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## General $\Omega$ -theorems for coefficients of $L$ -functions

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**Abstract:** We prove a general  $\Omega$ -theorem for the coefficients of polynomial combinations of  $L$ -functions from the Selberg class. As a consequence, we show that the real and imaginary parts of any linear combination of coefficients of such  $L$ -functions have infinitely many sign changes, provided some simple necessary conditions are satisfied.

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