ON THE MODIFIED q-GENOCCHI NUMBERS AND POLYNOMIALS AND THEIR APPLICATIONS

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Abstract. The main objective of this paper is to introduce the modified q-Genocchi polynomials and to define their generating function. In the paper, we show new relations, which are explicit formula, derivative formula, multiplication formula, and some others, for mentioned q-Genocchi polynomials. By applying Mellin transformation to the generating function of the modified q-Genocchi polynomials, we define q-Genocchi zeta-type functions which are interpolated by the modified q-Genocchi polynomials at negative integers.

1. Introduction, Definitions and Notations

As is well known, Genocchi polynomials are given by

$$\frac{2t}{e^t+1}e^{xt}=\sum_{n=0}^{\infty}G_n\left(x\right)\frac{t^n}{n!},\ \left|t\right|<\pi. \tag{1}$$

Putting $x=0$ into (1), we have the quantity $G_n\left(0\right):=G_n$ that stands for Genocchi numbers, some values of which are as follows:

$$G_0=0, G_1=1, G_2=-1, G_4=1, G_6=-3, G_8=17, G_{10}=-155, G_{12}=2073, \cdots$$

with $G_{2n+1}=0$ for $n\geq 1$. The even coefficients of Genocchi numbers can be computed by

$$G_{2n}=2\left(1-2^{2n}\right)B_{2n}$$

where B_n implies Bernoulli numbers and the last identity of the above is known as *Genocchi's* theorem (see [1], [2], [8], [12], [13], [18]). One of the most recent papers on the theory of Genocchi numbers and polynomials is the paper of A. F. Horadam [12], which deals mainly with the theory of Genocchi polynomials. Information on Bernoulli polynomials $B_n\left(x\right)$ and Euler polynomials $E_n\left(x\right)$, to which $G_n\left(x\right)$ may be related, has been derived in [12-14, 18-22, 24, 25]. While a lot of the properties of Genocchi polynomials bear a striking resemblance to the properties of Bernoulli and Euler polynomials, some properties are rather different. Note that Genocchi polynomials occur naturally in the areas of elementary number theory, complex analytic number theory, homotopy theory (stable homotopy groups of spheres), differential topology (differential structures on spheres), theory of modular forms

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(Eisenstein series), p-adic analytic number theory (p-adic L-functions), quantum physics (quantum groups).

We need the following notations and definitions for the sequel of this paper:

Suppose that p be a fixed odd prime number. Throughout this paper, we will employ the following notations:

$$\begin{aligned}&\mathbb{Z}_p, \text{ the ring of } p\text{-adic rational integers}\\&\mathbb{Q}_p, \text{ the field of } p\text{-adic rational numbers}\\&\mathbb{C}_p, \text{ denotes the completion of algebraic closure of } \mathbb{Q}_p\\&\mathbb{N}, \text{ the set of natural numbers.}\end{aligned}$$

In addition to the notation above, we will use the quantity $\mathbb{N}^*$ meaning $\mathbb{N}\cup\{0\}$.

Let υ_p be normalized exponential valuation of $\mathbb{C}_p$ such that

$$|p|_p = p^{-\upsilon_p(p)} = \frac{1}{p}.$$

When one talks of q-extension, q will be known as an indeterminate, either a complex number $q\in\mathbb{C}$, or a p-adic number $q\in\mathbb{C}_p$; it is always clear from the context. If $q\in\mathbb{C}$, then one usually assumes that $|q|<1$. If $q\in\mathbb{C}_p$, then one usually assumes that $|q-1|_p<1$, and hence $q^x=\exp(x\log q)$ for $x\in\mathbb{Z}_p$ (see [2], [3], [4], [8], [16], [24], [27], [28], [30], [31], [34], [35], [36], [37]).

In the theory of q-calculus for a real parameter $q\in(0,1)$, q-numbers are given by

$$[x]_q = \frac{1-q^x}{1-q} \text{ and } [x]_{-q} = \frac{1-(-q)^x}{1+q},$$

(for details, see [15]). Note that $\lim_{q\to 1}[x]_q = x$ for any x with $|x|_p\leq 1$ in the present p-adic case.

