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# New Symmetric Identities Involving $q$-Zeta Type Functions

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**Abstract:** The main object of this paper is to obtain several symmetric properties of the $q$-zeta type functions. As applications of these properties, we give some new interesting identities for the modified $q$-Genocchi polynomials. Finally, our applications are shown to lead to a number of interesting results which we state in the present paper.

**Keywords:** Genocchi numbers and polynomials, generating functions, $q$-Genocchi polynomials, Euler and $q$-Euler zeta functions, $q$-zeta type functions.

## 1 Introduction

Throughout this paper, we use the following standard notations:

$$\mathbb{N} := \{1,2,3,\cdots\} \quad \text{and} \quad \mathbb{N}_0 := \{0,1,2,\cdots\} = \mathbb{N}\cup\{0\}.$$

Also, as usual, $\mathbb{R}$ denotes the set of real numbers and $\mathbb{C}$ denotes the set of complex numbers.

The Genocchi polynomials $G_n(x)$ and the Genocchi numbers $G_n := G_n(0)$ are given by the following generating functions:

$$\sum_{n=0}^{\infty} G_n(x)\frac{t^n}{n!} = \left(\frac{2t}{e^t+1}\right)e^{xt}$$
and
$$\sum_{n=0}^{\infty} G_n\frac{t^n}{n!} = \frac{2t}{e^t+1}, \qquad (|t|<\pi), \tag{1}$$

respectively. In particular, the second generating function in (1) can be restated as follows:

$$e^{Gt+t}+e^{Gt} = 2t$$

by using the *umbral* (symbolic) convention exhibited by $G^n := G_n$. By utilizing the Taylor-Maclaurin expansion, one finds that

$$(G+1)^n + G_n = \begin{cases} 2 & (n=1) \\ 0 & (\text{otherwise}). \end{cases} \tag{2}$$

It follows from (2) that (see, for details, [29])

$$G_1 = 1,\ G_2 = -1,\ G_3 = 0,\ G_4 = 1,\ G_5 = 0,$$
$$G_6 = -3,\ G_7 = 0,\ G_8 = 17,\ldots$$

and (in general)

$$G_{2n+1} = 0 \qquad (n \in \mathbb{N}).$$

The history of the Genocchi polynomials $G_n(x)$ and the Genocchi numbers $G_n$ can be traced back to the Italian mathematician, Angelo Genocchi (1817–1889). From Genocchi to the present time, the Genocchi polynomials and the Genocchi numbers have been extensively studied in many different contexts in such branches of Mathematics as, for instance, Elementary Number Theory, Complex Analytic Number Theory, Homotopy Theory (especially stable Homotopy groups of spheres), Differential Topology (especially differential structures on spheres), Theory of Modular Forms (especially Eisenstein series), $p$-Adic Analytic Number

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Theory (especially $p$-adic $L$-functions) and Quantum Physics (especially quantum groups). Investigations involving the Genocchi polynomials and their associated combinatorial relations have received considerable attention in recent years (see, for details, [1], [2], [3], [6], [7], [8], [16], [24], [30] and [26]).

Araci *et al.* [6] studied the modified $q$-Genocchi polynomials which are given by the following generating function:

$$F_q(x,t) = \sum_{n=0}^{\infty} \mathscr{G}_{n,q}(x) \frac{t^n}{n!} = [2]_q t \sum_{m=0}^{\infty} (-q)^m e^{\left(x+[m]_q\right)t}, \quad (3)$$

where the $q$-number $[\lambda]_q$ is given by

$$[\lambda]_q := \frac{1-q^\lambda}{1-q} \qquad (0<q<1;\ \lambda \in \mathbb{C}), \quad (4)$$

so that, obviously, we have

$$\lim_{q\to 1^-} \{[\lambda]_q\} = \lambda \qquad (\lambda \in \mathbb{C}).$$

In the case when $x=0$ in (3), it leads to

$$\mathscr{G}_{n,q}(0) := \mathscr{G}_{n,q},$$

that is, to the modified $q$-Genocchi numbers $\mathscr{G}_{n,q}$. In addition to this, by letting $q \to 1^-$, $G_{n,q}$ reduces to the Genocchi numbers $G_n$:

$$\lim_{q\to 1^-} \{\mathscr{G}_{n,q}\} = G_n.$$

The Genocchi numbers $G_n(x)$ possess a number of important properties and are well known in Number Theory. In fact, these numbers are related to the values at negative integers of the Euler Zeta function defined by (see [20], [22], [23], [28], [29]; see also [31])

$$\zeta(s,x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(x+n)^s} = \Phi(-1,s,x) \quad (5)$$

$$\left(s \in \mathbb{C};\ x \in \mathbb{C}\setminus\mathbb{Z}_0^-;\ \mathbb{Z}_0^- := \{0,-1,-2,\ldots\}\right),$$

where $\Phi(z,s,a)$ denotes the widely- and extensively-studied general Hurwitz-Lerch Zeta function defined by (see, for example, [28, p. 121 *et seq.*] and [29, p. 194 *et seq.*]; see also [27], [31] and [32])

$$\Phi(z,s,a) := \sum_{n=0}^{\infty} \frac{z^n}{(n+a)^s} \quad (6)$$

$$\left(a \in \mathbb{C}\setminus\mathbb{Z}_0^-;\ s \in \mathbb{C} \text{ when } |z|<1;\ \Re(s)>1 \text{ when } |z|=1\right).$$

