On a general class of $q$-polynomials suggested by basic Laguerre polynomials

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Having defined a $q$-extension of the polynomial $L_n^{\alpha,\beta}(x)$, we investigate its fundamental properties such as $q$-generating relation, $q$-partial difference equation and recurrence relations. A generalized $q$-generating function for the said polynomial is also established. It has further been shown that the newly defined polynomial is closely related to the $q$-Laguerre polynomial $L_n^\beta(x;q)$. Certain interesting limiting cases in the form of the known results due to Prabhakar and Rekha [Math. Student, 40(1972), 311-317] and Prabhakar [Pacific J. Math. 35(1)(1970), 213-219] have also been discussed. Some of the main results proved in this paper are as under:

(a) A $q$-extension of $L_n^{\alpha,\beta}(x)$:

$$L_n^{\alpha,\beta}(x;q) = \frac{\Gamma_q(\alpha n + \beta + 1)}{(q;q)_n} \sum_{j=0}^{n} \frac{(q^{-n};q)_j}{(q;q)_j} \frac{(xq^n)^j q^{j(j-1)/2}}{\Gamma_q(\alpha j + \beta + 1)},$$  \hspace{1cm} (1)

where $\Re(\alpha) > 0$ and $\Re(\beta) > -1$.

(b) $q$-generating function:

$$\sum_{n=0}^{\infty} \frac{L_n^{\alpha,\beta}(x;q)t^n}{\Gamma_q(\alpha n + \beta + 1)} = e_q(t)\phi(\alpha, \beta + 1; q, -xt),$$  \hspace{1cm} (2)

where $\phi(\alpha, \beta + 1; q, -xt)$ is $q$-Bessel-Maitland function.