

Research Article

q -Analogues of the Bernoulli and Genocchi Polynomials and the Srivastava-Pintér Addition Theorems

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Received 24 April 2012; Revised 5 July 2012; Accepted 23 July 2012

Academic Editor: Lee Chae Jang

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The main purpose of this paper is to introduce and investigate a new class of generalized Bernoulli and Genocchi polynomials based on the q -integers. The q -analogues of well-known formulas are derived. The q -analogue of the Srivastava-Pintér addition theorem is obtained.

1. Introduction

Throughout this paper, we always make use of the following notation: \mathbb{N} denotes the set of natural numbers, \mathbb{N}_0 denotes the set of nonnegative integers, \mathbb{R} denotes the set of real numbers, and \mathbb{C} denotes the set of complex numbers.

The q -shifted factorial is defined by

$$(a; q)_0 = 1, \quad (a; q)_n = \prod_{j=0}^{n-1} (1 - q^j a), \quad n \in \mathbb{N}, \quad (a; q)_\infty = \prod_{j=0}^{\infty} (1 - q^j a), \quad |q| < 1, \quad a \in \mathbb{C}. \quad (1.1)$$

The q -numbers and q -numbers factorial is defined by

$$[a]_q = \frac{1 - q^a}{1 - q} \quad (q \neq 1); \quad [0]_q! = 1; \quad [n]_q! = [1]_q [2]_q \cdots [n]_q \quad n \in \mathbb{N}, \quad a \in \mathbb{C}, \quad (1.2)$$

respectively. The q -polynomial coefficient is defined by

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{(q; q)_n}{(q; q)_{n-k} (q; q)_k}. \quad (1.3)$$

The q -analogue of the function $(x + y)^n$ is defined by

$$(x + y)_q^n := \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q q^{(1/2)k(k-1)} x^{n-k} y^k, \quad n \in \mathbb{N}_0. \quad (1.4)$$

In the standard approach to the q -calculus two exponential function are used:

$$\begin{aligned} e_q(z) &= \sum_{n=0}^{\infty} \frac{z^n}{[n]_q!} = \prod_{k=0}^{\infty} \frac{1}{(1 - (1-q)q^k z)}, \quad 0 < |q| < 1, |z| < \frac{1}{|1-q|}, \\ E_q(z) &= \sum_{n=0}^{\infty} \frac{q^{(1/2)n(n-1)} z^n}{[n]_q!} = \prod_{k=0}^{\infty} (1 + (1-q)q^k z), \quad 0 < |q| < 1, z \in \mathbb{C}. \end{aligned} \quad (1.5)$$

From this form we easily see that $e_q(z)E_q(-z) = 1$. Moreover,

$$D_q e_q(z) = e_q(z), \quad D_q E_q(z) = E_q(qz), \quad (1.6)$$

where D_q is defined by

$$D_q f(z) := \frac{f(qz) - f(z)}{qz - z}. \quad (1.7)$$

The previous q -standard notation can be found in [1].

Carlitz has introduced the q -Bernoulli numbers and polynomials in [2]. Srivastava and Pintér proved some relations and theorems between the Bernoulli polynomials and Euler polynomials in [3]. They also gave some generalizations of these polynomials. In [4–6], Kim et al. investigated some properties of the q -Euler polynomials and Genocchi polynomials. They gave some recurrence relations. In [7], Cenkci et al. gave the q -extension of Genocchi numbers in a different manner. In [5], Kim gave a new concept for the q -Genocchi numbers and polynomials. In [8], Simsek et al. investigated the q -Genocchi zeta function and l -function by using generating functions and Mellin transformation. We also recall the definitions of the q -Bernoulli and the q -Genocchi polynomials of higher order (see [2, 9–12]):

$$\begin{aligned} (-t)^\alpha \sum_{n=0}^{\infty} \frac{([\alpha]_q)_n}{[n]_q!} q^{n+x} e^{t[n+x]_q} &= \sum_{n=0}^{\infty} B_{n,q}^{(\alpha)}(x) \frac{t^n}{n!}, \\ (2t)^\alpha \sum_{n=0}^{\infty} \frac{([\alpha]_q)_n}{[n]_q!} (-1)^n q^{n+x} e^{t[n+x]_q} &= \sum_{n=0}^{\infty} G_{n,q}^{(\alpha)}(x) \frac{t^n}{n!}. \end{aligned} \quad (1.8)$$

We propose the following definitions. We define the q -Bernoulli and the q -Genocchi polynomials of higher order in two variables x and y , using two q -exponential functions, which helps us easily prove some properties of these polynomials and q -analogue of the Srivastava and Pintér addition theorem.