Recently, Kim [20] defined the $q$-Euler Zeta function as follows:

$$\zeta_q(s,x) = [2]_q \sum_{n=0}^{\infty} \frac{(-1)^n q^n}{[x+n]_q^s}, \qquad \left(s \in \mathbb{C};\ x \in \mathbb{C}\setminus\mathbb{Z}_0^-\right). \quad (7)$$

On the other hand, Araci *et al* [6] introduced the $q$-Zeta type function $\widetilde{\zeta}_q(s,x)$ which is slightly different from Kim's $q$-Zeta function $\zeta_q(s,x)$ defined by (7):

$$\begin{aligned}\widetilde{\zeta}_q(s,x) &:= \frac{1}{\Gamma(s)} \int_0^\infty t^{s-2}\left\{-F_q(x,-t)\right\} dt \\ &= [2]_q \sum_{n=0}^{\infty} \frac{(-1)^n q^n}{\left(x+[n]_q\right)^s}, \quad (8) \\ &\left(s \in \mathbb{C};\ x \neq -[n]_q \quad (n \in \mathbb{N}_0)\right),\end{aligned}$$

where $F_q(x,-t)$ is given by (3). From (3) and (8), we find that (see [6])

$$\widetilde{\zeta}_q(-n,x) = \frac{\mathscr{G}_{n+1,q}(x)}{n+1} \qquad (n \in \mathbb{N}_0). \quad (9)$$

Moreover, by using (7) and (8), we have

$$q^{-sx} \widetilde{\zeta}_q\left(s, q^{-1}[x]_{q^{-1}}\right) = \zeta_q(s,x). \quad (10)$$

The Zeta functions play a crucially important rôle in Analytic Number Theory and have applications in such areas as (for example) physics, probability theory, applied statistics, complex analysis, mathematical physics, $p$-adic analysis and other related areas. In particular, the Zeta functions occur within the concept of knot theory, quantum field theory, applied analysis and number theory (see [9], [10], [11], [20], [21], [22], [23], [28] and [31]).

The distribution formula for the modified $q$-Genocchi polynomials is given by (see [6])

$$\begin{aligned}&\mathscr{G}_{n,q}\left(q^a [d]_q x\right) := \\ &\frac{[d]_q^{n-1}}{[d]_{-q}} \sum_{a=0}^{d-1} (-1)^a q^{a(n+1)} \mathscr{G}_{n,q^d}\left(x + \frac{[a]_q}{q^a [d]_q}\right), \quad (11) \\ &\text{for } d \equiv 1 \pmod 2.\end{aligned}$$

Araci *et al.* [8] derived several new identities for the $(h,q)$-Genocchi polynomials and gave symmetric identities of the $(h,q)$-Zeta type functions. Yuan He [14] gave symmetric identities for Carlitz's $q$-Bernoulli numbers (see also [12] and [13]). Kim also obtained symmetric identities for the $q$-Euler polynomials and derived the symmetric identities for the $q$-Euler Zeta function (see [15]). Simsek [25] gave the complete sum of products of $(h,q)$-extension of the Euler polynomials. Bagdasaryan investigated the elementary evaluation of the Zeta function and presented a real analytic approach to the values of the Riemann Zeta function (see, for details, [9] and [10]).

The symmetric identity of the Genocchi polynomials is given by Theorem 1 below (see [11]).

**Theorem 1.** *Let a and b be odd integers. Then we have*

$$\sum_{i=0}^{m}\binom{m}{i}a^{i-1}b^{m-i}G_i(bx)S_{m-i}(a) = \sum_{i=0}^{m}\binom{m}{i}b^{i-1}a^{m-i}G_i(ax)S_{m-i}(b), \tag{12}$$

*where*

$$S_m(a) := \sum_{j=0}^{a-1}(-1)^j j^m. \tag{13}$$

Motivated essentially by some of the aforecited investigations, the fundamental aim of this paper is to generalize Theorem 1 by presenting an interesting and potentially useful extension of the symmetry identity (12) to hold true for the modified $q$-Genocchi polynomials arising from the above-mentioned $q$-Zeta type functions. Several other related results are also considered.

