Day 5: ACQ, FO^k

ECI 2015
Buenos Aires

Fundamentos lógicos de bases de datos

(Logical foundations of databases)

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CNRS LaBRI

Recap

- **NP complexity class** (SAT, 3COL, ...)
- **Conjunctive Queries** (correspondence with SQL and Relational Algebra)
- **Homomorphisms and canonical structure**
- **Evaluation of CQ** (NP-completeness)
- **Containment, Equivalence, Minimisation of CQ** (NP-completeness)
- **Extension to functional dependencies** (chased canonical structure)

Acyclic CQ's

On graphs: CQ ϕ is **acyclic** if G_ϕ is tree-like

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underlying undirected graph is acyclic

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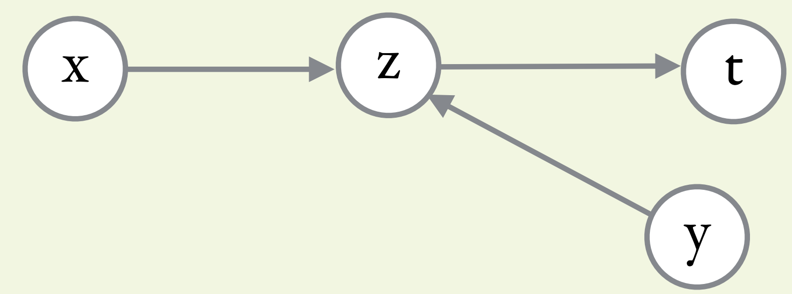
$\phi(x,y) = \exists z \,.\, E(x,z) \wedge E(z,t) \wedge E(y,z)$



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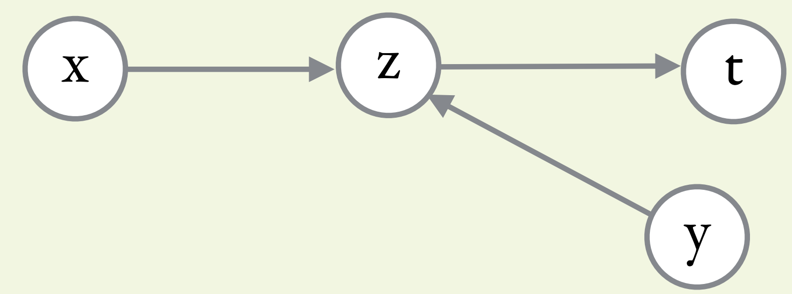

On arbitrary structures: a CQ ϕ is acyclic if it has a join tree

$$\phi(\bar{y}) = \exists \bar{z} \,.\, R_1(\bar{z}_1) \wedge \ldots \wedge R_m(\bar{z}_m)$$

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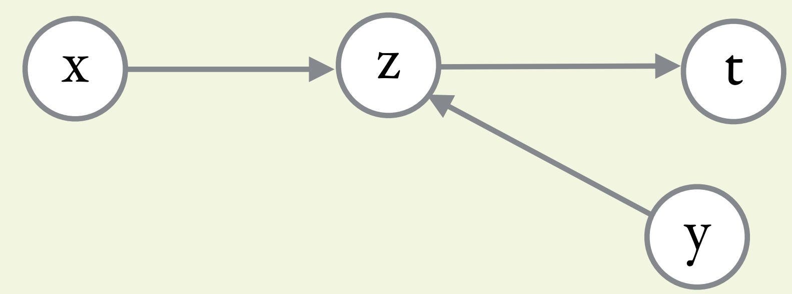

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A join tree is a tree T s.t.:

- nodes are the atoms $R_i(\bar{z}_i)$
- for every variable x of ϕ the set of $R_i(\bar{z}_i)$'s with $x \in \bar{z}_i$ forms a subtree of T

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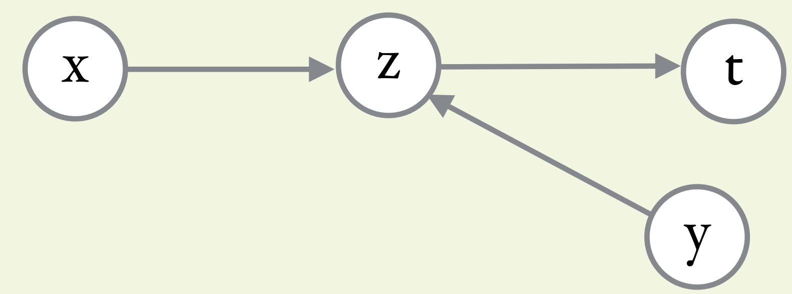

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If x occurs in two nodes, then it occurs in the path linking the two nodes.

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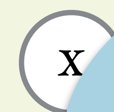

Alternatively, if its canonical hyper-graph is **α-acyclic.**

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Acyclic CQ's

A **reduced** hypergraph F =(V,E) is *α-acyclic* if for each U ⊆V, if $F|_U$ is connected and has more than one edge, then it has an **articulation set.**

On graphs: C

$\phi(x,y) = \exists z \,.\, E(x,z) \wedge E(z,t) \wedge E(y,z)$

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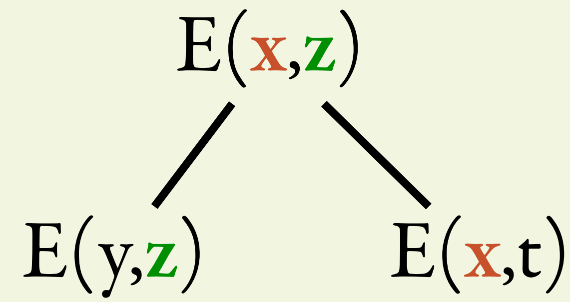

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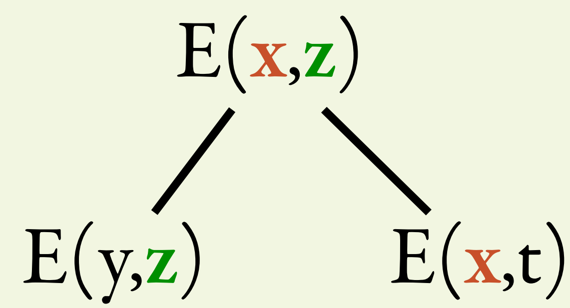

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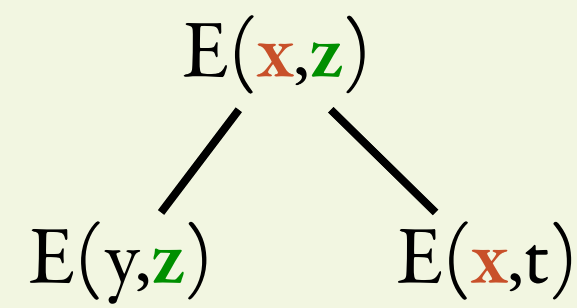

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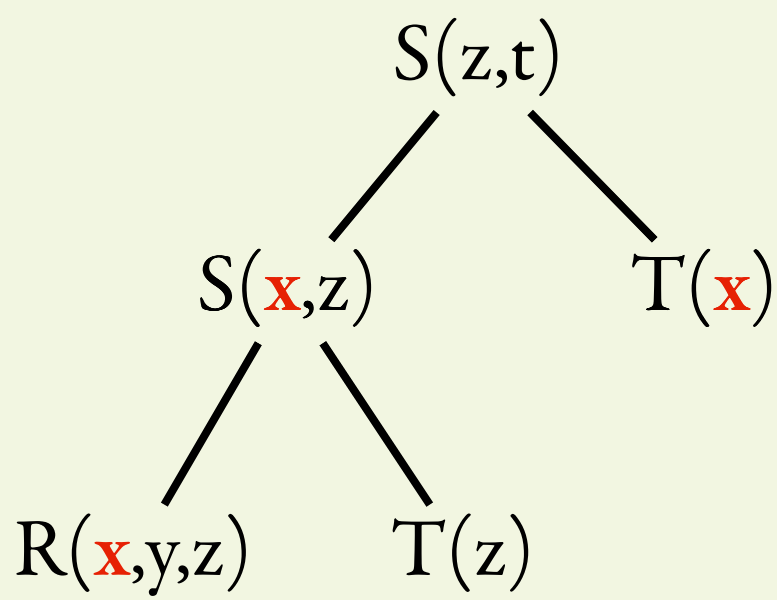

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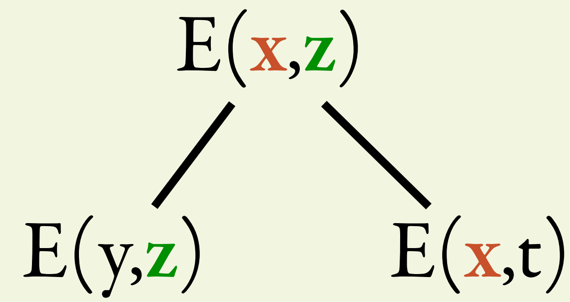

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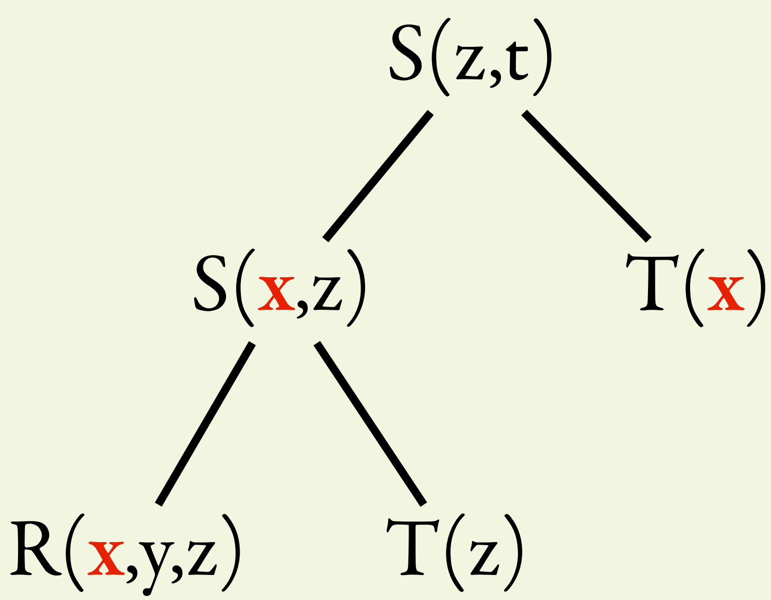