Definition 1.1. The q -Bernoulli numbers $\mathfrak{B}_{n,q}^{(\alpha)}$ and polynomials $\mathfrak{B}_{n,q}^{(\alpha)}(x, y)$ in x, y of order α are defined by means of the generating function functions:

$$\begin{aligned} \left(\frac{t}{e_q(t)-1}\right)^\alpha &= \sum_{n=0}^\infty \mathfrak{B}_{n,q}^{(\alpha)} \frac{t^n}{[n]_q!}, \quad |t| < 2\pi, \\ \left(\frac{t}{e_q(t)-1}\right)^\alpha e_q(tx)E_q(ty) &= \sum_{n=0}^\infty \mathfrak{B}_{n,q}^{(\alpha)}(x, y) \frac{t^n}{[n]_q!}, \quad |t| < 2\pi. \end{aligned} \tag{1.9}$$

Definition 1.2. The q -Genocchi numbers $\mathfrak{G}_{n,q}^{(\alpha)}$ and polynomials $\mathfrak{G}_{n,q}^{(\alpha)}(x, y)$ in x, y are defined by means of the generating functions:

$$\begin{aligned} \left(\frac{2t}{e_q(t)+1}\right)^\alpha &= \sum_{n=0}^\infty \mathfrak{G}_{n,q}^{(\alpha)} \frac{t^n}{[n]_q!}, \quad |t| < \pi, \\ \left(\frac{2t}{e_q(t)+1}\right)^\alpha e_q(tx)E_q(ty) &= \sum_{n=0}^\infty \mathfrak{G}_{n,q}^{(\alpha)}(x, y) \frac{t^n}{[n]_q!}, \quad |t| < \pi. \end{aligned} \tag{1.10}$$

It is obvious that

$$\begin{aligned} \mathfrak{B}_{n,q}^{(\alpha)} &= \mathfrak{B}_{n,q}^{(\alpha)}(0, 0), & \lim_{q \rightarrow 1^-} \mathfrak{B}_{n,q}^{(\alpha)}(x, y) &= B_n^{(\alpha)}(x + y), & \lim_{q \rightarrow 1^-} \mathfrak{B}_{n,q}^{(\alpha)} &= B_n^{(\alpha)}, \\ \mathfrak{G}_{n,q}^{(\alpha)} &= \mathfrak{G}_{n,q}^{(\alpha)}(0, 0), & \lim_{q \rightarrow 1^-} \mathfrak{G}_{n,q}^{(\alpha)}(x, y) &= G_n^{(\alpha)}(x + y), & \lim_{q \rightarrow 1^-} \mathfrak{G}_{n,q}^{(\alpha)} &= G_n^{(\alpha)}. \end{aligned} \tag{1.11}$$

Here $B_n^{(\alpha)}(x)$ and $E_n^{(\alpha)}(x)$ denote the classical Bernoulli, and Genocchi polynomials of order α are defined by

$$\left(\frac{t}{e^t-1}\right)^\alpha e^{tx} = \sum_{n=0}^\infty B_n^{(\alpha)}(x) \frac{t^n}{n!}, \quad \left(\frac{2}{e^t+1}\right)^\alpha e^{tx} = \sum_{n=0}^\infty G_n^{(\alpha)}(x) \frac{t^n}{n!}. \tag{1.12}$$

The aim of the present paper is to obtain some results for the q -Genocchi polynomials (properties of the q -Bernoulli polynomials are studied in [13]). The q -analogues of well-known results, for example, Srivastava and Pintér [3], can be derived from these q -identities. It should be mentioned that probabilistic proofs the Srivastava-Pintér addition theorems were given recently in [14]. The formulas involving the q -Stirling numbers of the second kind, q -Bernoulli polynomials and q -Bernstein polynomials, are also given. Furthermore some special cases are also considered.