Let $UD(\mathbb{Z}_p)$ be the space of uniformly differentiable functions on $\mathbb{Z}_p$. For a positive integer d with $(d,p)=1$, set

$$\begin{aligned}X &= X_d = \lim_{\overleftarrow{n}} \mathbb{Z}/dp^n\mathbb{Z},\ X_1=\mathbb{Z}_p\\X^* &= \bigcup_{\substack{0<a<dp\\(a,p)=1}} (a+dp\mathbb{Z}_p)\end{aligned}$$

and

$$a+dp^n\mathbb{Z}_p = \{x\in X \mid x\equiv a \pmod{dp^n}\},$$

where $a\in\mathbb{Z}$ satisfies the condition $0\leq a<dp^n$ cf. [2], [3], [4], [8], [16], [24], [27], [28], [30], [31], [34], [35], [36], [37].

The following p-adic q-Haar distribution is defined by Kim [28] as

$$\mu_q(x+p^n\mathbb{Z}_p) = \frac{q^x}{[p^n]_q}.$$

Thus, for $f\in UD(\mathbb{Z}_p)$, the p-adic q-integral on $\mathbb{Z}_p$ was defined by Kim as follows:

$$\begin{aligned}I_q(f) &= \int_{\mathbb{Z}_p} f(x)\,d\mu_q(x) \\&= \lim_{n\to\infty}\frac{1}{[p^n]_q}\sum_{x=0}^{p^n-1} f(x)\,q^x.\end{aligned} \tag{2}$$

On the one hand, the bosonic integral is considered as the bosonic limit $q \to 1$, $I_1(f) = \lim_{q\to 1} I_q(f)$, which are called *Volkenborn* integral. On the other hand, the *fermionic* p-adic q-integral on $\mathbb{Z}_p$ is considered by Kim as follows:

$$I_{-q}(f) = \lim_{t\to -q} I_t(f) = \int_{\mathbb{Z}_p} f(x)\, d\mu_{-q}(x). \tag{3}$$

By (3), we have the following well-known integral equation

$$qI_{-q}(f_1) + I_{-q}(f) = [2]_q f(0), \tag{4}$$

where $f_1(x)$ is a translation with $f(x+1)$ cf. [4], [23].

Recently, the modified q-Bernoulli polynomials are introduced by [16]

$$\widetilde{\beta}_{n,q}(x) = \int_{\mathbb{Z}_p} \left(x + [y]_q\right)^n d\mu_q(y) \text{ for } n \in \mathbb{N}^*.$$

In [35], the modified q-Euler polynomials are given by

$$\widetilde{\epsilon}_{n,q}(x) = \int_{\mathbb{Z}_p} q^{-y} \left(x + [y]_q\right)^n d\mu_{-q}(y) \text{ for } n \in \mathbb{N}^*.$$

These polynomials have interesting properties for doing study in the theory of Riemann-zeta function and Euler-zeta function. By the same motivation, let us now consider the following definition.

Definition 1.1. For $n \in \mathbb{N}^*$, the modified q-Genocchi polynomials are defined by

$$\frac{\mathcal{G}_{n+1,q}(x)}{n+1} = \int_{\mathbb{Z}_p} \left(x + [y]_q\right)^n d\mu_{-q}(y) \text{ for } n \in \mathbb{N}^*. \tag{5}$$

Remark 1.2. Setting $q \to 1$ in (5), it yields

$$\lim_{q\to 1}\left(\frac{\mathcal{G}_{n+1,q}(x)}{n+1}\right) := \frac{G_{n+1}(x)}{n+1} = \int_{\mathbb{Z}_p} (x+y)^n d\mu_{-1}(y)\ .$$

In the next section, we derive some new interesting identities for the modified q-Genocchi polynomials and get the generating function for the modified q-Genocchi polynomials. Next, by applying Mellin transformation to the generating function of the modified q-Genocchi polynomials, we define an analogue of Hurwitz Genocchi zeta function. It shows that this zeta function is interpolated by the modified q-Genocchi polynomials at negative integers.

2. On the modified q-Genocchi numbers and polynomials

In this part, we give the properties of the modified q-Genocchi numbers and polynomials and define the generating function of the modified q-Genocchi polynomials.

Substituting $x = 0$ in (5), we have

$$\frac{\mathcal{G}_{n+1,q}(0)}{n+1} := \frac{\mathcal{G}_{n+1,q}}{n+1} = \int_{\mathbb{Z}_p} [y]_q^n\, d\mu_{-q}(y) \text{ for } n \in \mathbb{N}^*. \tag{6}$$

It is clear that $\mathcal{G}_{0,q} = 0$. By (5) and (6), we have the following theorem.