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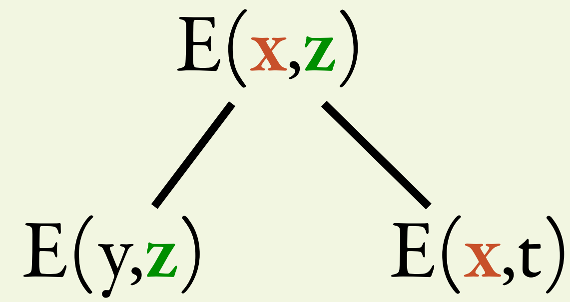

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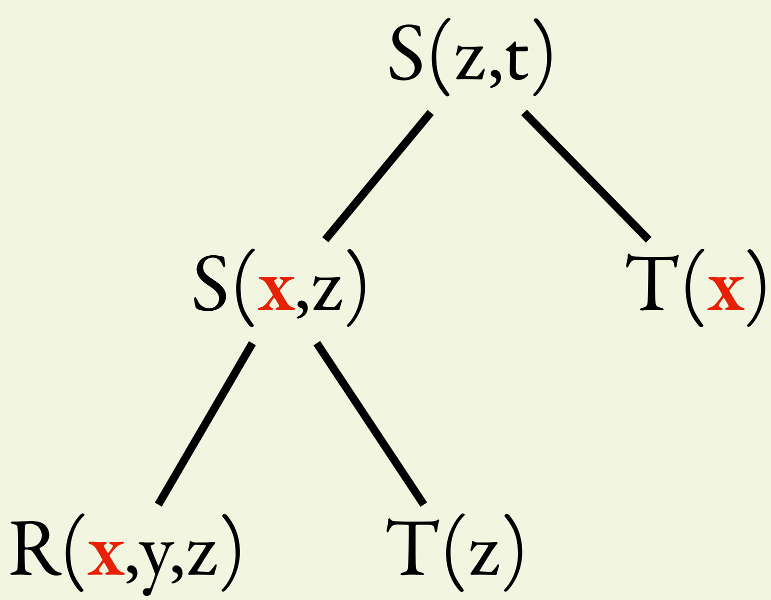

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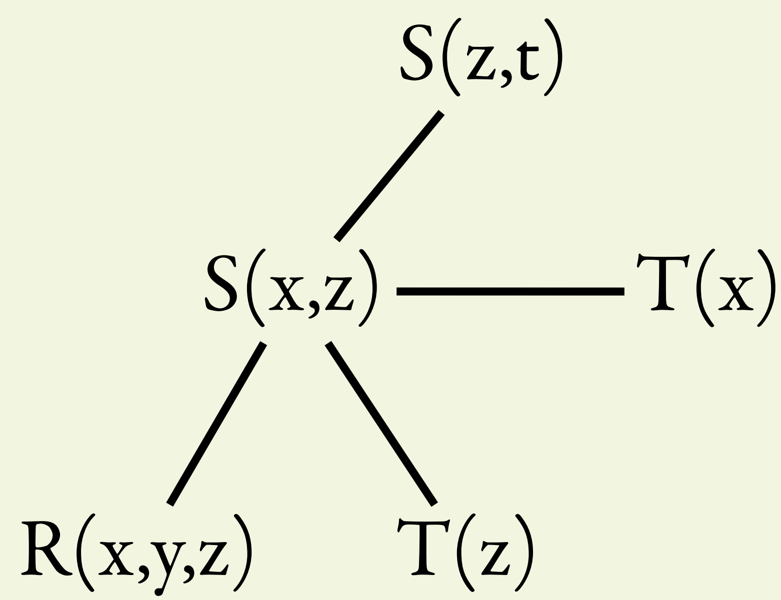

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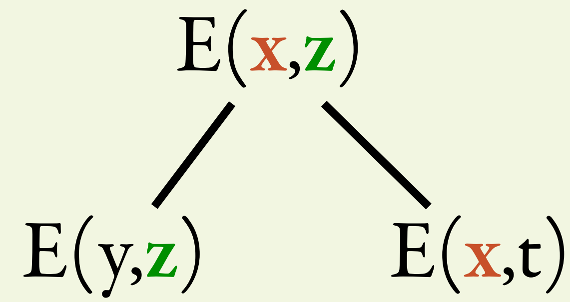

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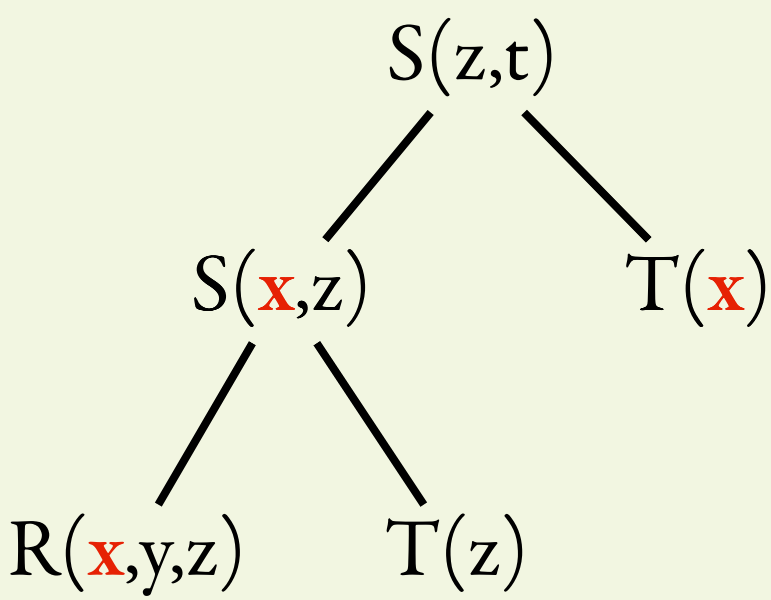

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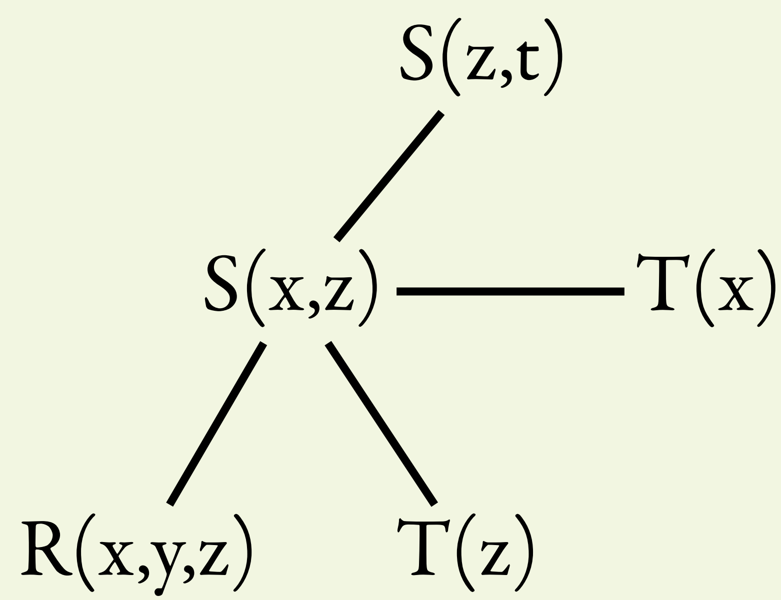

a join tree

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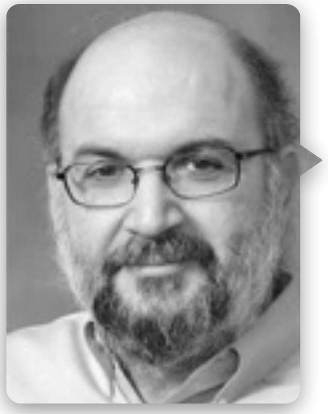

The **evaluation problem** for acyclic CQ sentences is in $O(|\phi|.|D|)$

[Yannakakis]

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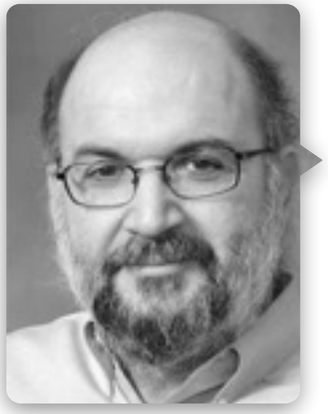

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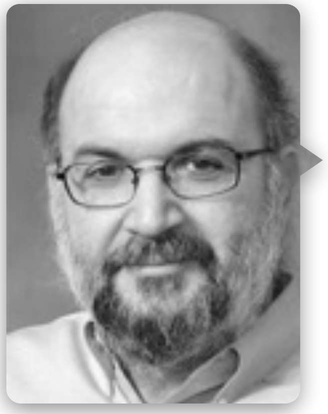
[Yannakakis]

The **semi-join**

$$R \ltimes_{\{i_1=j_1,\ldots,i_n=j_n\}} S = \{\ (x_1,\ldots,x_n) \in R \mid \text{there is } (y_1,\ldots,y_m) \in S \text{ where } x_{i_k} = y_{j_k} \text{ for all } k\}$$

Note: $R \ltimes_{\{i_1=j_1,\ldots,i_n=j_n\}} S \subseteq R$

Acyclic CQ's

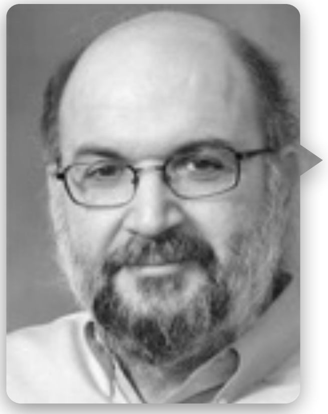


The **evaluation problem** for acyclic CQ sentences is in O(|ϕ|.|D|)

[Yannakakis]

1. Compute the join tree T for ϕ
2. Populate the nodes of T with tuples from corresponding relations of D
3. For every leaf $S(x_1,\ldots,x_n)$ with parent $R(y_1,\ldots,y_m)$ replace tuples of parent with
   $$R \ltimes_{\{i=j \mid x_i = y_j\}} S$$
   and delete the leaf $S(x_1,\ldots,x_n)$.
4. Repeat until we are left with one node. If it contains a non-empty relation, then D satisfies ϕ, otherwise it does not.