The following elementary properties of the q -Genocchi polynomials $\mathfrak{G}_{n,q}^{(\alpha)}(x, y)$ of order α are readily derived from Definition 1.2. We choose to omit the details involved.

Property 1.3. Special values of the q -Genocchi polynomials of order α :

$$\mathfrak{G}_{n,q}^{(0)}(x, 0) = x^n, \quad \mathfrak{G}_{n,q}^{(0)}(0, y) = q^{(1/2)n(n-1)} y^n. \quad (1.13)$$

Property 1.4. Summation formulas for the q -Genocchi polynomials of order α :

$$\begin{aligned} \mathfrak{G}_{n,q}^{(\alpha)}(x, y) &= \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q \mathfrak{G}_{k,q}^{(\alpha)}(x+y)_q^{n-k}, & \mathfrak{G}_{n,q}^{(\alpha)}(x, y) &= \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q \mathfrak{G}_{n-k,q}^{(\alpha-1)} \mathfrak{G}_{k,q}^{(\alpha)}(x, y), \\ \mathfrak{G}_{n,q}^{(\alpha)}(x, y) &= \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q q^{(n-k)(n-k-1)/2} \mathfrak{G}_{k,q}^{(\alpha)}(x, 0) y^{n-k} = \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q \mathfrak{G}_{k,q}^{(\alpha)}(0, y) x^{n-k}, & (1.14) \\ \mathfrak{G}_{n,q}^{(\alpha)}(x, 0) &= \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q \mathfrak{G}_{k,q}^{(\alpha)} x^{n-k}, & \mathfrak{G}_{n,q}^{(\alpha)}(0, y) &= \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q q^{(n-k)(n-k-1)/2} \mathfrak{G}_{k,q}^{(\alpha)} y^{n-k}. \end{aligned}$$

Property 1.5. Difference equations:

$$\begin{aligned} \mathfrak{G}_{n,q}^{(\alpha)}(1, y) + \mathfrak{G}_{n,q}^{(\alpha)}(0, y) &= 2[n]_q \mathfrak{G}_{n-1,q}^{(\alpha-1)}(0, y), \\ \mathfrak{G}_{n,q}^{(\alpha)}(x, 0) + \mathfrak{G}_{n,q}^{(\alpha)}(x, -1) &= 2[n]_q \mathfrak{G}_{n-1,q}^{(\alpha-1)}(x, -1). \end{aligned} \quad (1.15)$$

Property 1.6. Differential relations:

$$D_{q,x} \mathfrak{G}_{n,q}^{(\alpha)}(x, y) = [n]_q \mathfrak{G}_{n-1,q}^{(\alpha)}(x, y), \quad D_{q,y} \mathfrak{G}_{n,q}^{(\alpha)}(x, y) = [n]_q \mathfrak{G}_{n-1,q}^{(\alpha)}(x, qy). \quad (1.16)$$

Property 1.7. Addition theorem of the argument:

$$\mathfrak{G}_{n,q}^{(\alpha+\beta)}(x, y) = \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q \mathfrak{G}_{n-k,q}^{(\alpha)}(x, 0) \mathfrak{G}_{k,q}^{(\beta)}(0, y). \quad (1.17)$$

Property 1.8. Recurrence relationships:

$$\mathfrak{G}_{n,q}^{(\alpha)}\left(\frac{1}{m}, y\right) + \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q \left(\frac{1}{m} - 1\right)_q^{n-k} \mathfrak{G}_{k,q}^{(\alpha)}(0, y) = 2[n]_q \sum_{k=0}^{n-1} \begin{bmatrix} n-1 \\ k \end{bmatrix}_q \left(\frac{1}{m} - 1\right)_q^{n-1-k} \mathfrak{G}_{k,q}^{(\alpha-1)}(0, y). \quad (1.18)$$

2. Explicit Relationship between the q -Genocchi and the q -Bernoulli Polynomials

In this section we prove an interesting relationship between the q -Genocchi polynomials $\mathfrak{G}_{n,q}^{(\alpha)}(x, y)$ of order α and the q -Bernoulli polynomials. Here some q -analogues of known results will be given. We also obtain new formulas and their some special cases in the following.