Theorem 2.1. *(**Addition formula**) For $n \in \mathbb{N}^*$, we get*

$$\mathcal{G}_{n,q}(x) = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} \mathcal{G}_{k,q} = (\mathcal{G}_q + x)^n$$

where we have used the technique of umbral calculus as $(\mathcal{G}_q)^n := \mathcal{G}_{n,q}$.

On account of (5), we have

$$\begin{aligned}
\frac{\mathcal{G}_{n+1,q}(x)}{n+1} &= \int_{\mathbb{Z}_p} \left(x + [y]_q\right)^n d\mu_{-q}(y) \\
&= \left(\frac{1}{1-q}\right)^n \int_{\mathbb{Z}_p} ((1-q)x + 1 - q^y)^n d\mu_{-q}(y) \\
&= \sum_{k=0}^{n} \binom{n}{k} \left(\frac{1}{1-q}\right)^k x^{n-k} \int_{\mathbb{Z}_p} (1-q^y)^k d\mu_{-q}(y) \\
&= \sum_{k=0}^{n} \sum_{j=0}^{k} \binom{n}{k} \binom{k}{j} \left(\frac{1}{1-q}\right)^k x^{n-k} (-1)^j \int_{\mathbb{Z}_p} q^{jy} d\mu_{-q}(y) \\
&= [2]_q \sum_{k=0}^{n} \sum_{j=0}^{k} \binom{n}{k} \binom{k}{j} \left(\frac{1}{1-q}\right)^k \frac{x^{n-k}(-1)^j}{1+q^{j+1}}.
\end{aligned}$$

Therefore, we arrive at the following theorem that establishes an explicit formula for the modified q-Genocchi polynomials.

Theorem 2.2. *(**Explicit formula**) For $n \in \mathbb{N}^*$, we have*

$$\frac{\mathcal{G}_{n+1,q}(x)}{n+1} = [2]_q \sum_{k=0}^{n} \sum_{j=0}^{k} \binom{n}{k} \binom{k}{j} \left(\frac{1}{1-q}\right)^k \frac{x^{n-k}(-1)^j}{1+q^{j+1}}. \tag{7}$$

Let $F_q(x;t) = \sum_{n=0}^{\infty} \mathcal{G}_{n,q}(x) \frac{t^n}{n!}$ where $F_q(x;t)$ can be written by (5), as follows:

$$t \sum_{n=0}^{\infty} \left(\int_{\mathbb{Z}_p} \left(x + [y]_q\right)^n d\mu_{-q}(y) \right) \frac{t^n}{n!} = t \int_{\mathbb{Z}_p} e^{\left(x+[y]_q\right)t} d\mu_{-q}(y).$$

That is, the generating function of the modified q-Genocchi polynomials can be rewritten as

$$\sum_{n=0}^{\infty} \mathcal{G}_{n,q}(x) \frac{t^n}{n!} = t \int_{\mathbb{Z}_p} e^{\left(x+[y]_q\right)t} d\mu_{-q}(y). \tag{8}$$

On the other hand, by (7), we see that

$$\begin{aligned}
\sum_{n=0}^{\infty} \mathcal{G}_{n,q}(x) \frac{t^n}{n!} &= [2]_q t \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} \sum_{j=0}^{k} \binom{n}{k} \binom{k}{j} \left(\frac{1}{1-q}\right)^k \frac{x^{n-k}(-1)^j}{1+q^{j+1}} \right) \frac{t^n}{n!} \\
&= [2]_q t e^{xt} \left(\sum_{n=0}^{\infty} \left(\frac{1}{1-q}\right)^n \sum_{j=0}^{n} \binom{n}{j} \frac{(-1)^j}{1+q^{j+1}} \frac{t^n}{n!} \right) \\
&= [2]_q t \sum_{m=0}^{\infty} (-1)^m q^m e^{\left(x+[m]_q\right)t}.
\end{aligned}$$

Thus, we state the following theorem.

Theorem 2.3. *(**Generating function**) We have two forms of representation for the generating function of the modified q-Genocchi polynomials as follows:*

$$\begin{aligned} F_q(x;t) &= \sum_{n=0}^{\infty} \mathcal{G}_{n,q}(x) \frac{t^n}{n!} \\ &= t \int_{\mathbb{Z}_p} e^{\left(x+[y]_q\right)t} d\mu_{-q}(y) \\ &= [2]_q \, t \sum_{m=0}^{\infty} (-1)^m q^m e^{\left(x+[m]_q\right)t}. \end{aligned}$$

By Theorem 2.3, we arrive at

$$\sum_{n=0}^{\infty} \mathcal{G}_{n,q}(x) \frac{t^n}{n!} = \sum_{n=0}^{\infty} \left[[2]_q \sum_{m=0}^{\infty} (-1)^m q^m \left(x+[m]_q\right)^n \right] \frac{t^{n+1}}{n!}.$$

Equating the coefficients of t^n in the last identity above, we get the following theorem.