Acyclic CQ's



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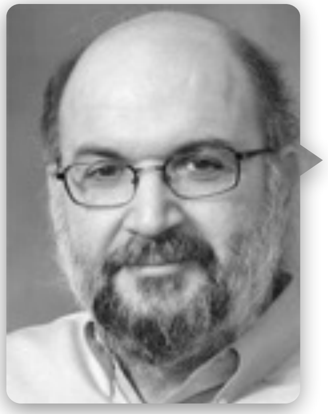

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remove all the tuples from the parent that do not match a tuple from the child

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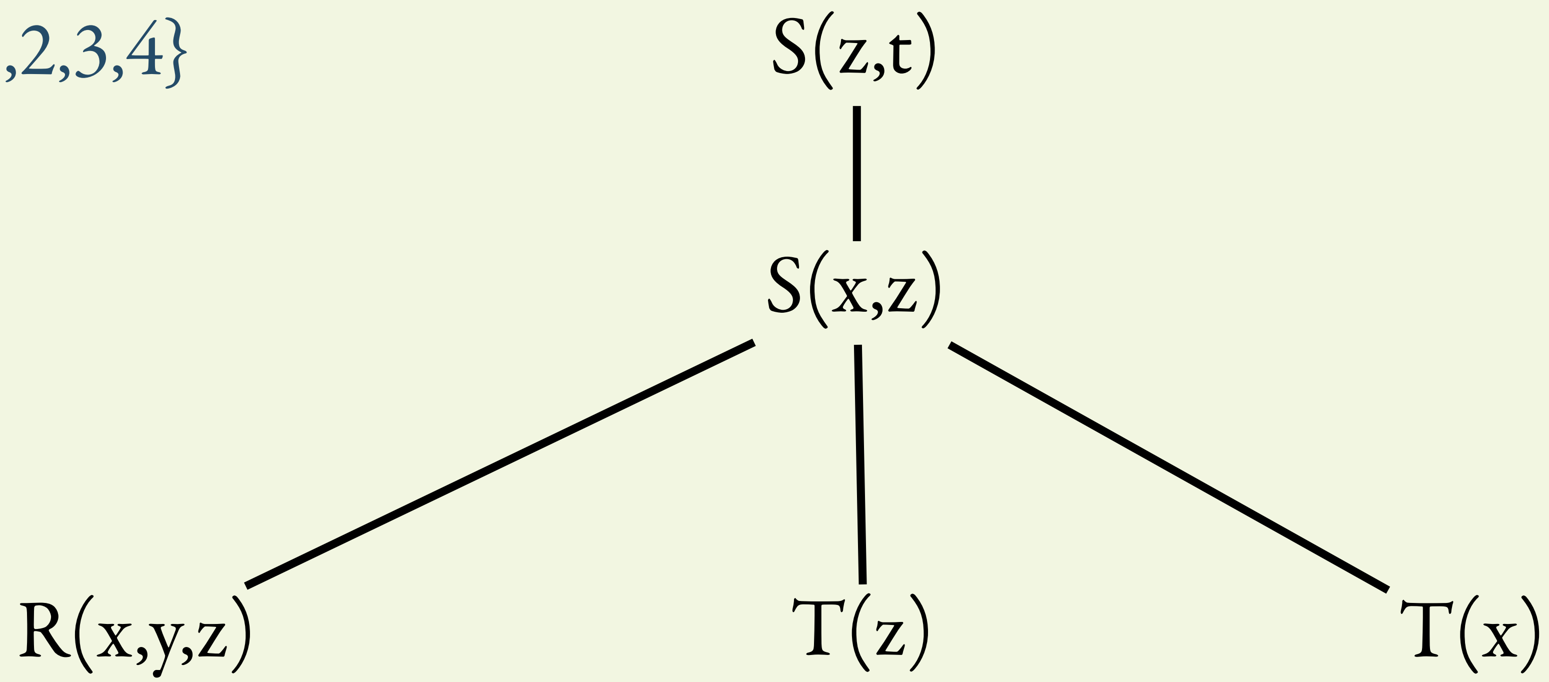

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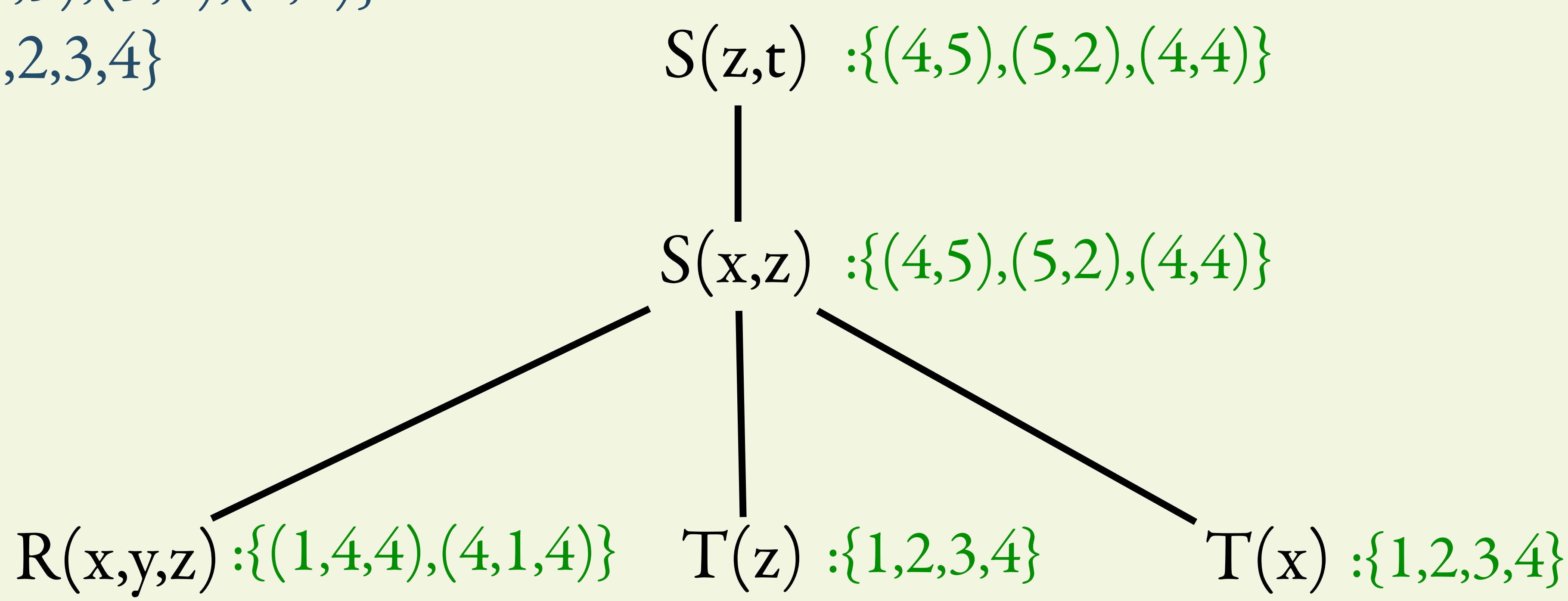

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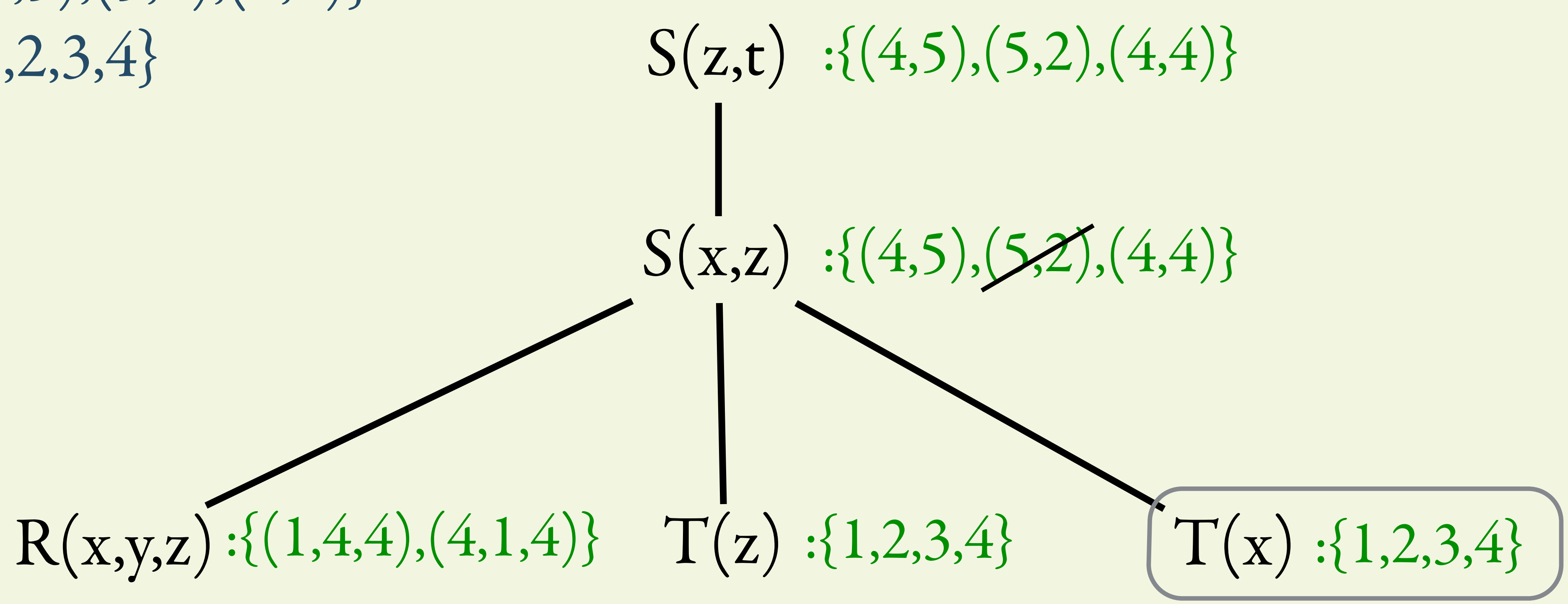