## 2 The $q$-Zeta Type Functions

In this section, we recall from (8) that

$$\widetilde{\zeta}_q(s,x) = [2]_q \sum_{m=0}^{\infty}\frac{(-1)^m q^m}{\left(x+[m]_q\right)^s}. \tag{14}$$

In view of (10), we consider (14) in the following form:

$$q^{-asbx-sbj}\widetilde{\zeta}_{q^a}\left(s,q^{-a}\left[bx+\frac{bj}{a}\right]_{q^{-a}}\right) = [2]_{q^a}\sum_{m=0}^{\infty}\frac{(-1)^m q^{ma}}{\left[m+bx+\frac{bj}{a}\right]_{q^a}^s}. \tag{15}$$

For non-negative integers $k$ and $i$ such that $m = bk+i$ with $0 \leqq i \leqq b-1$, if we suppose that $a \equiv 1 \pmod 2$ and $b \equiv 1 \pmod 2$), then we have

$$\begin{aligned} &q^{-asbx-sbj}\widetilde{\zeta}_{q^a}\left(s,q^{-a}\left[bx+\frac{bj}{a}\right]_{q^{-a}}\right) \\ &= [a]_q^s[2]_{q^a}\sum_{m=0}^{\infty}\frac{(-1)^m q^{ma}}{[ma+abx+bj]_{q^a}^s} \\ &= [a]_q^s[2]_{q^a}\sum_{m=0}^{\infty}\sum_{i=0}^{b-1}\frac{(-1)^{i+mb}q^{(i+mb)a}}{[(i+mb)a+abx+bj]_{q^a}^s} \\ &= [a]_q^s[2]_{q^a}\sum_{i=0}^{b-1}(-1)^i q^{ia}\sum_{m=0}^{\infty}\frac{(-1)^m q^{mba}}{[ab(m+x)+ai+bj]_{q^a}^s}, \end{aligned} \tag{16}$$

which readily yields

$$\begin{aligned} &\sum_{j=0}^{a-1}(-1)^j q^{jb}q^{-asbx-sbj}\widetilde{\zeta}_{q^a}\left(s,q^{-a}\left[bx+\frac{bj}{a}\right]_{q^{-a}}\right) \\ &= [a]_q^s[2]_{q^a}\sum_{j=0}^{a-1}(-1)^j q^{jb}\sum_{i=0}^{b-1}(-1)^i q^{ia}\times \\ &\times\sum_{m=0}^{\infty}\frac{(-1)^m q^{mba}}{[ab(m+x)+ai+bj]_{q^a}^s}. \end{aligned} \tag{17}$$

Upon replacing $a$ by $b$ and $j$ by $i$ in (16), we get

$$\begin{aligned} &q^{-asbx-as}\widetilde{\zeta}_{q^b}\left(s,q^{-b}\left[ax+\frac{ai}{b}\right]_{q^{-b}}\right) \\ &= [b]_q^s[2]_{q^b}\sum_{j=0}^{a-1}(-1)^j q^{jb}\times \\ &\times\sum_{m=0}^{\infty}\frac{(-1)^m q^{mba}}{[ab(m+x)+ai+bj]_{q^a}^s}. \end{aligned} \tag{18}$$

Thus, by applying (17) in (18), we obtain the following theorem.

**Theorem 2.** *For any odd integers a and b, we have*

$$\begin{aligned} &\frac{[2]_{q^b}}{[a]_q^s}\sum_{i=0}^{a-1}(-1)^i q^{ib(1-s)}\widetilde{\zeta}_{q^a}\left(s,q^{-a}\left[bx+\frac{bi}{a}\right]_{q^{-a}}\right) \\ &= \frac{[2]_{q^a}}{[b]_q^s}\sum_{i=0}^{b-1}(-1)^i q^{ia(1-s)}\times \\ &\times\widetilde{\zeta}_{q^b}\left(s,q^{-b}\left[ax+\frac{ai}{b}\right]_{q^{-b}}\right). \end{aligned} \tag{19}$$

**Remark 1.** Upon setting $b = 1$ in Theorem 2, we easily deduce that

$$\begin{aligned} &\widetilde{\zeta}_q\left(s,q^{-1}[ax]_{q^{-1}}\right) = \\ &\frac{[2]_q}{[2]_{q^a}[a]_q^s}\sum_{i=0}^{a-1}(-1)^i q^{i(1-s)}\widetilde{\zeta}_{q^a}\left(s,q^{-a}\left[x+\frac{i}{a}\right]_{q^{-a}}\right). \end{aligned} \tag{20}$$

Taking $a = 2$ in (20), we derive the following Corollary.