Theorem 2.4. *For $n \in \mathbb{N}^*$, we have*

$$\frac{\mathcal{G}_{n+1,q}(x)}{n+1} = [2]_q \sum_{m=0}^{\infty} (-1)^m q^m \left(x+[m]_q\right)^n. \tag{9}$$

Let us take $\frac{d}{dx}$, which is a derivative operator, in the both sides of expression in Theorem 2.3; we see that

$$\frac{d}{dx}\left[\mathcal{G}_{n,q}(x)\right] = n\mathcal{G}_{n-1,q}(x).$$

Therefore we obtain the following theorem.

Theorem 2.5. *(**Derivative property for** $\mathcal{G}_{n,q}(x)$) For $n \in \mathbb{N}$, we have*

$$\frac{d}{dx}\left[\mathcal{G}_{n,q}(x)\right] = n\mathcal{G}_{n-1,q}(x).$$

Polynomials $A_n(x)$ are called *Appell polynomials* [11], [29], [30] if they have the property

$$\frac{dA_n(x)}{dx} = nA_{n-1}(x).$$

Thus, by Theorem 2.5, the aforementioned modified q-Genocchi polynomials are *Appell polynomials*.

Let d be an odd positive integer. We are now in a position to state the property of multiplication for the modified q-Genocchi polynomials with the help of the aforementioned fermionic p-adic q-integral on $\mathbb{Z}_p$ in this paper:

$$\begin{aligned}
\frac{\mathcal{G}_{n+1,q}(x)}{n+1} &= \int_{\mathbb{Z}_p} \left(x+[y]_q\right)^n d\mu_{-q}(y) \\
&= \lim_{m\to\infty} \frac{1}{[dp^m]_{-q}} \sum_{y=0}^{dp^m-1} \left(x+[y]_q\right)^n (-q)^y \\
&= \lim_{m\to\infty} \frac{1}{[dp^m]_{-q}} \sum_{y=0}^{p^m-1} \sum_{a=0}^{d-1} \left(x+[a+dy]_q\right)^n (-q)^{a+dy} \\
&= \frac{1}{[d]_{-q}} \sum_{a=0}^{d-1} (-q)^a \lim_{m\to\infty} \frac{1}{[p^m]_{-q^d}} \sum_{y=0}^{p^m-1} \left(x+[a]_q+q^a [d]_q [y]_{q^d}\right)^n (-q)^{dy} \\
&= \frac{[d]_q^n}{[d]_{-q}} \sum_{a=0}^{d-1} (-1)^a q^{a(n+1)} \left[\lim_{m\to\infty} \frac{1}{[p^m]_{-(q^d)}} \sum_{y=0}^{p^m-1} \left(\frac{x+[a]_q}{q^a [d]_q} + [y]_{q^d}\right)^n \left(-q^d\right)^y\right] \\
&= \frac{[d]_q^n}{[d]_{-q}} \sum_{a=0}^{d-1} (-1)^a q^{a(n+1)} \int_{\mathbb{Z}_p} \left(\frac{x+[a]_q}{q^a [d]_q} + [y]_{q^d}\right)^n d\mu_{-q^d}(y)
\end{aligned}$$

where we have used the following two identities that are well known in the theory of q-calculus

$$[x+y]_q = [x]_q + q^x [y]_q \text{ and } [xy]_q = [x]_q [y]_{q^x} \text{ (see [15]).}$$

As a result of these applications, we have the following theorem.

Theorem 2.6. *(**Multiplication formula**) Let $d \equiv 1 \pmod 2$ and $n \in \mathbb{N}$, then we have*

$$\mathcal{G}_{n,q}\left(q^a [d]_q x\right) = \frac{[d]_q^{n-1}}{[d]_{-q}} \sum_{a=0}^{d-1} (-1)^a q^{a(n+1)} \mathcal{G}_{n,q^d}\left(x + \frac{[a]_q}{q^a [d]_q}\right).$$

3. On the modified q-Genocchi polynomials in connection with Zeta-type function

The Hurwitz zeta function is defined by

$$\zeta(s,x) = \sum_{n=0}^{\infty} \frac{1}{(n+x)^s}, \text{ for } s \in \mathbb{C}. \tag{10}$$