Theorem 2.1. For $n \in \mathbb{N}_0$, the following relationship

$$\begin{aligned} \mathfrak{G}_{n,q}^{(\alpha)}(x, y) = \sum_{k=0}^n \frac{1}{m^{n-k-1} [k+1]_q} & \left[2[k+1]_q \sum_{j=0}^k \binom{k}{j}_q \frac{1}{m^{k-j}} \mathfrak{G}_{j,q}^{(\alpha-1)}(x, -1) \right. \\ & \left. - \sum_{j=0}^{k+1} \binom{k+1}{j}_q \frac{1}{m^{k+1-j}} \mathfrak{G}_{j,q}^{(\alpha)}(x, -1) - \mathfrak{G}_{k+1,q}^{(\alpha)}(x, 0) \right] \mathfrak{B}_{n-k,q}(0, my) \end{aligned} \tag{2.1}$$

holds true between the q -Genocchi and the q -Bernoulli polynomials.

Proof. Using the following identity:

$$\left(\frac{2t}{e_q(t) + 1} \right)^\alpha e_q(tx) E_q(ty) = \left(\frac{2t}{e_q(t) + 1} \right)^\alpha e_q(tx) \cdot \frac{e_q(t/m) - 1}{t} \cdot \frac{t}{e_q(t/m) - 1} \cdot E_q\left(\frac{t}{m} my\right), \tag{2.2}$$

we have

$$\begin{aligned} \sum_{n=0}^{\infty} \mathfrak{G}_{n,q}^{(\alpha)}(x, y) \frac{t^n}{[n]_q!} &= \frac{m}{t} \sum_{n=0}^{\infty} \left(\sum_{k=0}^n \binom{n}{k}_q \frac{1}{m^{n-k}} \mathfrak{G}_{k,q}^{(\alpha)}(x, 0) - \mathfrak{G}_{n,q}^{(\alpha)}(x, 0) \right) \frac{t^n}{[n]_q!} \sum_{n=0}^{\infty} \mathfrak{B}_{n,q}(0, my) \frac{t^n}{m^n [n]_q!} \\ &= \sum_{n=1}^{\infty} \left(\sum_{k=0}^n \binom{n}{k}_q \frac{1}{m^{n-1-k}} \mathfrak{G}_{k,q}^{(\alpha)}(x, 0) \right. \\ & \quad \left. - m \mathfrak{G}_{n,q}^{(\alpha)}(x, 0) \right) \frac{t^{n-1}}{[n]_q!} \sum_{n=0}^{\infty} \mathfrak{B}_{n,q}(0, my) \frac{t^n}{m^n [n]_q!} \\ &= \sum_{n=0}^{\infty} \left(\sum_{k=0}^{n+1} \binom{n+1}{k}_q m^k \mathfrak{G}_{k,q}^{(\alpha)}(x, 0) \right. \\ & \quad \left. - m^{n+1} \mathfrak{G}_{n+1,q}^{(\alpha)}(x, 0) \right) \frac{t^n}{m^n [n+1]_q!} \sum_{n=0}^{\infty} \mathfrak{B}_{n,q}(0, my) \frac{t^n}{m^n [n]_q!} \\ &= \sum_{n=0}^{\infty} \sum_{k=0}^n \frac{1}{m^n [k+1]_q} \left(\sum_{j=0}^{k+1} \binom{k+1}{j}_q m^j \mathfrak{G}_{j,q}^{(\alpha)}(x, 0) \right. \\ & \quad \left. - m^{k+1} \mathfrak{G}_{k+1,q}^{(\alpha)}(x, 0) \right) \mathfrak{B}_{n-k,q}(0, my) \frac{t^n}{[n]_q!}. \end{aligned} \tag{2.3}$$

It remains to use Property 1.8. □

Since $\mathfrak{G}_{n,q}^{(\alpha)}(x, y)$ is not symmetric with respect to x and y , we can prove a different form of the previously mentioned theorem. It should be stressed out that Theorems 2.1 and 2.2 coincide in the limiting case when $q \rightarrow 1^-$.