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R(x,y,z) :{(1,4,4),(4,1,4)}

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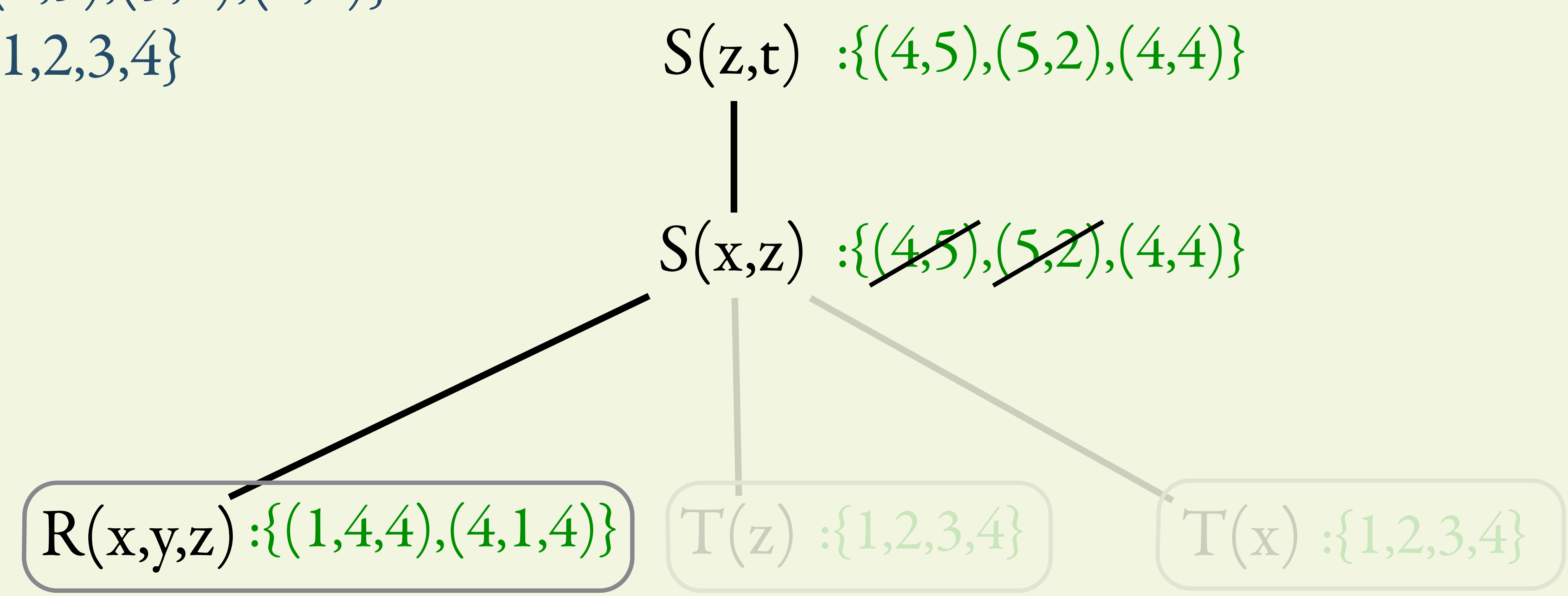

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$S(z,t)\ :\{(4,5),\cancel{(5,2)},(4,4)\} \neq \varnothing$

$S(x,z)\ :\{\cancel{(4,5)},\cancel{(5,2)},(4,4)\}$

$R(x,y,z)\ :\{(1,4,4),(4,1,4)\}$ $T(z)\ :\{1,2,3,4\}$ $T(x)\ :\{1,2,3,4\}$

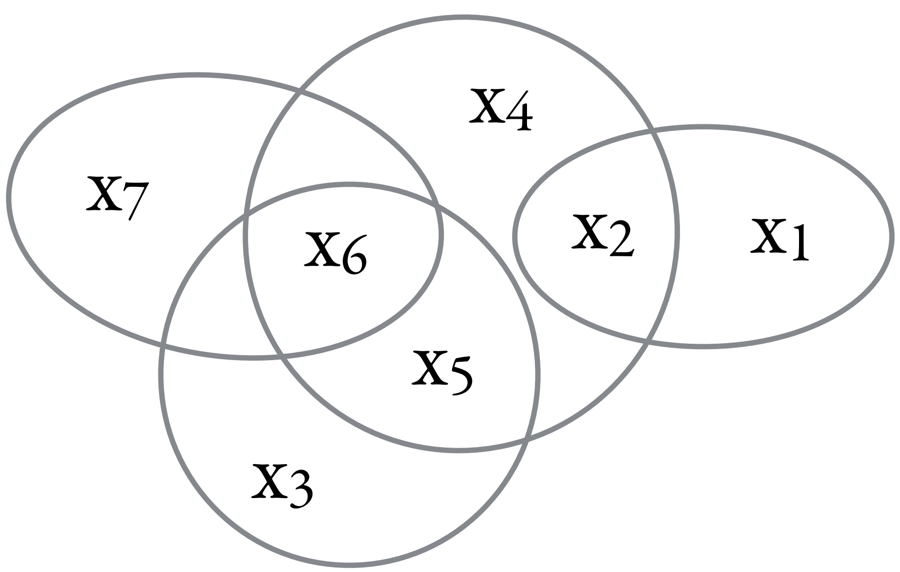
How to compute a join tree?

GYO reducts [Graham, Yu, Ozsoyoglu]

An ear of a hypergraph (V,E) is a hyperedge e in E such that one of the following conditions holds:

(1) There is a witness e' in E, such that e' ≠ e and each vertex from e is either
  (a) **only** in e or
  (b) in e'; or

(2) **e** has no intersection with any other hyperedge.



ears?

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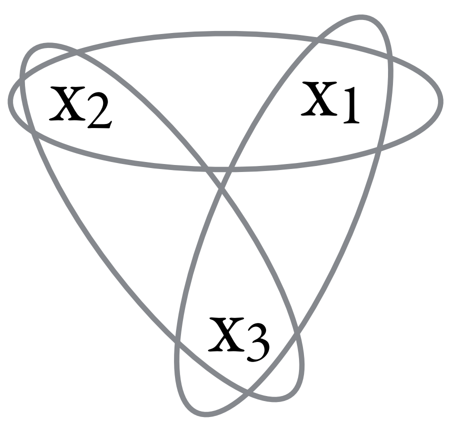

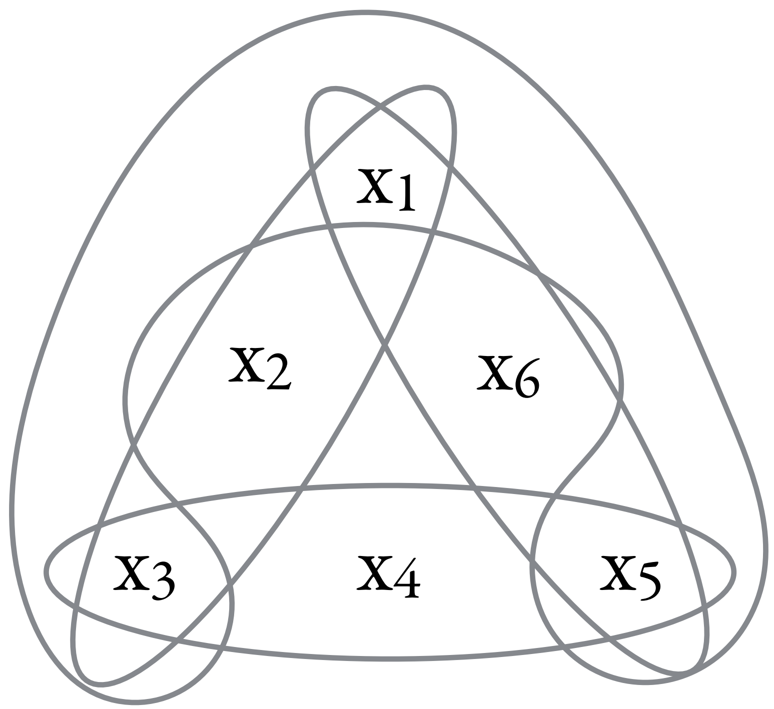

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Theorem: TFAE

- The GYO **reduct** of a hyper graph **G** is empty
- A CQ ϕ having **G** as underlying canonical hyper-graph is acyclic
- The hyper graph **G** is α-acyclic

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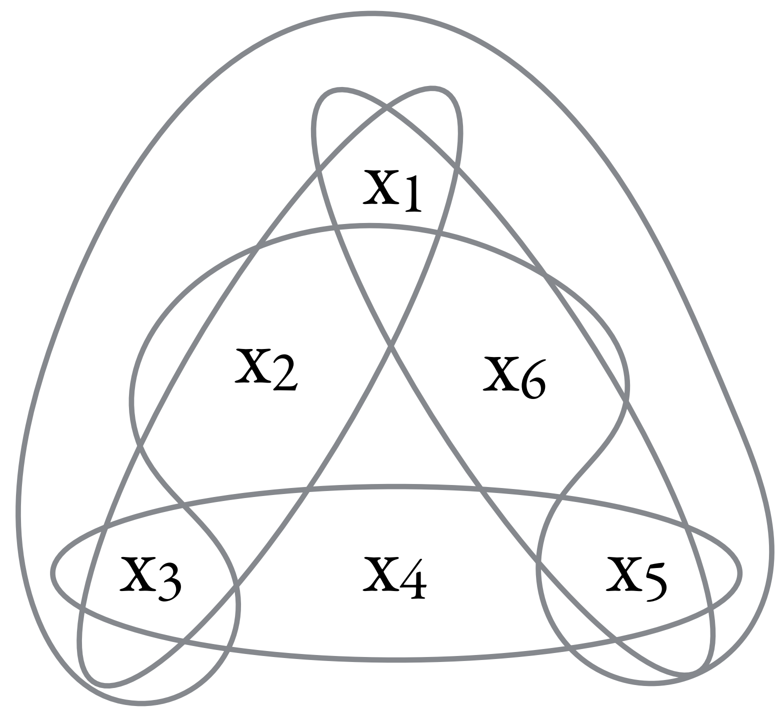
We can test acyclicity by computing the GYO reduct!

Ears?



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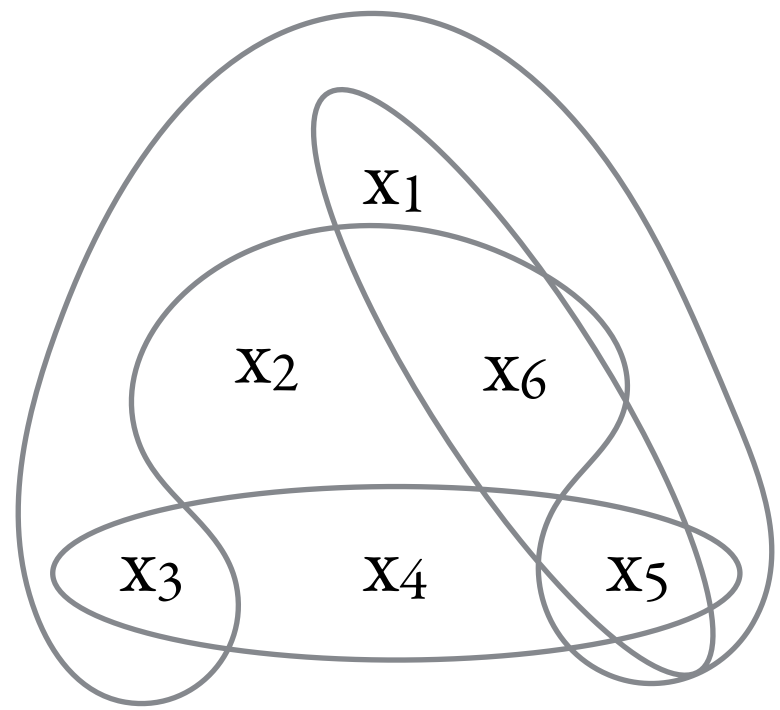

