Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH **Open Access**

Some identities on Bernoulli and Euler polynomials arising from orthogonality of Legendre polynomials

Dae San Kim[1], Seog-Hoon Rim[2] and Taekyun Kim[3*]

*Correspondence: tkkim@kw.ac.kr
[3]Department of Mathematics, Kwangwoon University, Seoul, 139-701, Republic of Korea
Full list of author information is available at the end of the article

Abstract

The purpose of this paper is to investigate some interesting identities on the Bernoulli and Euler polynomials arising from the orthogonality of Legendre polynomials in the inner product space $\mathbb{P}_n$.

1 Introduction

As is well known, the Legendre polynomial $P_n(x)$ is a solutions of the following differential equation:

$$(1-x^2)u''-2xu'+n(n+1)u=0 \quad \text{(see [1–7])},$$

where $n=0,1,2,\ldots$.

It is a polynomial of degree n. If n is even or odd, then $P_n(x)$ is accordingly even or odd. They are determined up to constant and normalized so that $P_n(1)=1$.

Rodrigues' formula is given by

$$P_n(x)=\frac{1}{2^n n!}\left\{\left(\frac{d}{dx}\right)^n(x^2-1)^n\right\}, \quad n\in\mathbb{Z}_+. \tag{1.1}$$

Integrating by parts, we can derive

$$\int_{-1}^{1}P_m(x)P_n(x)\,dx=\frac{2}{2n+1}\delta_{m,n} \quad \text{(see [1–7])}, \tag{1.2}$$

where $\delta_{m,n}$ is the Kronecker symbol.

By (1.1), we get

$$P_n(x)=\frac{1}{2^n}\sum_{k=0}^{[\frac{n}{2}]}(-1)^k\binom{n}{k}\binom{2n-2k}{n}x^{n-2k}. \tag{1.3}$$

The generating function is given by

$$\left(1-2xt+t^2\right)^{-\frac{1}{2}}=\sum_{n=0}^{\infty}P_n(x)t^n. \tag{1.4}$$

The Bernoulli polynomial is defined by a generating function to be

$$\frac{t}{e^t-1}e^{xt}=e^{B(x)t}=\sum_{n=0}^{\infty}B_n(x)\frac{t^n}{n!}\quad(\text{see [8–13]})\tag{1.5}$$

with the usual convention about replacing $B^n(x)$ by $B_n(x)$.

In the special case, $x=0$, $B_n(0)=B_n$ are called the *Bernoulli numbers*.

From (1.5), we have

$$B_n(x)=\sum_{l=0}^{n}\binom{n}{l}B_{n-l}x^l\quad(\text{see [10–26]}).\tag{1.6}$$

As is well known, the *Euler numbers* are defined by

$$E_0=1,\qquad (E+1)^n+E_n=2\delta_{0,n}\quad(\text{see [10–13]})\tag{1.7}$$

with the usual convention about replacing E^n by E_n.

The Euler polynomials are defined as

$$E_n(x)=\sum_{l=0}^{n}\binom{n}{l}E_{n-l}x^l\quad(\text{see [27–31]}).\tag{1.8}$$

Let $\mathbb{P}_n=\{p(x)\in\mathbb{O}[x]\mid\deg p(x)\le n\}$. Then $\mathbb{P}_n$ is an inner product space with respect to the inner product $\langle\cdot,\cdot\rangle$ with

$$\langle q_1(x),q_2(x)\rangle=\int_{-1}^{1}q_1(x)q_2(x)\,dx,$$

where $q_1(x),q_2(x)\in\mathbb{P}_n$.

From (1.2), we can show that $\{P_0(x),P_1(x),\dots,P_n(x)\}$ is an orthogonal basis for $\mathbb{P}_n$. In this paper, we derive some interesting identities on the Bernoulli and Euler polynomials from the orthogonality of Legendre polynomials in $\mathbb{P}_n$.