Theorem 2.2. For $n \in \mathbb{N}_0$, the following relationship

$$\begin{aligned} \mathfrak{G}_{n,q}^{(\alpha)}(x, y) &= \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q \frac{1}{m^{n-k-1} [k+1]_q} \left[2[k+1]_q \sum_{j=0}^k \begin{bmatrix} k \\ j \end{bmatrix}_q \left(\frac{1}{m} - 1 \right)_q^{k-j} \mathfrak{G}_{j,q}^{(\alpha-1)}(0, y) \right. \\ &\quad \left. - \sum_{j=0}^{k+1} \begin{bmatrix} k+1 \\ j \end{bmatrix}_q \left(\frac{1}{m} - 1 \right)_q^{k+1-j} \mathfrak{G}_{j,q}^{(\alpha)}(0, y) - \mathfrak{G}_{k+1,q}(0, y) \right] \\ &\quad \times \mathfrak{B}_{n-k,q}(mx, 0) \end{aligned} \quad (2.4)$$

holds true between the q -Genocchi and the q -Bernoulli polynomials.

Proof. The proof is based on the following identity:

$$\left(\frac{2t}{e_q(t) + 1} \right)^\alpha e_q(tx) E_q(ty) = \left(\frac{2t}{e_q(t) + 1} \right)^\alpha E_q(ty) \cdot \frac{e_q(t/m) - 1}{t} \cdot \frac{t}{e_q(t/m) - 1} \cdot e_q\left(\frac{t}{m} mx\right). \quad (2.5)$$

□

Next we discuss some special cases of Theorems 2.1 and 2.2. By noting that

$$\mathfrak{G}_{j,q}^{(0)}(0, y) = q^{(1/2)j(j-1)} y^j, \quad \mathfrak{G}_{j,q}^{(0)}(x, -1) = (x-1)_q^j, \quad (2.6)$$

we deduce from Theorems 2.1 and 2.2 Corollary 2.3 below.

Corollary 2.3. For $n \in \mathbb{N}_0$, $m \in \mathbb{N}$ the following relationship

$$\begin{aligned} \mathfrak{G}_{n,q}(x, y) &= \sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q \frac{1}{m^{n-k-1} [k+1]_q} \left[2[k+1]_q \sum_{j=0}^k \begin{bmatrix} k \\ j \end{bmatrix}_q \left(\frac{1}{m} - 1 \right)_q^{k-j} q^{(1/2)j(j-1)} y^j \right. \\ &\quad \left. - \sum_{j=0}^{k+1} \begin{bmatrix} k+1 \\ j \end{bmatrix}_q \left(\frac{1}{m} - 1 \right)_q^{k+1-j} \mathfrak{G}_{j,q}(0, y) - \mathfrak{G}_{k+1,q}(0, y) \right] \\ &\quad \times \mathfrak{B}_{n-k,q}(mx, 0), \end{aligned}$$

$$\begin{aligned} \mathfrak{G}_{n,q}(x, y) = \sum_{k=0}^n \binom{n}{k}_q \frac{1}{m^{n-k-1}[k+1]_q} & \left[2[k+1]_q \sum_{j=0}^k \binom{k}{j}_q \frac{1}{m^{k-j}} (x-1)_q^j \right. \\ & \left. - \sum_{j=0}^{k+1} \binom{k+1}{j}_q \frac{1}{m^{k+1-j}} \mathfrak{G}_{j,q}(x, -1) - \mathfrak{G}_{k+1,q}(x, 0) \right] \\ & \times \mathfrak{B}_{n-k,q}(0, my) \end{aligned} \tag{2.7}$$

holds true between the q -Bernoulli polynomials and q -Euler polynomials.

