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An Operator Calculus for the Askey-Wilson Operator

Mourad E.H. Ismail[*]

Department of Mathematics, University of South Florida, Tampa, Florida 33620-5700, USA
ismail@math.usf.edu

In fond memory of a dear friend: Gian-Carlo Rota

Abstract. We introduce a translation operator which commutes with the Askey-Wilson operator and develop the corresponding theory of polynomials of binomial type and the Sheffer classification.

Keywords: continuous q-Hermite polynomials, q-translation, q-calculus, q-shift-invariant operators, generating functions, operator calculus, Sheffer sequences, polynomials of q-binomial type, q-A type zero

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