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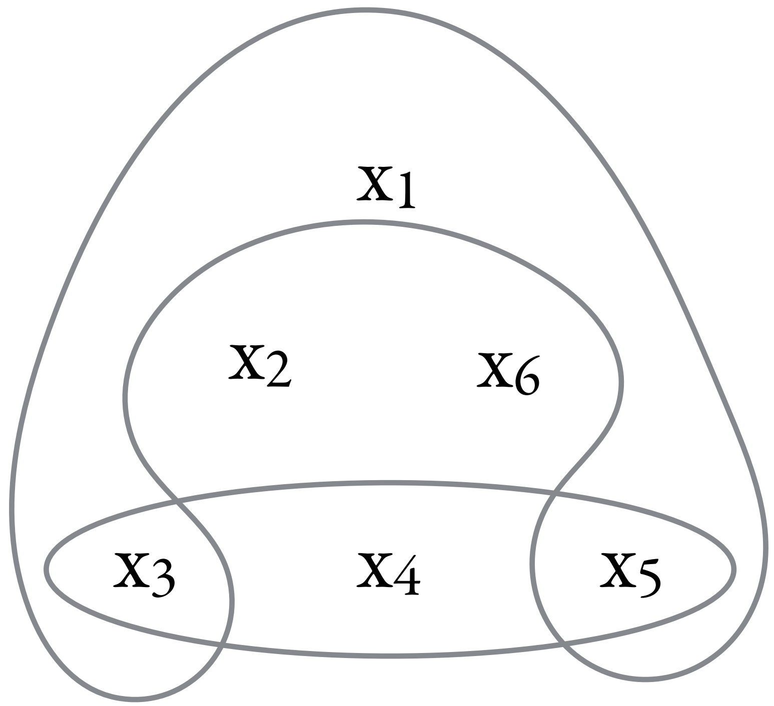

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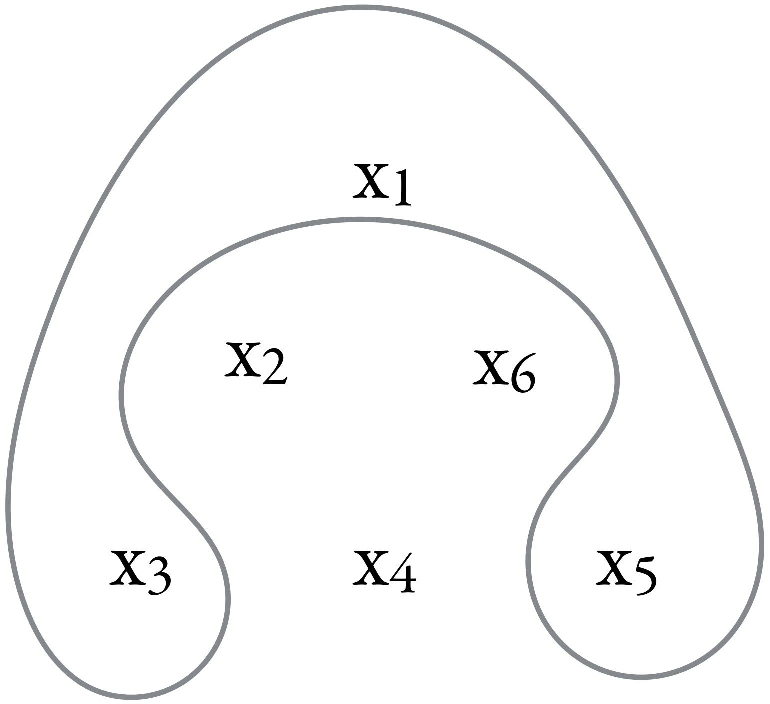
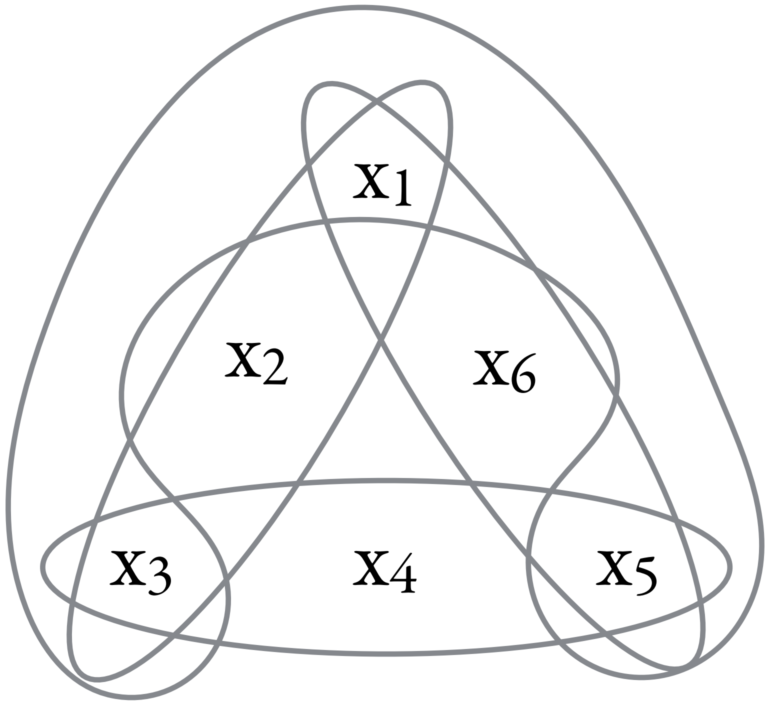


Ears?



Acyclic!

Acyclic CQ's

How to compute a join tree?

GYO algorithm [Graham, Yu, Ozsoyoglu]

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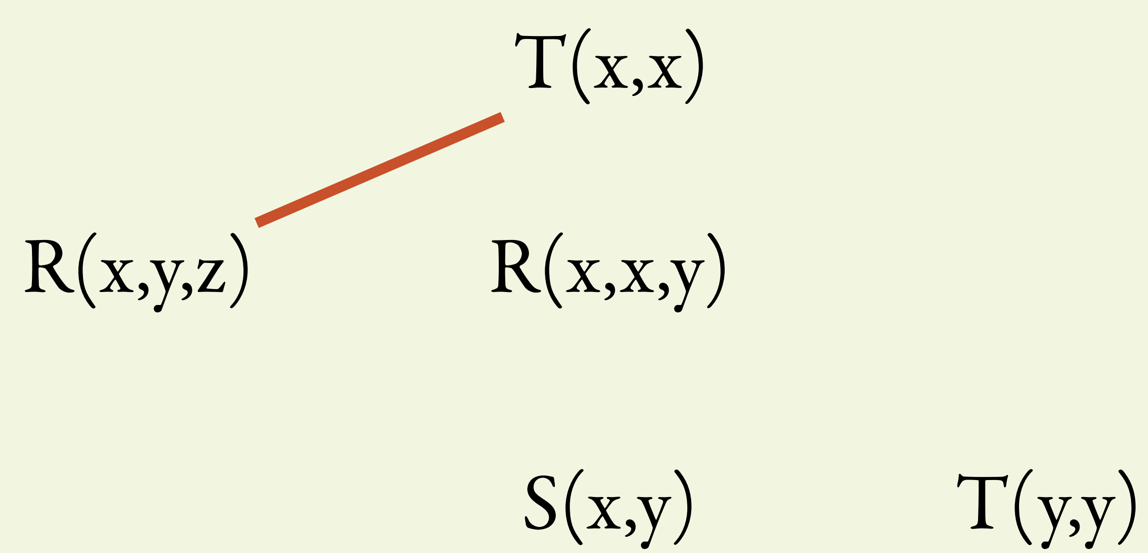

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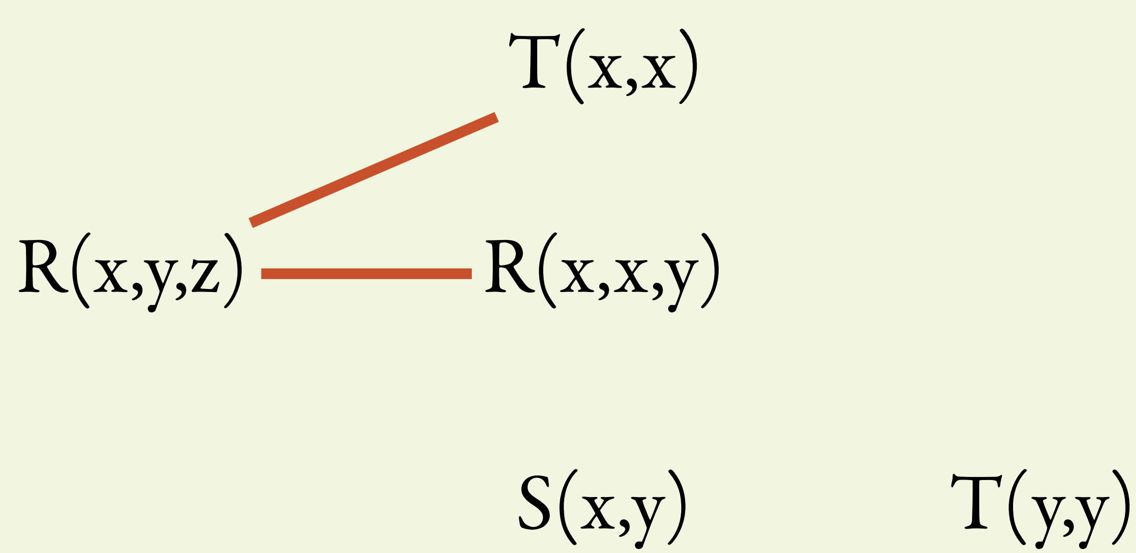