**Corollary 1.** *For any odd integer a, we have*

$$\begin{aligned} &\widetilde{\zeta}_q\left(s,q^{-1}[2x]_{q^{-1}}\right) = \frac{[2]_q}{[2]_{q^2}[2]_q^s}\times \\ &\left[\widetilde{\zeta}_{q^2}\left(s,q^{-2}[x]_{q^{-2}}\right) - q^{b(1-s)}\widetilde{\zeta}_{q^2}\left(s,q^{-2}\left[x+\frac{1}{2}\right]_{q^{-2}}\right)\right]. \end{aligned} \tag{21}$$

**Remark 2.** If we take $s = -n$ in Theorem 2, we get the following symmetric property of the modified $q$-Genocchi polynomials.

**Theorem 3.** *For any odd integers a and b, we have*

$$[2]_{q^b}[a]_q^{n-1}\sum_{i=0}^{a-1}(-1)^i q^{ib(n+1)}\times \times \mathscr{G}_{n,q^a}\left(q^{-a}\left[bx+\frac{bi}{a}\right]_{q^{-a}}\right) = = [2]_{q^a}[b]_q^{n-1}\sum_{i=0}^{b-1}(-1)^i q^{ia(n+1)}\times \times \mathscr{G}_{n,q^b}\left(q^{-b}\left[ax+\frac{ai}{b}\right]_{q^{-b}}\right). \quad (22)$$

We now take $b=1$ and replace $x$ by $\frac{x}{a}$ in Theorem 3. We thus restate the distribution formula for the modified $q$-Genocchi polynomials as follows:

$$\mathscr{G}_{n,q}\left(-[-x]_q\right) = \frac{[2]_q}{[2]_{q^a}}[a]_q^{n-1}\sum_{i=0}^{a-1}(-1)^i q^{i(n+1)}\times \times \mathscr{G}_{n,q^a}\left(q^{-a}\left[\frac{x+i}{a}\right]_{q^{-a}}\right), \quad (2\nmid a). \quad (23)$$

We next find from (3) that

$$\sum_{n=0}^{\infty}\mathscr{G}_{n,q}(x+y)\frac{t^n}{n!} = [2]_q t\sum_{m=0}^{\infty}(-q)^m e^{(x+y+[m]_q)t} = \left(\sum_{m=0}^{\infty}y^m\frac{t^m}{m!}\right)\left(\sum_{n=0}^{\infty}\mathscr{G}_{n,q}(x)\frac{t^n}{n!}\right),$$

which, by applying the Cauchy product, yields

$$\sum_{n=0}^{\infty}\mathscr{G}_{n,q}(x+y)\frac{t^n}{n!} = \sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}\binom{n}{k}\mathscr{G}_{k,q}(x)y^{n-k}\right)\frac{t^n}{n!}. \quad (24)$$

Thus, by comparing the coefficients of $\frac{t^n}{n!}$ on both sides of this last equation (24), we get the following Corollary.

**Corollary 2.** *For $n\in\mathbb{N}_0$, we obtain*

$$\mathscr{G}_{n,q}(x+y) = \sum_{k=0}^{n}\binom{n}{k}\mathscr{G}_{k,q}(x)y^{n-k}. \quad (25)$$

By using Theorem 3 and (25), we can derive Theorem 4 below.

**Theorem 4.** *For any odd integers a and b, we have*

$$[2]_{q^b}[a]_q^{n-1}\sum_{k=0}^{n}\binom{n}{k}[a]_{q^{-1}}^{k-n}[b]_{q^{-1}}^{n-k}\times \times \mathscr{G}_{k,q}\left(q^{-a}[bx]_{q^{-a}}\right)S_{n-k:q^{-b}}^{(-n-1)}(a) = = [2]_{q^a}[b]_q^{n-1}\sum_{k=0}^{n}\binom{n}{k}[b]_{q^{-1}}^{k-n}[a]_{q^{-1}}^{n-k}\times \times \mathscr{G}_{k,q}\left(q^{-b}[ax]_{q^{-b}}\right)S_{n-k:q^{-a}}^{(-n-1)}(b). \quad (26)$$

*where*

$$S_{m:q}^{(j)}(a) := \sum_{i=0}^{a-1}(-1)^i q^{ji}[i]_q^m. \quad (27)$$

**Remark 3.** Letting $q\to 1^-$ in Theorem 4, we can deduce the known symmetry identity (12).

## 3 Concluding Remarks and Observations

In this article, we have derived several symmetric properties of the $q$-Zeta type function $\widetilde{\zeta}_q(s,x)$ defined by (8). As applications of these properties, we give new interesting symmetry identities for the modified $q$-Genocchi polynomials $\mathscr{G}_{n,q}(x)$ which are defined by (3). In the limit when $q\to 1^-$, this last result (Theorem 4) is shown to yield the known symmetry identity (12) for the Genocchi polynomials $G_n(x)$.

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