2 Some identities on the Bernoulli and Euler polynomials

For $q(x)\in\mathbb{P}_n$, let

$$q(x)=\sum_{k=0}^{n}C_kP_k(x).\tag{2.1}$$

Then, from (1.2), we have

$$\begin{aligned}\langle q(x),P_k(x)\rangle&=C_k\langle P_k(x),P_k(x)\rangle\\&=C_k\int_{-1}^{1}\{P_k(x)\}^2\,dx\\&=\frac{2}{2k+1}C_k.\end{aligned}\tag{2.2}$$

By (2.2), we get

$$\begin{aligned}C_k &= \frac{2k+1}{2}\langle q(x), P_k(x)\rangle = \frac{2k+1}{2}\int_{-1}^{1} P_k(x)q(x)\,dx \\ &= \left(\frac{2k+1}{2}\right)\frac{1}{2^k k!}\int_{-1}^{1}\left(\frac{d^k}{dx^k}\left(x^2-1\right)^k\right)q(x)\,dx \\ &= \left(\frac{2k+1}{2^{k+1}k!}\right)\int_{-1}^{1}\left(\frac{d^k}{dx^k}\left(x^2-1\right)^k\right)q(x)\,dx. \end{aligned} \tag{2.3}$$

Therefore, by (2.1) and (2.3), we obtain the following proposition.

Proposition 2.1 *For $q(x) \in \mathbb{P}_n$, let*

$$q(x) = \sum_{k=0}^{n} C_k P_k(x).$$

Then

$$C_k = \frac{2k+1}{2^{k+1}k!}\int_{-1}^{1}\left(\frac{d^k}{dx^k}\left(x^2-1\right)^k\right)q(x)\,dx.$$

Let us assume that $q(x) = x^n \in \mathbb{P}_n$.

From Proposition 2.1, we have

$$\begin{aligned}C_k &= \frac{2k+1}{2^{k+1}k!}\int_{-1}^{1}\left(\frac{d^k}{dx^k}\left(x^2-1\right)^k\right)x^n\,dx \\ &= \frac{2k+1}{2^{k+1}}(-1)^k\binom{n}{k}\int_{-1}^{1}\left(x^2-1\right)^k x^{n-k}\,dx \\ &= \frac{2k+1}{2^{k+1}}\binom{n}{k}\left(1+(-1)^{n-k}\right)\int_{0}^{1}\left(1-x^2\right)^k x^{n-k}\,dx. \end{aligned} \tag{2.4}$$

For $n-k \equiv 0 \pmod 2$, by (2.4), we get

$$\begin{aligned}C_k &= \frac{2k+1}{2^{k+1}}\binom{n}{k}\int_{0}^{1}(1-y)^k y^{\frac{n-k-1}{2}}\,dy \\ &= \frac{2k+1}{2^{k+1}}\binom{n}{k}B\left(k+1, \frac{n-k+1}{2}\right) \\ &= \frac{2k+1}{2^{k+1}}\binom{n}{k}\frac{\Gamma(k+1)\Gamma(\frac{n-k+1}{2})}{\Gamma(\frac{n+k+1}{2}+1)} \\ &= \frac{2k+1}{2^{k+1}}\binom{n}{k}\frac{k!\Gamma(\frac{n-k+1}{2})}{(\frac{n+k+1}{2})(\frac{n+k-1}{2})\cdots(\frac{n-k+1}{2})\Gamma(\frac{n-k+1}{2})} \\ &= \frac{2k+1}{2^{k+1}}\binom{n}{k}k!2^{k+1}\frac{(n-k)!(n+k+2)(n+k)\cdots(n-k+2)}{(n+k+2)!} \\ &= \frac{(2k+1)2^{k+1}}{(n+k+2)!}\times\frac{n!(\frac{n+k+2}{2})!}{(\frac{n-k}{2})!}. \end{aligned} \tag{2.5}$$

Here the beta function $B(x,y)$ is defined by

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt \quad \bigl(\mathrm{Re}(x), \mathrm{Re}(y) > 0\bigr),$$

and it is well known that

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)},$$

where $\Gamma(s) = \int_0^\infty t^{s-1}e^{-t}\,dt$ ($\mathrm{Re}(s) > 0$) is the gamma function.

