



## A Conjectured Representation of Genocchi Numbers

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Reference

1. P. Erdős, On sets of distances of  $n$  points, this MONTHLY, 53 (1946) 248-250.

RESEARCH PROBLEMS

EDITED BY VICTOR KLEE

*In this Department the Monthly presents easily stated research problems dealing with notions ordinarily encountered in undergraduate mathematics. Each problem should be accompanied by relevant references (if any are known to the author) and by a brief description of known partial results. Material should be sent to Victor Klee, Department of Mathematics, University of Washington, Seattle, WA 98105.*

A CONJECTURED REPRESENTATION OF GENOCCHI NUMBERS

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Genocchi numbers are defined by the formula

$$(1) \quad (G + 1)^N + G_N = 1, \quad N > 1, \quad \text{with } G_1 = 1,$$

where after expansion  $G^i$  is to be replaced by  $G_i$  for each  $i \leq N$ . We remark that the Genocchi numbers can also be generated by the formula [1, pp. 250-263]:

$$(2) \quad \frac{2t}{e^t + 1} = \sum_{N=0}^{\infty} \frac{G_N}{N!} t^N.$$

The first few Genocchi numbers are  $G_1 = +1, G_2 = -1, G_4 = +1, G_6 = -3, G_8 = +17, G_{10} = -155, G_{12} = +2073, G_{14} = -38227, G_{16} = +929569$ , etc. with  $G_{2N+1} = 0$ . We conjecture that

$$(3) \quad G_{2N} = (-1)^N \sum 1^2 \sum 2^2 \sum 3^2 \dots \sum (N - 1)^2,$$

where the  $\sum$  notation used in (3) has the following meaning:

$$\begin{aligned} \sum k^2 &= k^2 - (k - 1)^2 \\ \sum k^2 \sum (k + 1)^2 &= k^2 \sum (k + 1)^2 - (k - 1)^2 \sum k^2 \\ &= k^2 \{ (k + 1)^2 - k^2 \} - (k - 1)^2 \{ k^2 - (k - 1)^2 \} \end{aligned}$$

and in general we have the recurrence

$$(4) \quad \begin{aligned} \sum k^2 \sum (k + 1)^2 \sum (k + 2)^2 \dots \sum (k + N)^2 \\ = k^2 \sum (k + 1)^2 \sum (k + 2)^2 \dots \sum (k + N)^2 \\ - (k - 1)^2 \sum k^2 \sum (k + 1)^2 \dots \sum (k + N - 1)^2. \end{aligned}$$

We note that  $(N+1) \sum$ 's on the left hand side of (4) are reduced to  $N \sum$ 's and the process can be continued till there are no  $\sum$ 's left. The  $\sum$  notation used

can be easily understood by actually calculating the first few Genocchi numbers:

$$\begin{aligned} G_4 &= \sum 1^2 = 1. \\ G_6 &= - \sum 1^2 \sum 2^2 = - 1^2 \sum 2^2 + 0^2 \sum 1^2 = - 1^2(2^2 - 1^2) = - 3. \\ G_8 &= \sum 1^2 \sum 2^2 \sum 3^2 = 1^2 \sum 2^2 \sum 3^2 = 1^2(2^2 \sum 3^2 - 1^2 \sum 2^2) \\ &= 1^2[2^2(3^2 - 2^2) - 1^2(2^2 - 1^2)] = + 17. \\ G_{10} &= - \sum 1^2 \sum 2^2 \sum 3^2 \sum 4^2 \\ &= - 1^2[2^2(3^2 \sum 4^2 - 2^2 \sum 3^2) - 1^2(2^2 \sum 3^2 - 1^2 \sum 2^2)]; \end{aligned}$$

using (5) and simplifying we get  $G_{10} = -155$ .

The formula (3) has been verified to be true for all values of  $G$ 's up to  $G_{14}$ .

For similar  $\sum$  notation as used in this paper, though slightly different, reference may be made to [2].

#### References

1. Anatole Lucas, *Théorie des Nombres*, Vol. 1., Gauthier Villars, Paris, 1891.
2. C. Krishnamachary and R. M. Bhimsenrao, On a table for calculating Eulerian numbers based on a new method, *Proc. London Math. Soc.*, Ser. 2, Vol. 22. (1921) 73-80.

### CLASSROOM NOTES

EDITED BY DAVID DRASIN

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#### INTEGRATION OF TOTAL DIFFERENTIAL EQUATIONS

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Consider the total differential equation

$$(1) \quad P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0,$$

where we shall suppose first of all that

$$Q(x, y, z) = P(y, z, x) \quad \text{and} \quad R(x, y, z) = P(z, x, y).$$

By treating  $x$  in (1) as a constant and integrating, we get

$$(2) \quad U(x, y, z) = \text{const.} = f(x).$$

Assuming (1) is integrable, there exists an  $f$  such that (2) is a solution of (1). The problem is to find  $f$ .

To do this, we have, by the symmetry of (1),

$$(3) \quad U(y, z, x) = f(y)$$