapporto di Ricerca UDMI/7/2000/RR, 1–17.

Consider the system of ordinary differential equations

$$\ddot x = -x f(x), \qquad \ddot y = -y g(x)\,,$$

where f, g are smooth functions defined in a neighbourhood of the origin in $\mathbb{R}$, and $f(0) > 0$, $g(0) > 0$. The origin is obviously an equilibrium for the system. The problem is to decide when it is a stable equilibrium in the sense of Lyapunov. In the paper [29], joint work with Ângelo Barone-Netto e Mauro de Oliveira Cesar (University of São Paulo, Brasil), we prove first an abstract necessary and sufficient condition for stability, and then we extract from it a number of either necessary or sufficient condition for either stability or instability in terms of polynomial inequalities involving $f(0), f'(0), f''(0),\ g(0), g'(0), g''(0)$. As one of the applications, we also prove that for the system

$$\ddot{x} = -x + x^3\,, \qquad \ddot{y} = -y \cdot (1 - x^2 + x^4)$$

any solution that starts with $(x_0, \dot{x}_0, y_0, \dot{y}_0)$ close enough to 0 is globally bounded in time, but there exists a sequence of initial points $(x_{0,n}, \dot{x}_{0,n}, y_{0,n}, \dot{y}_{0,n})$ converging to 0 such that for the corresponding solution $(x_n(t), y_n(t))$ we have that

$$\lim_{n\to+\infty} \sup_{t\geq 0} \bigl|y_n(t)\bigr| = +\infty\,.$$

4.10 Miscellaneous papers

In the article

[5] Gianluca Gorni, Filippo La Cava, *Compensazione di una rete geodetica libera. Bollettino di Geodesia e Scienze Affini,* Anno XLVI, No. 4, (1987), 333–367,

joint work with Filippo La Cava, (Istituto Geografico Militare, Firenze), we examine from a geometric perspective the deterministic nonlinear and linearized model and the linearized stochastic model for the problem of the least-square fitting of a geodetic net without a priori known points.

In the note

[6] Gianluca Gorni, *A geometric approach to l'Hôpital's rule. American Mathematical Monthly* **97,** No. 6 (1990), 518–523, ZB `705.26007`,

we give a proof of L'Hôpital's rule that uses the formula

$$\frac{f'(x)}{g'(x)} = (f \circ g^{-1})'(g(x))$$

with its geometric meaning. The counterexamples to wrong generalizations of the rule are interpreted geometrically.

In the note

[7] Gianluca Gorni, *Uma prova da unicidade no problema de Dirichlet. Matemática Universitaria,* Sociedade Brasileira de Matemática **8** (1988), 43–45,

we first prove the elementary fact that in any neighbourhood of a strict local extremum of a C^2 real function there is a point where the hessian matrix is strictly defined (positive or negative). The result, or a variant, is applied to the uniqueness of the regular solutions to the Dirichlet problem $\Delta u = 0$ on Ω, u given and continuous on $\partial\Omega$, where Ω is an open bounded set in $\mathbb{R}^n$.

The paper

[15] Gianluca Gorni, *Periodic Orbits for Convex Hamiltonians. Bulletin of the Faculty of Science and Engineering of Chuo University* **35** (1992), 101–117. MR `94d:58123`, ZB `779.58033`,

reworks and develops a theorem by Mawhin and Willem on the existence of periodic orbits for Hamiltonian systems, adding a characterization in terms of the dual action, and gives uniqueness of the solution under certain condition on the second derivative of the Hamiltonian.