By Proposition 2.1 and (2.5), we get

$$x^n = \sum_{0\le k\le n, n-k\equiv 0 \pmod 2} \frac{(2k+1)n!2^{k+1}(\frac{n+k+2}{2})!}{(n+k+2)!(\frac{n-k}{2})!} P_k(x). \tag{2.6}$$

From (1.5), we can easily derive the following equation (2.7):

$$x^n = \frac{1}{n+1}\sum_{l=0}^{n}\binom{n+1}{l}B_l(x) \quad (n \in \mathbb{Z}_+). \tag{2.7}$$

Therefore, by (2.6) and (2.7), we obtain the following Proposition 2.2.

Proposition 2.2 *For $n \in \mathbb{Z}_+$, we have*

$$\sum_{l=0}^{n}\frac{B_l(x)}{(n+1-l)!l!} = \sum_{0\le k\le n, n-k\equiv 0 \pmod 2} \frac{(2k+1)2^{k+1}(\frac{n+k+2}{2})!}{(n+k+2)!(\frac{n-k}{2})!} P_k(x).$$

Let us take $q(x) = B_n(x) \in \mathbb{P}_n$. By Proposition 2.1, we get

$$\begin{aligned}
C_k &= \frac{2k+1}{2^{k+1}k!}\int_{-1}^{1}\left(\frac{d^k}{dx^k}(x^2-1)^k\right)B_n(x)\,dx \\
&= \frac{(-1)^k(2k+1)}{2^{k+1}}\binom{n}{k}\int_{-1}^{1}(x^2-1)^k B_{n-k}(x)\,dx \\
&= \frac{(-1)^k(2k+1)}{2^{k+1}}\binom{n}{k}\sum_{l=0}^{n-k}\binom{n-k}{l}B_{n-k-l}\int_{-1}^{1}(x^2-1)^k x^l\,dx \\
&= \frac{2k+1}{2^{k+1}}\binom{n}{k}\sum_{l=0}^{n-k}\binom{n-k}{l}B_{n-k-l}\bigl(1+(-1)^l\bigr)\int_0^1(1-x^2)^k x^l\,dx.
\end{aligned} \tag{2.8}$$

For $l \in \mathbb{Z}_+$ with $l \equiv 0 \pmod 2$, we have

$$\begin{aligned}
C_k &= \frac{2k+1}{2^{k+1}}\binom{n}{k}\sum_{0\le l\le n-k, l \text{ is even}}\binom{n-k}{l}B_{n-k-l}\int_0^1(1-y)^k y^{\frac{l-1}{2}}\,dy \\
&= \frac{2k+1}{2^{k+1}}\binom{n}{k}\sum_{0\le l\le n-k, l \text{ is even}}\binom{n-k}{l}B_{n-k-l}\frac{\Gamma(k+1)\Gamma(\frac{l+1}{2})}{\Gamma(\frac{2k+l+1}{2}+1)} \\
&= (2k+1)2^{k+1}n!\sum_{0\le l\le n-k, l\equiv 0 \pmod 2}\frac{B_{n-k-l}}{(n-k-l)!}\times\frac{(\frac{2k+l+2}{2})!}{(2k+l+2)!(\frac{l}{2})!}.
\end{aligned} \tag{2.9}$$

In [14], we showed that

$$B_n(x) = \sum_{k=0}^{n-2} \binom{n}{k} B_{n-k}E_k(x) + E_n(x) = \sum_{k=0,k\neq n-1}^{n} \binom{n}{k} B_{n-k}E_k(x). \tag{2.10}$$

Therefore, by Proposition 2.1, (2.9) and (2.10), we obtain the following theorem.