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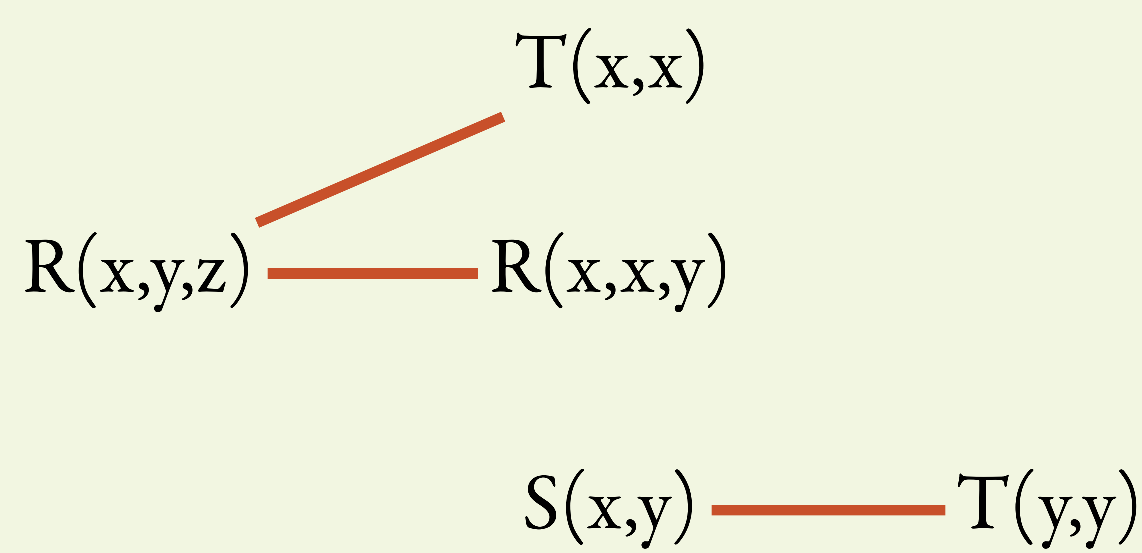

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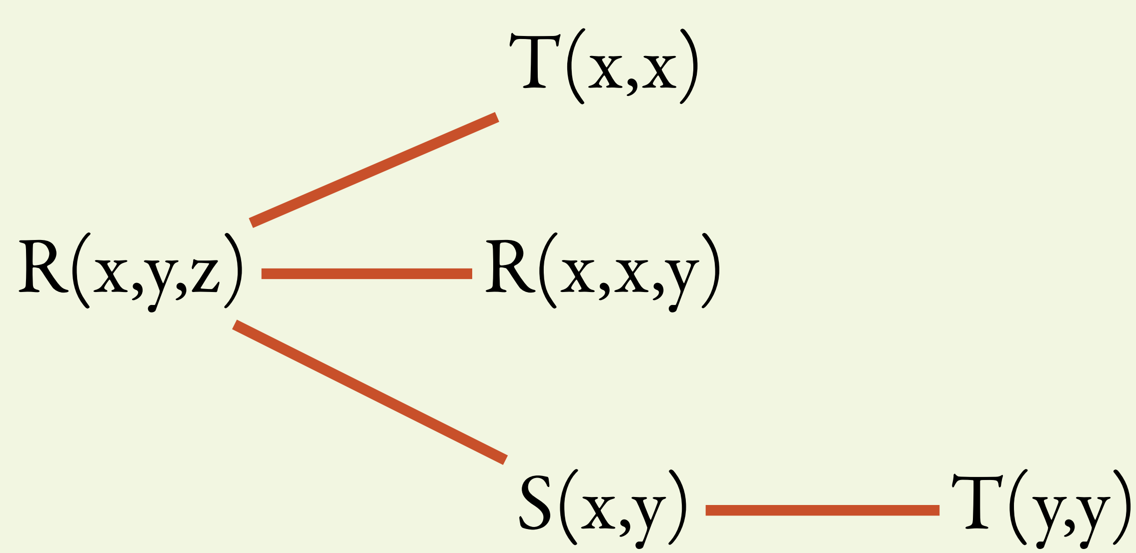

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Remove ears until you're left with only one!

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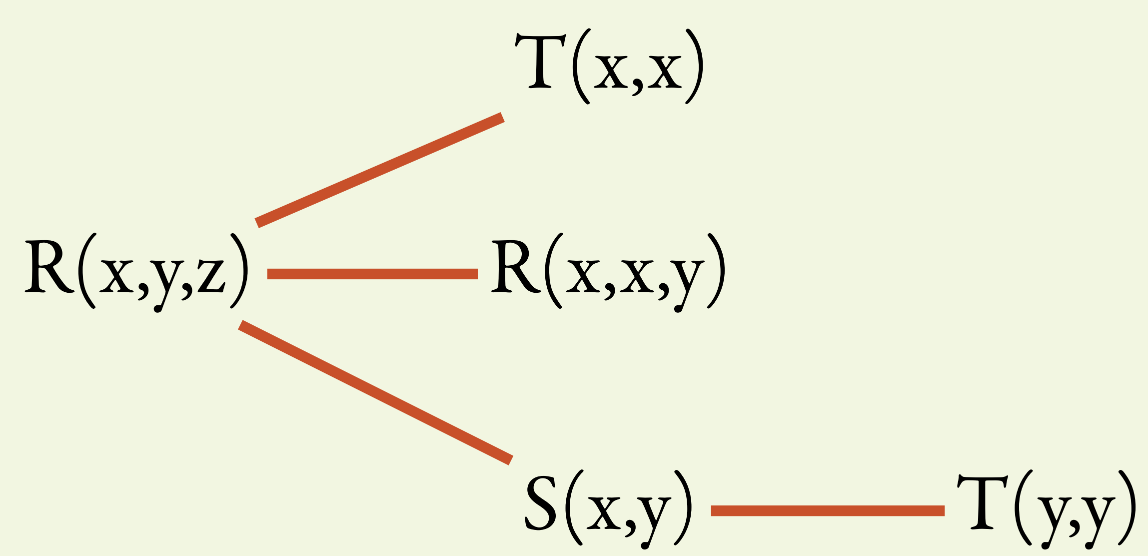

Acyclic CQ's

[Gottlob, Leone, Scarcello]

- Evaluation problem for boolean ACQ's is LOGCFL-complete
- NL $\subseteq$ **LOGCFL** $\subseteq$ AC^1 $\subseteq$ NC^2 $\subseteq$ P

the class of problems logspace-reducible to a context-free language

Beyond acyclic CQ's

- **(Hyper-)Treewidth** = a measure of the cyclicity of (hyper-)graphs

  tw(Q) = tw(canonical hypergraph of Q)

  tw = 1 are forests,

  tw = 2 are graphs without K_4 as a minor, ...

- For fixed k,

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Bounded tree width queries = a class of CQ's so that for some k, every query has tw ≤ k

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CQ's with bounded treewidth can be evaluated in PTIME

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Containment of CQ's with bounded treewidth is in PTIME

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CQ's with bounded treewidth can be evaluated i

$\phi \subseteq \psi$ iff $\psi(G_\phi) \neq \varnothing$

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A class *C* of CQ's can be evaluated in PTIME iff they have bounded tree width!

For graphs, assuming W[1]≠ FPT, and C a r.e. class of graphs. [Grohe, Schwentick, Segoufin]

Querying with semi-joins

The semi-join

$$R \ltimes_{\{i_1=j_1,\dots,i_n=j_n\}} S = \{ (x_1,\dots,x_n) \in R \mid \text{there is } (y_1,\dots,y_m) \in S \text{ where } x_{i_k} = y_{j_k} \text{ for all } k\}$$

The **semi-join algebra (SA)**: variant of RA with operations:

$\ltimes$, $\cup$, π, σ, $\setminus$, *dupcol*

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Output at most linear in the database. Further,

The **evaluation problem** for SA is in $O(|\phi|.|D|)$

Logical characterisation: "stored-tuples guarded fragment of FO"

Acyclic CQs:

- every intermediate relation is **linear** in $|D|$
- we apply $|\phi|$ semi-joins

What if we allow intermediate relations to be **polynomial** in $|D|$?

Bounded variable FO

Def.

FO^k = The fragment of FO restricted to k variable names

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$$= \forall y. \Big(E(x,y) \implies \exists z\, \exists w \,(E(y,z) \wedge E(z,w)) \Big) \in \mathrm{FO}^4$$

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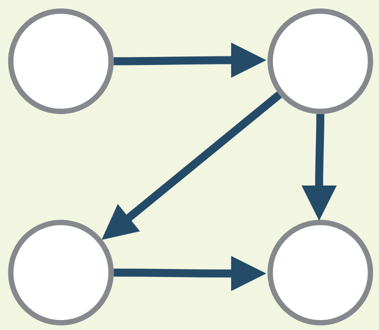

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Question: in FO^2 ?

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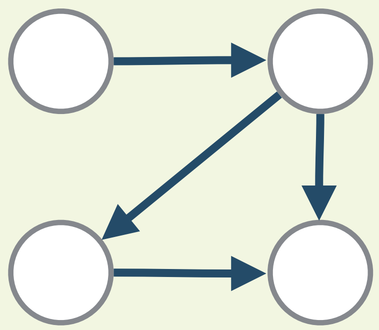

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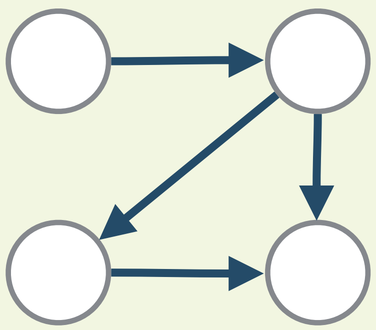

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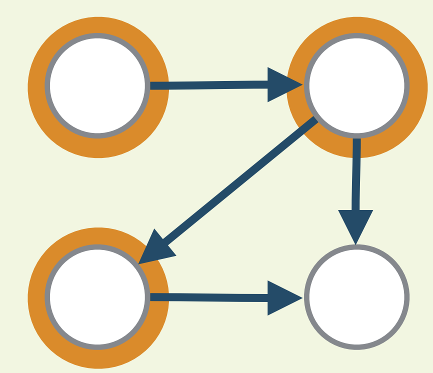

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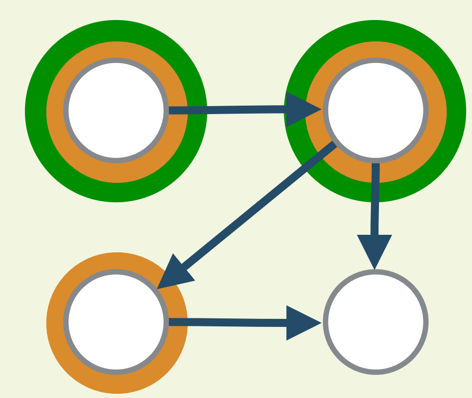

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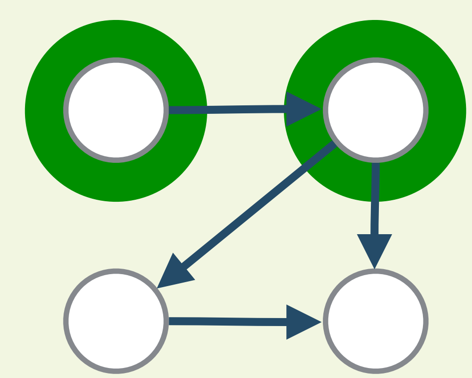