Corollary 2.4. For $n \in \mathbb{N}_0$, $m \in \mathbb{N}$ the following relationship holds true:

$$G_n(x+y) = \sum_{k=0}^n \binom{n}{k} \frac{2}{k+1} \left((k+1)y^k - G_{k+1,q}(y) \right) B_{n-k}(x), \tag{2.8}$$

$$\begin{aligned} G_n(x+y) = \sum_{k=0}^n \binom{n}{k} \frac{1}{m^{n-k-1}(k+1)} & \left[2(k+1)G_k\left(y + \frac{1}{m} - 1\right) \right. \\ & \left. - G_{k+1}\left(y + \frac{1}{m} - 1\right) - G_{k+1}(y) \right] B_{n-k,q}(mx) \end{aligned} \tag{2.9}$$

between the classical Genocchi polynomials and the classical Bernoulli polynomials.

Note that the formula (2.9) is new for the classical polynomials.

In terms of the q -Genocchi numbers $\mathfrak{G}_{k,q}^{(\alpha)}$, by setting $y = 0$ in Theorem 2.1, we obtain the following explicit relationship between the q -Genocchi polynomials $\mathfrak{G}_{k,q}^{(\alpha)}$ of order α and the q -Bernoulli polynomials.

Corollary 2.5. The following relationship holds true:

$$\begin{aligned} \mathfrak{G}_{n,q}^{(\alpha)}(x, 0) = \sum_{k=0}^n \binom{n}{k}_q \frac{1}{m^{n-k-1}[k+1]_q} & \left[2[k+1]_q \sum_{j=0}^k \binom{k}{j}_q \left(\frac{1}{m} - 1\right)_q^{k-j} \mathfrak{G}_{j,q}^{(\alpha-1)} \right. \\ & \left. - \sum_{j=0}^{k+1} \binom{k+1}{j}_q \left(\frac{1}{m} - 1\right)_q^{k+1-j} \mathfrak{G}_{j,q}^{(\alpha)} - \mathfrak{G}_{k+1,q}^{(\alpha)} \right] \mathfrak{B}_{n-k,q}(mx, 0). \end{aligned} \tag{2.10}$$

Corollary 2.6. For $n \in \mathbb{N}_0$ the following relationship holds true:

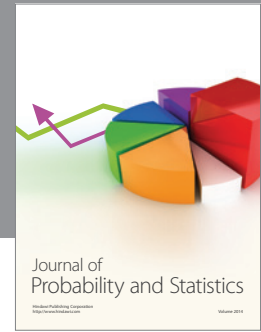
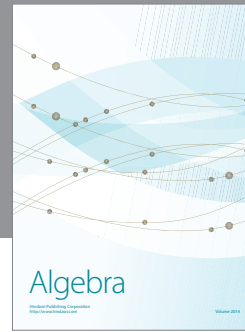
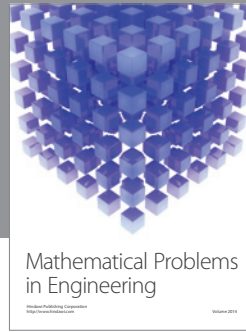
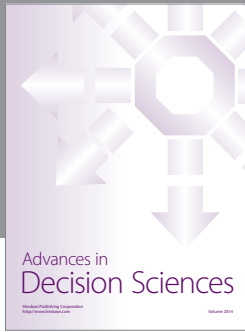
$$\mathfrak{G}_{n,q}(x, y) = \sum_{k=0}^n \binom{n}{k}_q \frac{2}{[k+1]_q} \left[[k+1]_q q^{(1/2)k(k-1)} y^k - \mathfrak{G}_{k+1,q}(0, y) \right] \mathfrak{B}_{n-k,q}(x, 0). \tag{2.11}$$

Corollary 2.7. For $n \in \mathbb{N}_0$ the following relationship holds true:

$$\begin{aligned}\mathfrak{G}_{n,q}(x, 0) &= -\sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q \frac{2}{[k+1]_q} \mathfrak{G}_{k+1,q} \mathfrak{B}_{n-k,q}(x, 0), \\ \mathfrak{G}_{n,q} &= -\sum_{k=0}^n \begin{bmatrix} n \\ k \end{bmatrix}_q \frac{2}{[k+1]_q} \mathfrak{G}_{k+1,q} \mathfrak{B}_{n-k,q}.\end{aligned}\tag{2.12}$$

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