Theorem 2.3 *For $n \in \mathbb{Z}_+$, we have*

$$\frac{1}{n!} \sum_{k=0,k\neq n-1}^{n} \binom{n}{k} B_{n-k}E_k(x) = \sum_{k=0}^{n} \left(\sum_{0\le l\le n-k, l\equiv 0 \pmod 2} \frac{(2k+1)2^{k+1}(\frac{l+2k+2}{2})! B_{n-k-l}}{(n-k-l)!(l+2k+2)!(\frac{l}{2})!} \right) P_k(x).$$

By the same method of Theorem 2.3, we easily see that

$$\frac{E_n(x)}{n!} = \sum_{k=0}^{n} \left(\sum_{0\le l\le n-k, l\equiv 0 \pmod 2} \frac{(2k+1)2^{k+1}(\frac{l+2k+2}{2})! B_{n-k-l}}{(n-k-l)!(l+2k+2)!(\frac{l}{2})!} \right) P_k(x). \tag{2.11}$$

Let us take $q(x) = \sum_{k=0}^{n} B_k(x)B_{n-k}(x) \in \mathbb{P}_n$. Then we see that

$$\begin{aligned} &\sum_{k=0}^{n} B_k(x)B_{n-k}(x) \\ &\quad = (n+1) \sum_{k=0}^{n} \frac{\binom{n}{k}}{n-k+1} \left\{ \sum_{l=k}^{n} B_{l-k}B_{n-l} + B_{n-1-k} \right\} E_k(x) + \frac{(n^2-1)n}{12} E_{n-2}(x). \end{aligned} \tag{2.12}$$

The equation (2.12) was proved in [14].

By (2.12) and Proposition 2.2, we have

$$\begin{aligned} C_k = \frac{2k+1}{2^{k+1}k!} \Bigg\{ &(n+1) \sum_{l=0}^{n} \frac{\binom{n}{l}}{n-l+1} \left(\sum_{m=l}^{n} B_{m-l}B_{n-m} + B_{n-1-l} \right) \\ &\times \int_{-1}^{1} E_l(x) \left(\frac{d^k}{dx^k} (x^2-1)^k \right) dx \\ &+ \frac{(n^2-1)n}{12} \int_{-1}^{1} \left(\frac{d^k}{dx^k} (x^2-1)^k \right) E_{n-2}(x)\, dx \Bigg\}. \end{aligned} \tag{2.13}$$

Integrating by parts, we get

$$\begin{aligned} &\int_{-1}^{1} E_l(x) \left(\frac{d^k}{dx^k} (x^2-1)^k \right) dx \\ &\quad = \sum_{j=0}^{l} \binom{l}{j} E_{l-j} \int_{-1}^{1} x^j \frac{d^k}{dx^k} (x^2-1)^k \, dx \\ &\quad = \sum_{j=k}^{l} \binom{l}{j} E_{l-j} \frac{(-1)^k j!}{(j-k)!} \int_{-1}^{1} x^{j-k} (x^2-1)^k \, dx \\ &\quad = \sum_{j=k}^{l} \binom{l}{j} E_{l-j} \frac{j!}{(j-k)!} \left(1+(-1)^{j-k}\right) \int_{0}^{1} x^{j-k} \left(-x^2+1\right)^k dx. \end{aligned} \tag{2.14}$$

Then we see that

$$\begin{aligned}
&\int_{-1}^{1} E_l(x)\left(\frac{d^k}{dx^k}\left(x^2-1\right)^k\right)dx \\
&= \sum_{j=k,j-k\equiv 0 \pmod 2}^{l} \binom{l}{j} E_{l-j}\frac{j!}{(j-k)!}\int_0^1 t^{\frac{j-k-1}{2}}(1-t)^k\,dt \\
&= \sum_{j=k,j-k\equiv 0 \pmod 2}^{l} \binom{l}{j} E_{l-j}\frac{j!}{(j-k)!}\frac{\Gamma(\frac{j-k+1}{2})\Gamma(k+1)}{\Gamma(\frac{j+k+1}{2}+1)} \\
&= \sum_{j=k,j-k\equiv 0 \pmod 2}^{l} \binom{l}{j} E_{l-j}\frac{j!k!}{(j-k)!}\times\frac{(j-k)!2^{2k+2}}{(j+k+2)!}\times\frac{(\frac{j+k+2}{2})!}{(\frac{j-k}{2})!} \\
&= \sum_{j=k,j-k\equiv 0 \pmod 2}^{l} \binom{l}{j} E_{l-j}\frac{j!k!2^{2k+2}}{(j+k+2)!}\times\frac{(\frac{j+k+2}{2})!}{(\frac{j-k}{2})!}. \qquad (2.15)
\end{aligned}$$

