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# A generalization of Gauss sums and its applications to Siegel modular forms and *L*-functions associated with the vector space of quadratic forms

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## Introduction

The main purpose of this paper is the study of Gauss sums associated with symmetric matrices over finite fields and its applications to twisting operators on Siegel modular forms and $L$-functions associated with the vector space of symmetric matrices. Let $F_q$ be the finite field with $q$ elements and $\psi$ a character of $F_q^\times$. In Introduction, we assume the characteristic $p$ of $F_q$ is not 2. We are interested mainly in the case where $\psi^2$ is the trivial character, and we denote by $\chi_p$ the character of order 2 and by $\chi_0$ the trivial character of $F_q^\times$. Let $\Lambda_n(F_q)$ be the set of symmetric matrices of degree $n$ with coefficients in $F_q$ and $\Lambda_{nr}(F_q)$ the subset consisting of elements of rank $r$. For $N \in \Lambda_n(F_q)$, we define a Gauss sum $W_n^n(N, \psi)$ by

$$W_n^n(N, \psi) = \sum_S \psi(\det S) e_p(\mathrm{tr}(NS)),$$

where $e_p(*) = \exp(2\pi\sqrt{-1}\, \mathrm{tr}_{F_q/F_p}(*)/p)$ and $S$ runs through $\Lambda_{nn}(F_q)$. When $\psi^2 = 1$, we can define more Gauss sums. For $S \in \Lambda_{nr}(F_q)$, there exists $g \in \mathrm{GL}_n(F_q)$ such that ${}^t g S g = \begin{pmatrix} S' & 0 \\ 0 & 0 \end{pmatrix}$ with $S' \in \Lambda_{rr}(F_q)$. For $\psi = \chi_p$ or $\chi_0$, we set

$$\psi(S) = \psi(\det S').$$

Then $\psi(S)$ is independent of the choice of $g$. For $r$, $0 \leqq r \leqq n$, and $\psi = \chi_p$ or $\chi_0$, we define

$$W_r^n(N, \psi) = \sum_S \psi(S) e_p(\mathrm{tr}(NS)),$$

where $S$ runs through all $\Lambda_{nr}(F_q)$. The value of $W_n^n(N, \psi)$ for $N \in \Lambda_{nn}(F_q)$ has been determined explicitly by Murakami [3]. In § 1, we give an explicit formula of $W_n^n(N, \psi)$ for all $N \in \Lambda_n(F_q)$

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