Annals of Combinatorics 13 (2009) 65-85
0218-0006/09/010065-21
DOI 10.1007/s00026-009-0367-4

Annals of Combinatorics

Production Matrices and Riordan Arrays

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Received October 22, 2006

AMS Subject Classification: 05A15, 05C38

Abstract. We translate the concept of succession rule and the ECO method into matrix notation, introducing the concept of *production matrix*. This allows us to combine our method with other enumeration techniques using matrices, such as the method of Riordan matrices. Finally we treat the case of rational production matrices, i.e., those leading to rational generating functions.

Keywords: ECO method, production matrices, Riordan arrays

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* L. Ferrari and S. Rinaldi have been partially supported by MIUR project: *Linguaggi formali e automi: metodi, modelli e applicazioni.*

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