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New Approach to q-Euler Polynomials of Higher Order

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Abstract. In this paper, we present new generating functions related to q-Euler numbers and polynomials of higher order. Using those generating functions, we present new identities involving q-Euler numbers and polynomials of higher order.

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1. INTRODUCTION/PRELIMINARIES

Let $\mathbb{C}$ be the complex number field. Assume that $q \in \mathbb{C}$ with $|q| < 1$ and define the *q-number* by

$$[x]_q = (1 - q^x)/(1 - q).$$

The *q-factorial* is given by

$$[n]_q! = [n]_q[n-1]_q \cdots [2]_q[1]_q,$$

and the *q-binomial* formulas are known to be

$$(x:q)_n = \prod_{i=1}^{n}(1 - xq^{i-1}) = \sum_{i=0}^{n}\binom{n}{i}_q q^{\binom{i}{2}}(-x)^i$$

(see [3, 14, 15]) and

$$\frac{1}{(x:q)_n} = \prod_{i=1}^{n}\left(\frac{1}{1 - xq^{i-1}}\right) = \sum_{i=0}^{\infty}\binom{n+i-1}{i}_q x^i$$

(see [3, 5, 14, 15]), where

$$\binom{n}{i}_q = \frac{[n]_q!}{[n-i]_q![i]_q!} = \frac{[n]_q[n-1]_q \cdots [n-i+1]_q}{[i]_q!}.$$

The *Euler polynomials* are defined by

$$(2/(e^t + 1))e^{xt} = \sum_{n=0}^{\infty} E_n(x)t^n/n!$$

for $|t| < \pi$. In the special case $x = 0$, the values $E_n(= E_n(0))$ are referred to as the nth Euler numbers. In this paper, we consider q-extensions of Euler numbers and polynomials of higher order. Barnes' multiple Bernoulli polynomials are also defined by

$$\frac{t^r}{\prod_{j=1}^{r}(e^{a_j t} - 1)}e^{xt} = \sum_{n=0}^{\infty} B_n(x, r|a_1, \ldots, a_r)\frac{t^n}{n!}, \tag{1}$$

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