It is easy to show that

$$\begin{aligned}
&\int_{-1}^{1}\left(\frac{d^k}{dx^k}\left(x^2-1\right)^k\right)E_{n-2}(x)\,dx \\
&= \sum_{j=0}^{n-2}\binom{n-2}{j}E_{n-2-j}\int_{-1}^{1}\left(\frac{d^k}{dx^k}\left(x^2-1\right)^k\right)x^j\,dx \\
&= \sum_{j=k}^{n-2}\binom{n-2}{j}E_{n-2-j}\left(1+(-1)^{j-k}\right)(-1)^k\frac{j!}{(j-k)!}\int_0^1\left(x^2-1\right)^k x^{j-k}\,dx \\
&= \sum_{k\le j\le n-2,j-k\equiv 0 \pmod 2}\binom{n-2}{j}E_{n-2-j}\frac{j!k!2^{2k+2}}{(j+k+2)!}\times\frac{(\frac{j+k+2}{2})!}{(\frac{j-k}{2})!}. \qquad (2.16)
\end{aligned}$$

Therefore, by (2.13), (2.14), (2.15) and (2.16), we get

$$\begin{aligned}
C_k &= (2k+1)2^{k+1}\left\{(n+1)\sum_{l=k}^{n}\frac{\binom{n}{k}}{n-l+1}\left(\sum_{m=l}^{n}B_{m-l}B_{n-m}+B_{n-1-l}\right)\right. \\
&\quad\times\sum_{k\le j\le l,j-k\equiv 0 \pmod 2}\binom{l}{j}E_{l-j}\frac{j!}{(j+k+2)!}\times\frac{(\frac{j+k+2}{2})!}{(\frac{j-k}{2})!} \\
&\quad\left.+\frac{(n^2-1)n}{12}\sum_{k\le j\le n-2,j-k\equiv 0 \pmod 2}\binom{n-2}{j}E_{n-2-j}\frac{j!}{(j+k+2)!}\times\frac{(\frac{j+k+2}{2})!}{(\frac{j-k}{2})!}\right\}. \qquad (2.17)
\end{aligned}$$

Therefore, by Proposition 2.1 and (2.17), we obtain the following theorem.

Theorem 2.4 *For $n \in \mathbb{Z}_+$, we have*

$$\begin{aligned}
&\sum_{k=0}^{n}B_k(x)B_{n-k}(x) \\
&= \sum_{k=0}^{n}(2k+1)2^{k+1}\left\{(n+1)\sum_{l=k}^{n}\frac{\binom{n}{k}}{n-l+1}\left(\sum_{m=l}^{n}B_{m-l}B_{n-m}+B_{n-1-l}\right)\right.
\end{aligned}$$

$$\times \sum_{k\le j\le l, j-k\equiv 0 \pmod 2} \binom{l}{j} E_{l-j} \frac{j!}{(j+k+2)!} \times \frac{(\frac{j+k+2}{2})!}{(\frac{j-k}{2})!}$$
$$+ \frac{(n^2-1)n}{12} \sum_{k\le j\le n-2, j-k\equiv 0 \pmod 2} \binom{n-2}{j} E_{n-2-j} \frac{j!}{(j+k+2)!} \times \frac{(\frac{j+k+2}{2})!}{(\frac{j-k}{2})!} \Bigg\} P_k(x).$$