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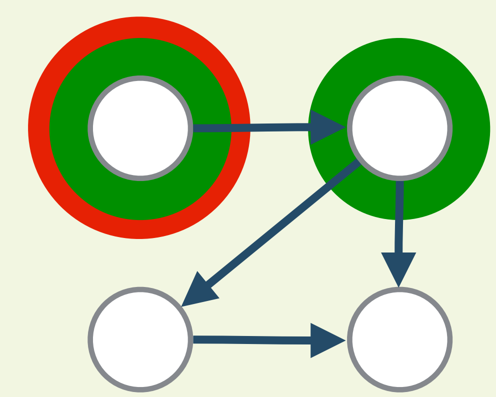

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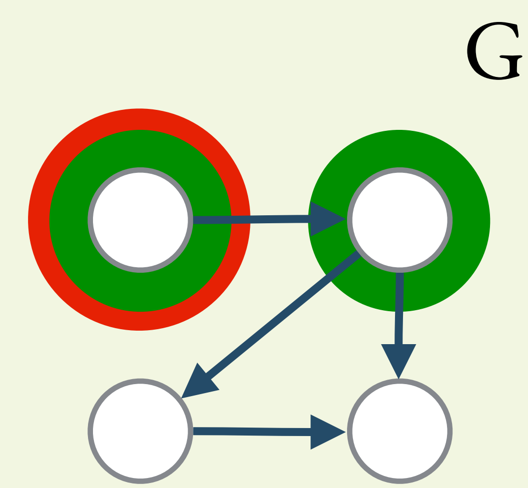

The evaluation problem for FO^k is in PTIME (combined c.)

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⋮

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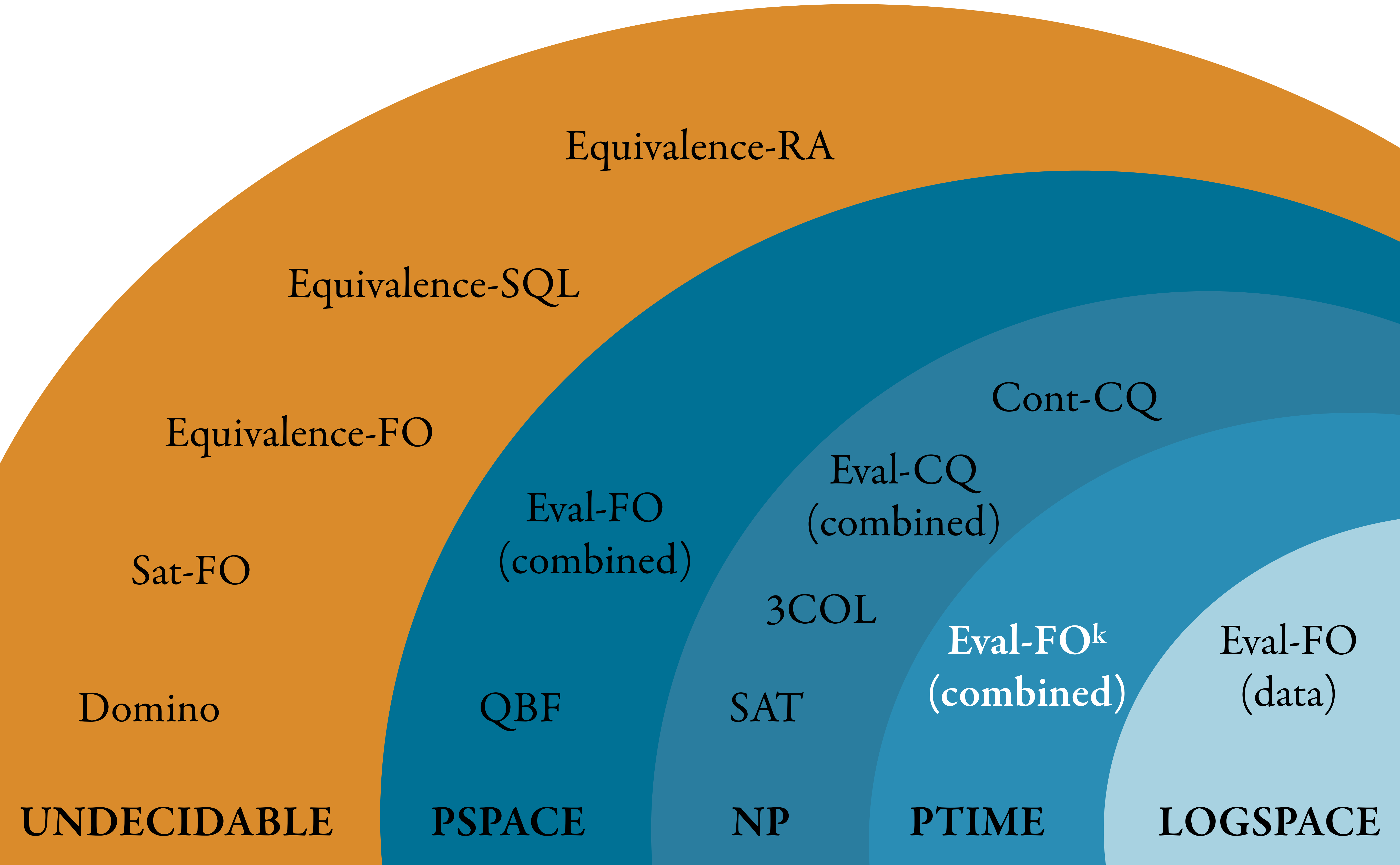
- Satisfiability for FO^k ⇝ Undecidable if $k \geq 3$ (Domino)

  ⇝ NEXPTIME-complete if $k=2$

Recap



Equivalence-RA
Equivalence-SQL
Equivalence-FO
Sat-FO
Domino
Eval-FO (combined)
QBF
Cont-CQ
Eval-CQ (combined)
3COL
SAT
Eval-FO^k (combined)
Eval-FO (data)
UNDECIDABLE
PSPACE
NP
PTIME
LOGSPACE

Descriptive complexity

What properties can be checked efficiently? E.g. 3COL can be tested in NP

Metatheorem

"A property can be expressed in [insert some logic here]
iff
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Some more cool stuff…

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⇝ "A property is FO-definable iff it can be tested in AC^0"

⇝ "A property is ∃SO-definable iff it can be tested in NP" [Fagin 73]

⇝ Open problem: which logic captures PTIME?

Some more cool stuff…

Recursion

Datalog (semantics based on least fixpoint)

```
Ancestor(X,Y) :- Parent(X,Z), Ancestor(Z,Y)
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Can we enhance query languages with recursion ? E.g. express reachability properties

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⇝ Incomparable with FO (has recursion, but is monotone)

⇝ Evaluation is in PTIME (for data complexity, but also for bounded arity)

Semi-structured data

Tree-structured or graph-structures dbs in place of relational dbs.

XML, XPath, Stream processing, …

```
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    <author>Matthew Gambardella</author>
    <year>2000</year>
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    <author>David Hunter</author>
    <author>David Gibbons</author>
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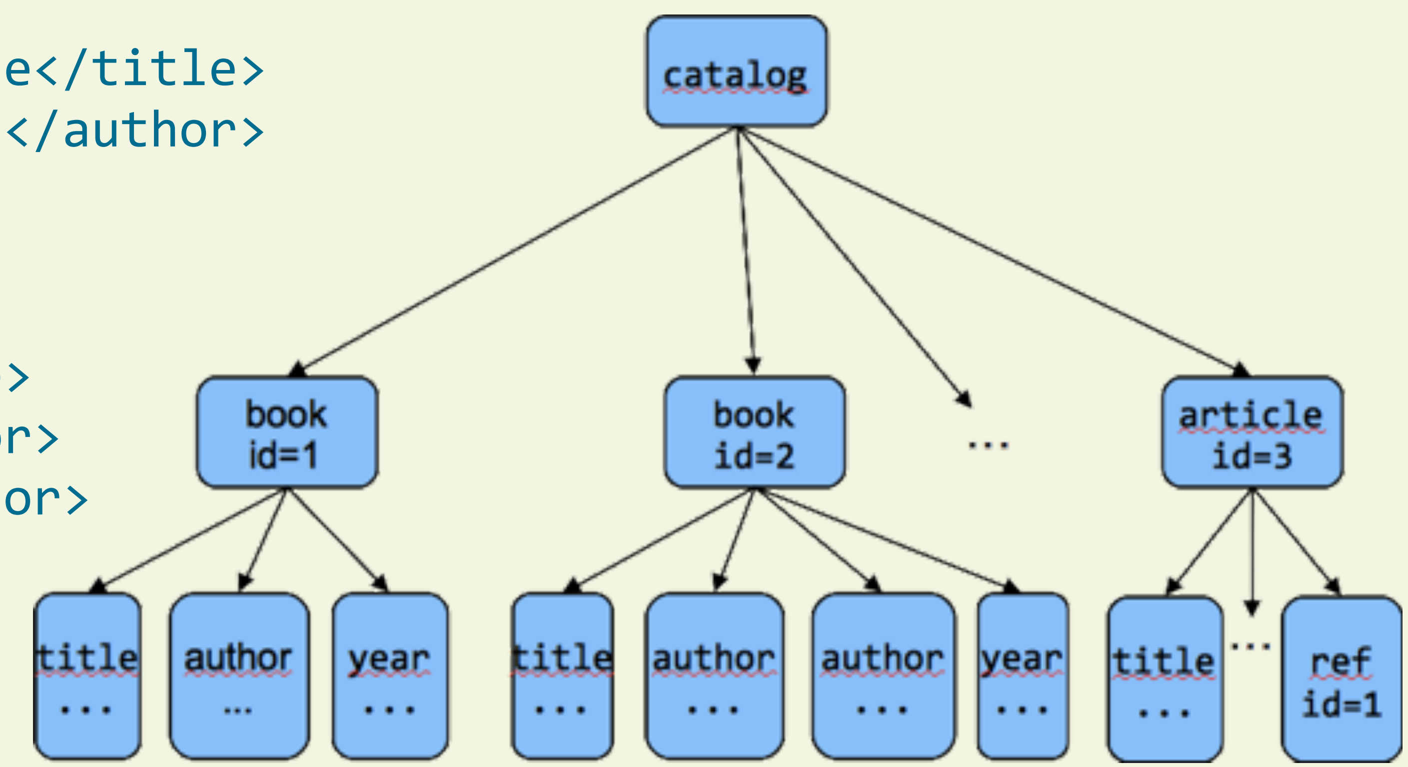

