New Approach to q-Euler Polynomials of Higher Order

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Abstract. In this paper, we present new generating functions related to q-Euler numbers and polynomials of higher order. Using these generating functions, we present new identities involving q-Euler numbers and polynomials of higher order.

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1. INTRODUCTION/PRELIMINARIES

Let $C$ be the complex number field. Assume that \( q \in C \) with \( |q| < 1 \) and define the q-number by

\[ [x]_q = (1 - q^x)/(1 - q). \]

The q-factorial is given by

\[ [n]_q! = [x]_q[x - 1]_q \cdots [2]_q[1]_q, \]

and the q-binomial formulas are known to be

\[ (x:y)_q = \prod_{i=0}^{n-1} (1 - xq^i) = \sum_{i=0}^{\infty} \binom{n}{i}_q (-x)_q^i \]

(see [3, 14, 15]) and

\[ \frac{1}{(x:y)_q} = \prod_{i=0}^{n-1} \left( 1 - \frac{x}{1 - xq^i} \right) = \sum_{i=0}^{\infty} \binom{n+i-1}{i}_q y^i \]

(see [3, 5, 14, 15]), where

\[ \binom{n}{k}_q = \frac{[n]_q!}{[n-k]_q! [k]_q!}. \]

The Euler polynomials are defined by

\[ 2/(e^x + 1)e^x = \sum_{n=0}^{\infty} E_n(x)x^n/n! \]

for \( |x| < \pi \). In the special case \( x = 0 \), the values \( E_n(0) = E_n \) are referred to as the nth Euler numbers. In this paper, we consider q-extensions of Euler numbers and polynomials of higher order. Barnes’ multiple Bernoulli polynomials are also defined by

\[ \prod_{i=1}^{m}(e^{x_i} - 1) = \sum_{n=0}^{\infty} B_n(x, r; a_1, \ldots, a_n) x^n/n! \]

(1)

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