Remark 2.5 The extended Laguerre polynomials are given by

$$L_n^{\alpha}(x) = \sum_{r=0}^{n} \frac{(-1)^r}{r!} \binom{n+\alpha}{n-r} x^r \quad (\alpha > -1).$$

By the same method, we get

$$L_n^{\alpha}(x) = \sum_{k=0}^{n} \sum_{0\le l\le n-k, l\equiv 0 \pmod 2} \frac{(-1)^{k+l}(2k+1)2^{k+1}\binom{n+\alpha}{n-k-l}(\frac{l+2k+2}{2})!}{(l+2k+2)!(\frac{l}{2})!} P_k(x)$$

and

$$H_n(x) = \sum_{k=0}^{n} \sum_{0\le l\le n-k, l\equiv 0 \pmod 2} \frac{(2k+1)2^{2k+l+1} n! (\frac{l+2k+2}{2})! H_{n-k-l}}{(n-k-l)!(l+2k+2)!(\frac{l}{2})!} P_k(x),$$

where $H_n(x)$ is the Hermite polynomial of degree n (see [7]).

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally in this paper. They read and approved the final manuscript.

Author details
[1]Department of Mathematics, Sogang University, Seoul, 121-742, Republic of Korea. [2]Department of Mathematics Education, Kyungpook National University, Taegu, 702-701, Republic of Korea. [3]Department of Mathematics, Kwangwoon University, Seoul, 139-701, Republic of Korea.

Acknowledgements
The authors would like to express their sincere gratitude to referee for his/her valuable comments and information. This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science, and Technology 2012R1A1A2003786.

Received: 26 June 2012 Accepted: 25 September 2012 Published: 9 October 2012

References
1. Carlitz, L: Some integrals containing products of Legendre polynomials. Arch. Math. **12**, 334-340 (1961)
2. Carlitz, L: Some congruence properties of the Legendre polynomials. Math. Mag. **34**, 387-390 (1960/1961)
3. Carlitz, L: Some arithmetic properties of the Legendre polynomials. Acta Arith. **4**, 99-107 (1958)
4. Al-Salam, WA, Carlitz, L: Finite summation formulas and congruences for Legendre and Jacobi polynomials. Monatshefte Math. **62**, 108-118 (1958)
5. Carlitz, L: Some arithmetic properties of the Legendre polynomials. Proc. Camb. Philos. Soc. **53**, 265-268 (1957)
6. Carlitz, L: Note on Legendre polynomials. Bull. Calcutta Math. Soc. **46**, 93-95 (1954)
7. Carlitz, L: Congruence properties of the polynomials of Hermite, Laguerre and Legendre. Math. Z. **59**, 474-483 (1954)
8. Ding, D, Yang, J: Some identities related to the Apostol-Euler and Apostol-Bernoulli polynomials. Adv. Stud. Contemp. Math. **20**(1), 7-21 (2010)
9. Kim, G, Kim, B, Choi, J: The DC algorithm for computing sums of powers of consecutive integers and Bernoulli numbers. Adv. Stud. Contemp. Math. **17**(2), 137-145 (2008)
10. Kim, T: A note on q-Bernstein polynomials. Russ. J. Math. Phys. **18**(1), 73-82 (2011)
11. Kim, T: Some identities on the q-Euler polynomials of higher order and q-Stirling numbers by the fermionic p-adic integral on $\mathbb{Z}_p$. Russ. J. Math. Phys. **16**(4), 484-491 (2009)
12. Kim, T: Symmetry of power sum polynomials and multivariate fermionic p-adic invariant integral on $\mathbb{Z}_p$. Russ. J. Math. Phys. **16**(1), 93-96 (2009)