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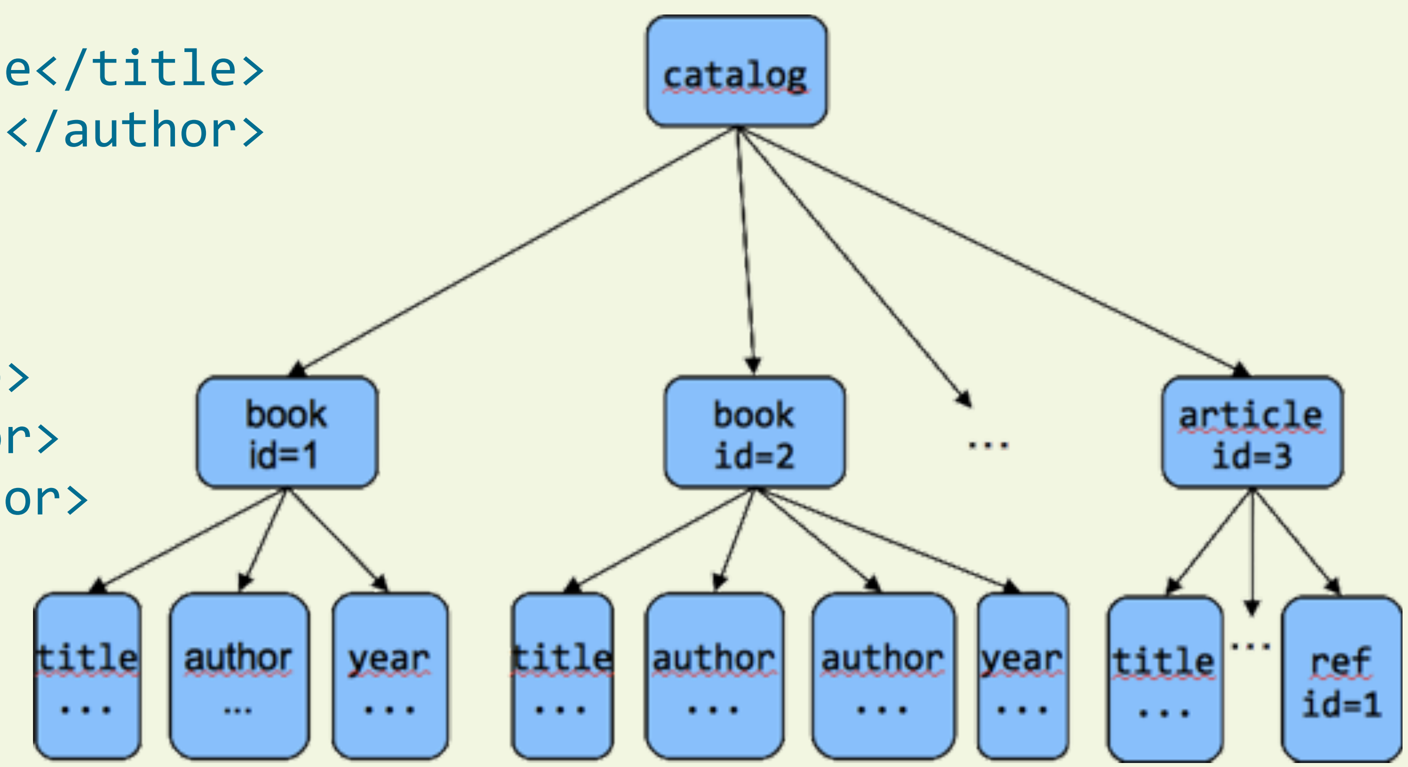


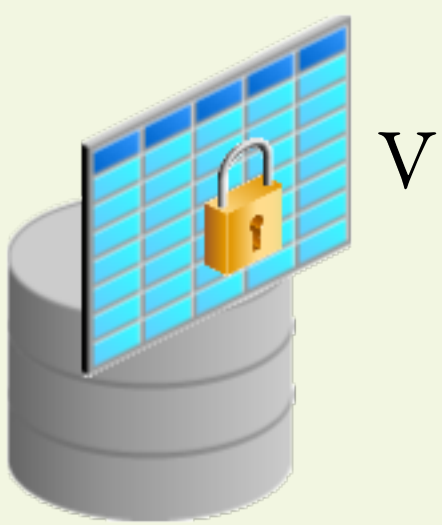
⇝ Evaluation of XPath is in linear time (data complexity) [Bojanczyk, Parys 08]

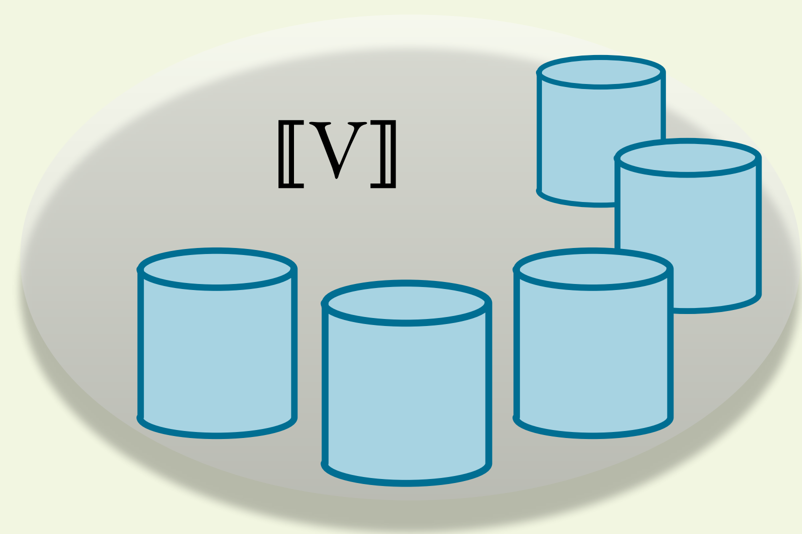
⇝ Satisfiability for $FO^2[\downarrow,\sim]$ is decidable [Bojanczyk, Muscholl, Schwentick, Segoufin 09]

Some more cool stuff…

Incomplete information

Certain Query Answers (CQA)





$$\phi[\![V]\!] = \bigcap_{D \in [\![V]\!]} \phi(D)$$

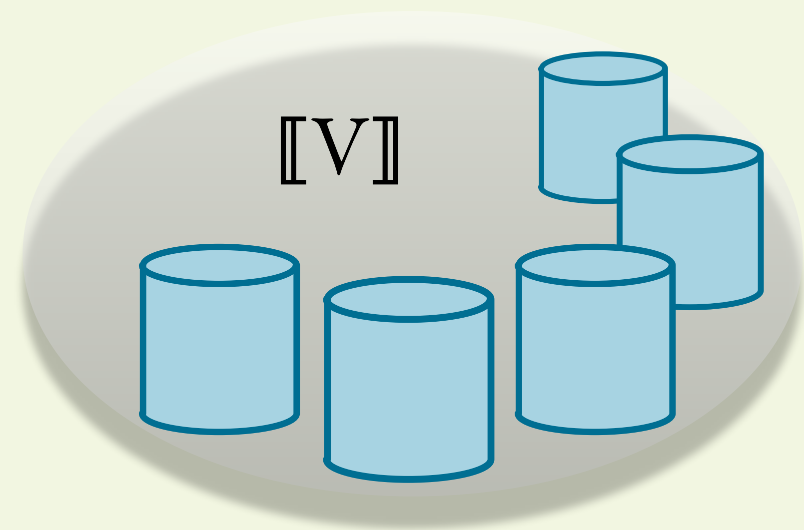
Some more cool stuff…

Incomplete information

How to correctly treat NULL values, missing tuples, noisy data ?

Certain Query Answers (CQA)

V



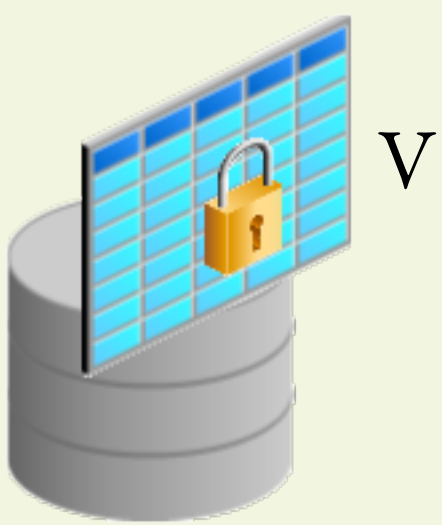
$$\phi[\![V]\!] = \bigcap_{D \in [\![V]\!]} \phi(D)$$

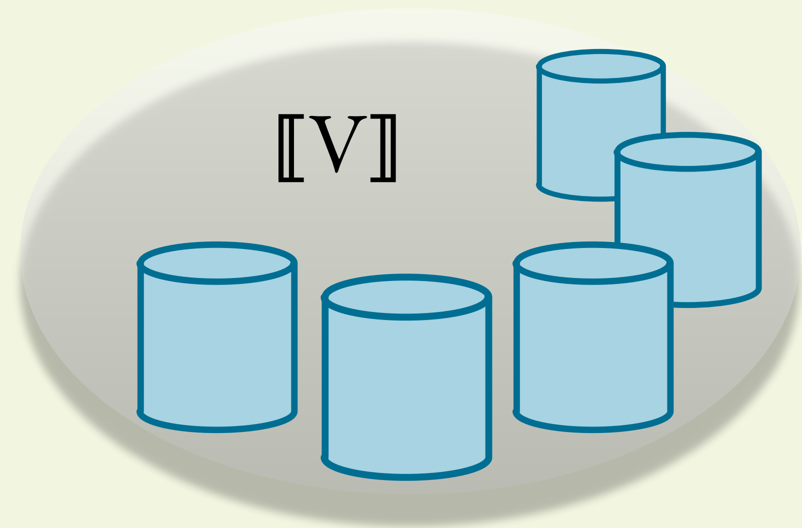
Some more cool stuff...

Incomplete information

How to correctly treat NULL values, missing tuples, noisy data ?

Certain Query Answers (CQA)





$$\phi[\![V]\!] = \bigcap_{D \in [\![V]\!]} \phi(D)$$

⇝ CQA computable in PTIME w.r.t. view size. [Abiteboul, Kanellakis, Grahne 91]

Recap

- Relational Algebra = simple SQL = FO on active domain
- Evaluation, Satisfiability, Equivalence, Containment
- Data / Combined complexity
- Expressiveness of FO: EF games, 0-1 law, Rado structure, Locality
- Conjunctive Queries: Homomorphism lemma, Canonical structures
- Acyclic CQ
- FO^k

Bibliography

- Abiteboul, Hull, Vianu, “Foundations of Databases”, Addison-Wesley, 1995.

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