Putting $x=1$ in (10), yields to

$$\zeta(s,1) := \zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

where $\zeta(s)$ is known as Riemann zeta function to be convergence for $\operatorname{Re}(s) > 1$ (cf. [5], [6], [7], [17], [26]). Note that the Bernoulli numbers interpolate by the Riemann zeta function, which plays a crucial role in analytic number theory and has many applications in physics, probability and applied statistics. Firstly, Leonard Euler introduced the Riemann zeta function of real argument without using complex analysis. By (10), we have the following relation: For $n \in \mathbb{N}$,

$$\zeta(1-n) = -\frac{B_n}{n} \text{ (see [5], [6], [7], [17], [26]).}$$

So we consider an analogue of zeta function by applying Mellin transformation to the generating function of the modified q-Genocchi polynomials. From those consideration, we write the following:

$$\frac{1}{\Gamma(s)}\int_0^\infty t^{s-2}F_q(x;-t)\,dt$$

this identity yields to

$$[2]_q\sum_{m=0}^\infty \frac{(-1)^m q^m}{\left(x+[m]_q\right)^s}. \tag{11}$$

As q approaches to 1 in (11), it becomes

$$2\sum_{m=0}^\infty \frac{(-1)^m}{(x+m)^s}. \tag{12}$$

The Equation (12) is known as Genocchi zeta function [36]. It shows that the Equation (11) seems to be the new q-analogue of Genocchi zeta function. So we give the following definition.

Definition 3.1. Let $s\in\mathbb{C}$ with $\operatorname{Re}(s)>1$ and $0<x\leq 1$. The new q-analogue of the Hurwitz Genocchi zeta-type function is expressed by

$$\zeta_q(s,x)=[2]_q\sum_{m=0}^\infty \frac{(-1)^m q^m}{\left(x+[m]_q\right)^s}. \tag{13}$$

By (9) and (13), we have

$$\zeta_q(-n,x)=\frac{\mathcal{G}_{n+1,q}(x)}{n+1}. \tag{14}$$

The Equation (14) seems to be interpolating for the modified q-Genocchi polynomials at negative integers. Now we define partial q-Hurwitz Genocchi zeta-type function as follows: For $F\equiv 1\,(\operatorname{mod} 2)$

$$\mathcal{H}_q(s,x:a,F)=[2]_q\sum_{\substack{m\equiv a(\operatorname{mod} F)\\ m>0}} \frac{(-1)^m q^m}{\left(x+[m]_q\right)^s}. \tag{15}$$

By (15), we have

$$\begin{aligned}
\mathcal{H}_q(s,x:a,F) &= [2]_q\sum_{m=0}^\infty \frac{(-1)^{mF+a} q^{mF+a}}{\left(x+[mF+a]_q\right)^s}\\
&= (-1)^a q^a[2]_q\sum_{m=0}^\infty \frac{(-1)^m q^{mF}}{\left(x+[a]_q+q^a[F]_q[m]_{q^F}\right)^s}\\
&= \frac{(-1)^a q^{a(1-s)}[2]_q}{[F]_q^s[2]_{q^F}}\left[[2]_{q^F}\sum_{m=0}^\infty \frac{(-1)^m q^{mF}}{\left(\frac{x+[a]_q}{q^a[F]_q}+[m]_{q^F}\right)^s}\right]\\
&= \frac{(-1)^a q^{a(1-s)}[2]_q}{[F]_q^s[2]_{q^F}}\zeta_{q^F}\left(s,\frac{x+[a]_q}{q^a[F]_q}\right).
\end{aligned}$$

Therefore we procure the following theorem.

Theorem 3.2. *For $F \equiv 1 \,(\mathrm{mod}\, 2)$ and $0 \leq a < F$, we have*

$$\mathcal{H}_q(s, x : a, F) = \frac{(-1)^a q^{a(1-s)} [2]_q}{[F]_q^s [2]_{q^F}} \zeta_{q^F}\left(s, \frac{x + [a]_q}{q^a [F]_q}\right). \tag{16}$$

Putting $s = -n$ in (16) yields to

$$\mathcal{H}_q(-n, x : a, F) = \frac{(-1)^a q^{a(n+1)} [2]_q [F]_q^n}{[2]_{q^F}} \frac{\mathcal{G}_{n+1,q^F}\left(\frac{x+[a]_q}{q^a [F]_q}\right)}{n+1}.$$

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