13. Kim, T: q-Bernoulli numbers and polynomials associated with Gaussian binomial coefficients. Russ. J. Math. Phys. **15**(1), 51-57 (2008)
14. Kim, T, Kim, DS, Dolgy, DV, Rim, SH: Some identities on the Euler numbers arising from Euler basis polynomials. ARS Comb. **109** (2013, in press)
15. Kudo, A: A congruence of generalized Bernoulli number for the character of the first kind. Adv. Stud. Contemp. Math. **2**, 1-8 (2000)
16. Leyendekkers, JV, Shannon, AG, Wong, GCK: Integer structure analysis of the product of adjacent integers and Euler's extension of Fermat's last theorem. Adv. Stud. Contemp. Math. **17**(2), 221-229 (2008)
17. Ozden, H, Cangul, IN, Simsek, Y: Remarks on q-Bernoulli numbers associated with Daehee numbers. Adv. Stud. Contemp. Math. **18**(1), 41-48 (2009)
18. Rim, S-H, Lee, SJ: Some identities on the twisted (h,q)-Genocchi numbers and polynomials associated with q-Bernstein polynomials. Int. J. Math. Math. Sci. **2011**, Art. ID 482840 (2011)
19. Rim, SH, Jin, JH, Moon, EJ, Lee, SJ: Some identities on the q-Genocchi polynomials of higher-order and q-Stirling numbers by the fermionic p-adic integral on $\mathbb{Z}_p$. Int. J. Math. Math. Sci. **2010**, Art. ID 860280 (2010)
20. Ryoo, CS: A note on the Frobenius-Euler polynomials. Proc. Jangjeon Math. Soc. **14**(4), 495-501 (2011)
21. Ryoo, CS: Some identities of the twisted q-Euler numbers and polynomials associated with q-Bernstein polynomials. Proc. Jangjeon Math. Soc. **14**(2), 239-248 (2011)
22. Ryoo, CS: Some relations between twisted q-Euler numbers and Bernstein polynomials. Adv. Stud. Contemp. Math. **21**(2), 217-223 (2011)
23. Simsek, Y, Acikgoz, M: A new generating function of (q–) Bernstein-type polynomials and their interpolation function. Abstr. Appl. Anal. **2010**, Art. ID 769095 (2010)
24. Simsek, Y: Generating functions of the twisted Bernoulli numbers and polynomials associated with their interpolation functions. Adv. Stud. Contemp. Math. **16**(2), 251-278 (2008)
25. Simsek, Y: Complete sum of products of (h,q)-extension of Euler polynomials and numbers. J. Differ. Equ. Appl. **16**(11), 1331-1348 (2010)
26. Zhang, Z, Yang, H: Some closed formulas for generalized Bernoulli-Euler numbers and polynomials. Proc. Jangjeon Math. Soc. **11**(2), 191-198 (2008)
27. Acikgoz, M, Erdal, D, Araci, S: A new approach to q-Bernoulli numbers and q-Bernoulli polynomials related to q-Bernstein polynomials. Adv. Differ. Equ. **2010**, Art. ID 951764 (2010)
28. Araci, S, Erdal, D, Seo, J: A study on the fermionic p-adic q-integral representation on $\mathbb{Z}_p$ associated with weighted q-Bernstein and q-Genocchi polynomials. Abstr. Appl. Anal. **2011**, Art. ID 649248 (2011)
29. Bayad, A: Modular properties of elliptic Bernoulli and Euler functions. Adv. Stud. Contemp. Math. **20**(3), 389-401 (2010)
30. Bayad, A, Kim, T: Identities for the Bernoulli, the Euler and the Genocchi numbers and polynomials. Adv. Stud. Contemp. Math. **20**(2), 247-253 (2010)
31. Bayad, A, Kim, T: Identities involving values of Bernstein, q-Bernoulli, and q-Euler polynomials. Russ. J. Math. Phys. **18**(2), 133-143 (2011)

doi:10.1186/1029-242X-2012-227
Cite this article as: Kim et al.: **Some identities on Bernoulli and Euler polynomials arising from orthogonality of Legendre polynomials.** *Journal of Inequalities and Applications* 2012 **2012**:227.